

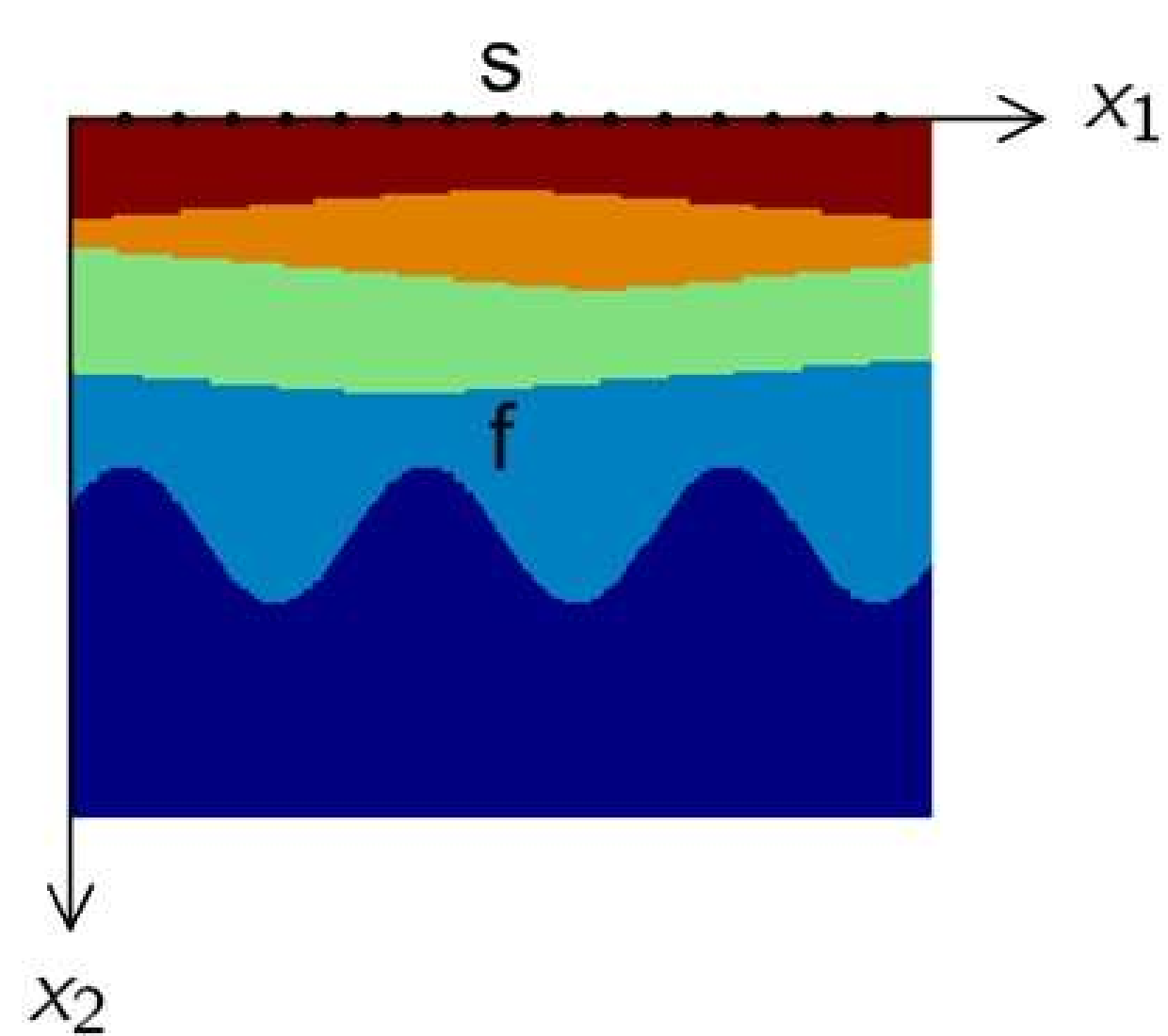
Adjoint method in time domain ultrasound tomography

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Ultrasound Tomography



Imaging with common source gathers



$$\left. \begin{aligned} \frac{\partial^2 u_s}{\partial t^2} &= c^2 \Delta u_s && \text{in } x_2 > 0, \quad c^2 = \frac{c_0^2}{1+f} \\ \frac{\partial u_s}{\partial x_2} &= p(x-s)q(t) && \text{on } x_2 = 0 \\ u &= 0 && \text{for } t < 0 \end{aligned} \right\} R_s(f) = u|_{x_2=0}$$

Adjoint method for common source gathers

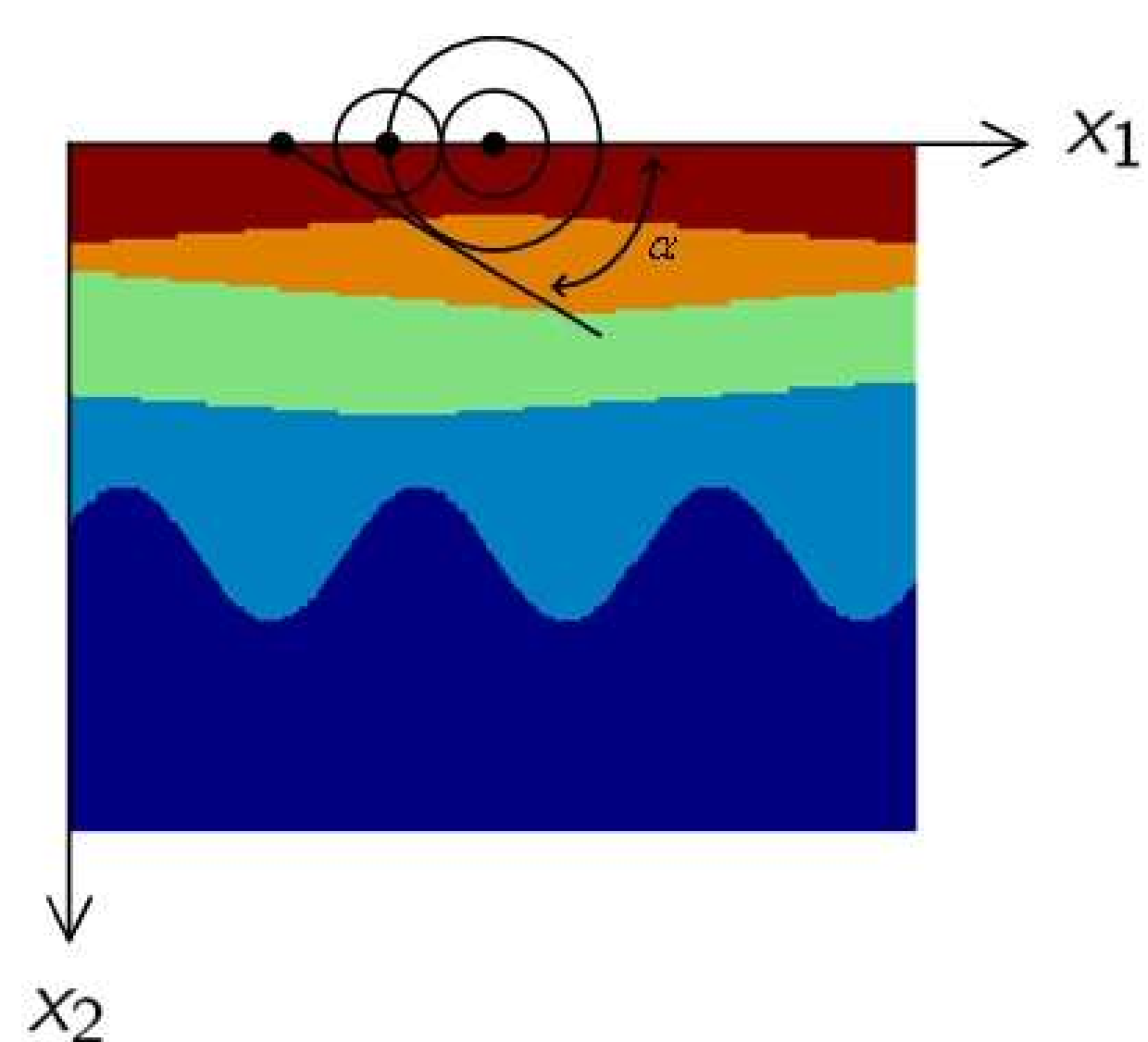
Time domain adjoint method for common source gathers:

$$R_s(f) = u_s|_{x_2=0} = g_s$$

$$f \leftarrow f - \omega (R'_s(f)) * (R_s(f) - g_s) \quad \text{for all sources } s$$

Possible, but too time consuming for a large number of sources:
Many forward/backward calculations per sweep.

Plane wave stacking



$$\text{Common source gather: } \frac{\partial u_s}{\partial x_2} = p(x-s)q(t)$$

$$\text{Plane wave stacking: } \frac{\partial u_\alpha}{\partial x_2} = \sum_s p(x-s)q(t - \sin(\alpha)s/c_0)$$

$$R_\alpha(f)(x_1, t) = u_\alpha(x_1, 0, t) = \sum_s R_s(f)(x_1, 0, t - \sin(\alpha)s/c_0)$$

Adjoint method after plane wave stacking

Preprocessing by plane wave stacking:

$$R_\alpha(f) = u_\alpha|_{x_2=0} = g_\alpha$$

$$f \leftarrow f - \omega (R'_\alpha(f)) * (R_\alpha(f) - g_\alpha) \quad \text{for } \alpha = \pm 45^\circ$$

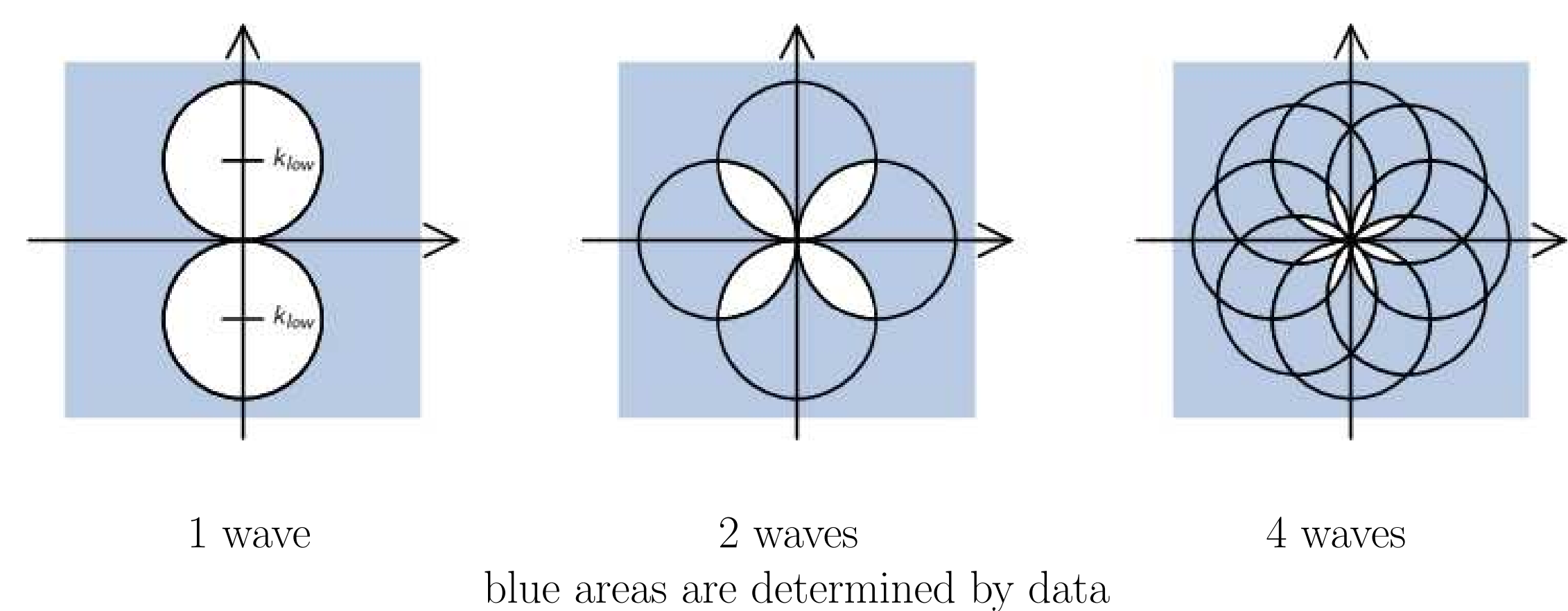
Only 2 forward/backward calculations per sweep.

Fourier Analysis of resolution for one downgoing plane wave

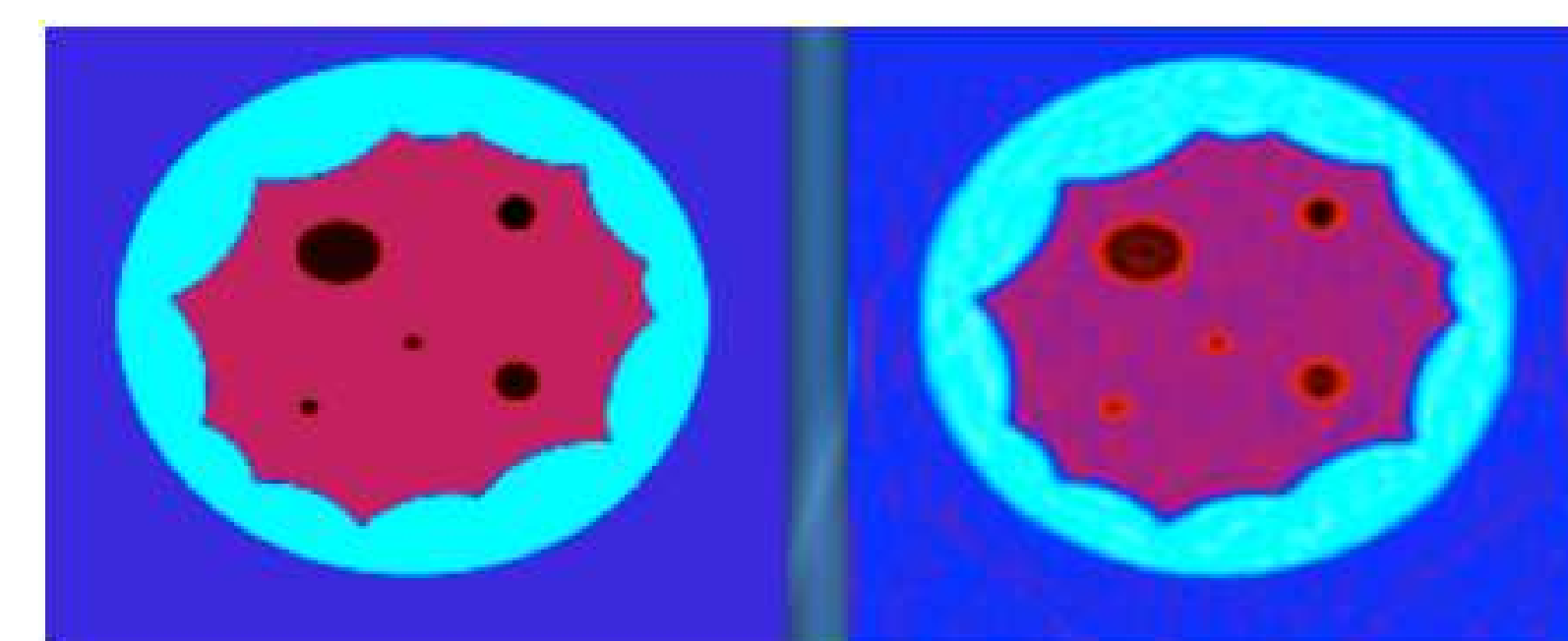
$$(R_\alpha f)^\wedge(\xi_1, \omega) = ik^2 \hat{q}(\omega) \hat{f}(\xi_1, -k \pm \sqrt{k^2 - \xi_1^2})$$

$$\alpha = 0^\circ \quad \begin{array}{l} - \text{reflection} \quad k = \frac{\omega}{c_0} \\ + \text{transmission} \end{array}$$

Fourier Coverage for several incoming waves



Example 1: SLC breast phantom



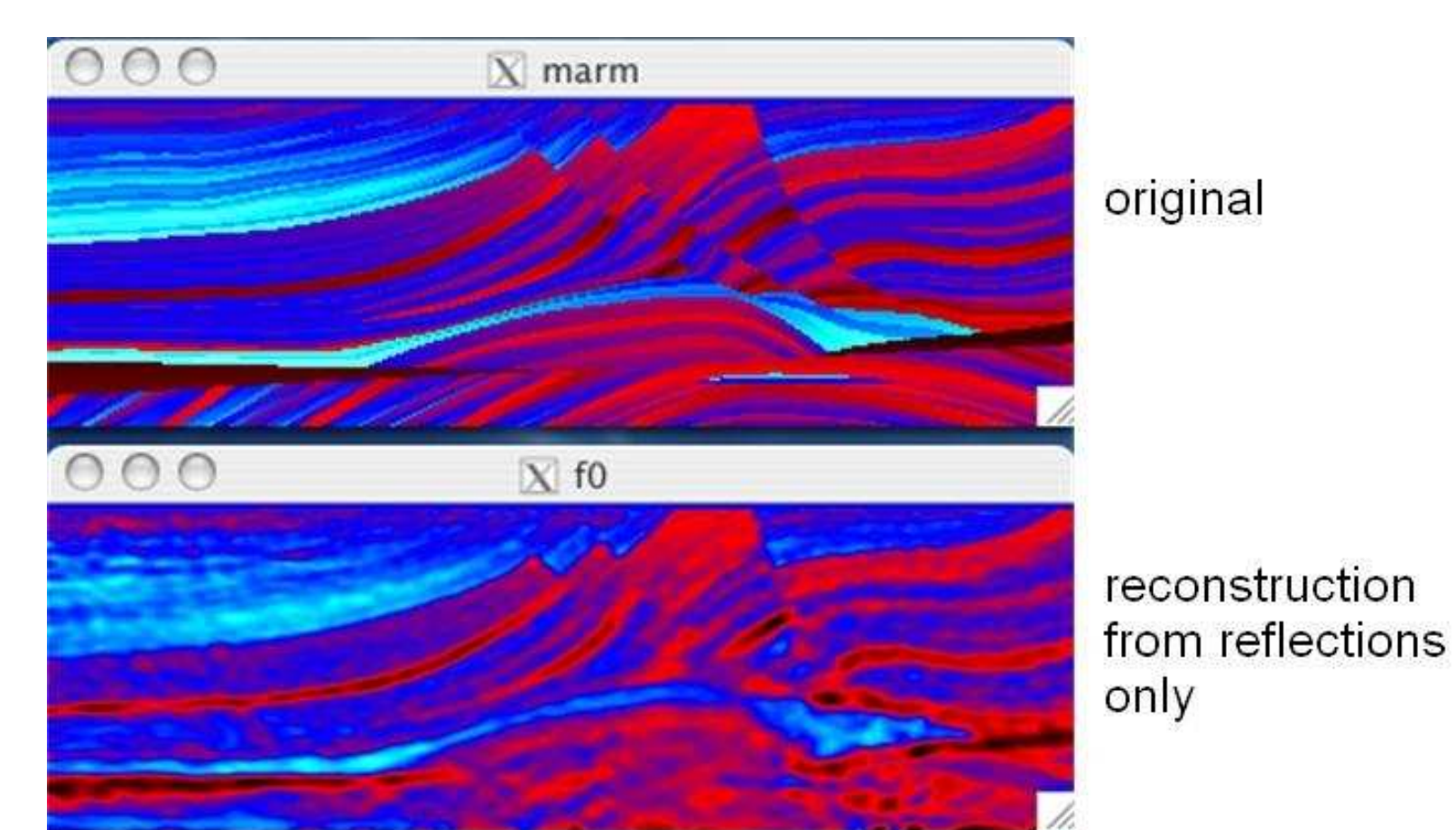
original

reconstruction

Computing time: < 1 minute on a 3Ghz double processor PC

Example 2: Marmousi data set

$$\text{Source wavelet: } q(t) = e^{-(t/\tau)^2/2}, \quad \tau = 0.06s$$



original

reconstruction
from reflections
only

Computing time: 2 minutes on a 3Ghz double processor PC

References

SCHULTZ, P.S. - CLAERBOUT, I.F.: Velocity estimation and downward continuation by wavefront synthesis. Geophysics 43, 691-714 (1978).
NATTERER, F.: Ultrasonic image reconstruction via plane wave stacking, Preprint.