Thermoacoustic Tomography – Inherently 3D Reconstruction

An Inherently 3D Generalized Radon Inversion Problem
Outline

• **WHY TCT** – *images, ...*
• Ties to wave equation
• Physics - forward problem
• Inversion formulae for complete data
• Inverting *incomplete* data

Backup
• Wave Fronts
• Recon Background
  – Xray CT
  – Spherical Transforms

---

Xrays *propagate* straight through, image recovery stable.

Sound waves also *propagate*, permitting stable inversion.

*NIR diffuses.* . .
Images across modalities

Prototype TCT already competes w/conventional scans!

US

Xray projection

MRI slice

TCT slice

TransScan R&D

UW-Madison

Kruger/OptoSonics, Inc.
Ductal Carcinoma in situ (DCIS)

9/7/2000 11:17:05 AM
Study: IBC-077-LJR-070500
Description: MammoCalcifications RT
Set: Right Breast (128/4096)
Coronal: 1.20mm (Slice 126)

9/7/2000 11:17:28 AM
Study: IBC-077-LJR-070500
Description: MammoCalcifications RT
Set: Right Breast (128/4096)
Coronal: 2.40mm (Slice 132)

Images courtesy R. Kruger, OptoSonics, Inc.
TCT Changes During Chemo (TCT V2.4)

Baseline | 7 weeks | Pre-Surgery
--- | --- | ---

Longitudinal changes during primary chemotherapy. Tumor mass (arrows) appears to have decreased markedly.

Images courtesy R. Kruger, OptoSonics, Inc.
Fibrocyctic Breast
(Extra Dense Breast)

6/16/00 2:06:47 PM
Study: IBC-006-CDO-040600
Description: TCT V3.0.1
Set: Left Breast (64 Angles)
Coronal: 26.40mm (Slice 84)

6/16/00 2:06:47 PM
Study: IBC-006-CDO-040600
Description: TCT V3.0.1
Set: Left Breast (64 Angles)
Sagittal: 5.40mm (Slice 119)

cyst

Images courtesy R. Kruger, OptoSonics, Inc.
Thermoacoustics (Kruger, Wang, . . .)

- RF/NIR heating →
- thermal expansion →
- pressure waves →
- US signal

Measured Data

\[
(R_{TCT} f)(p, r) = r^2 \int f(p + r\theta) \, d\theta
\]

- Integrate \( f \) over spheres
- Centers of spheres on sphere
- Partial data only for mammography
\( \rho \)-filtered inversion (complete data)

- Backproject data (thanks to V. Palamodov!)
- Switch order of integration
- Use co-area formula
- \( f \ast \) Riesz potential \( \rightarrow \)
- \( f \) after high-pass filter

\[
M = \{ z | \phi(z) = 0 \}
\]

\[
\int_{z \in M} h(z) d^{n-1}z = \int_{R^n} h(z) |\nabla \phi(z)| \delta(\phi(z)) d^n z
\]
\[ p - \text{filtered inversion (complete data)} \]

\[
\int_{|p|=1} \frac{1}{|x-p|} (R_{TCT} f)(p, |x-p|) dp
\]

\[
= \int_{|p|=1} \frac{1}{|x-p|} \left[ \int_{|\theta|=1} f(p + |x-p|\theta)|x-p|^2 d\theta \right] dp
\]

\[
= \int_{|p|=1} \frac{1}{|x-p|} \left[ \int_{y \in \mathbb{R}^3} f(y) \delta \left( |y-p|^2 - |x-p|^2 \right) |2(y-p)| dy \right] dp
\]

\[
= \int_{y \in \mathbb{R}^3} f(y) \left[ \int_{|p|=1} 2\delta \left( |y-p|^2 - |x-p|^2 \right) dp \right] dy
\]

\[
= \int_{y \in \mathbb{R}^3} f(y) \left[ \int_{p \in \mathbb{R}^3} 2\delta \left( |p|^2 - 1 \right) \delta \left( |y-p|^2 - |x-p|^2 \right) dp \right] dy
\]
\(\rho-\text{filtered inversion (}\delta\text{-identity)\)}

\[\int_{p\in R^3} 2\delta \left(|p|^2 - 1\right) \delta \left(|y - p|^2 - |x - p|^2\right) d^3p = \int_{|x - p| = |y - p|} \frac{1}{|y - x|} \delta \left(|p|^2 - 1\right) d^2p\]

\[= \frac{1}{|y - x|} \int_{p_1^2 + p_2^2 = 1 - h^2} \frac{1}{\sqrt{1 - h^2}} ds\]

\[= \frac{2\pi}{|y - x|}\]

\[\int_{|p| = 1} \frac{1}{|x - p|} \left(R_{TCT} f\right)(p, |x - p|) dp = 2\pi \int_{y \in R^3} \frac{f(y)}{|y - x|} dy\]
Thermoacoustic Tomography – Inherently 3D Reconstruction

Inversion Formulae (complete data)

\[
f(x) = \frac{-1}{8\pi^2} \Delta_x \left[ f * R_{sz,2} \right](x)
\]

\[
= \frac{-1}{8\pi^2} \Delta_x \int_{|p|=1} \frac{1}{|x-p|} (R_{TCT} f)(p, |x-p|) dp
\]

\[
\cdot \cdot \cdot
\]

\[
= \frac{-1}{8\pi^2} \int_{|p|=1} \frac{1}{|x-p|} (R_{TCT} f)'(p, |x-p|) dp
\]
Numerical Results (G Ambartsoumian)
256x256 images from $(N_\phi,N_\theta,N_r) = (400,200,200)$

FBP with $1/\rho$ weighting  

FBP w/experimental weighting
FBP and $\rho$-filtered images windowed to $[-0.09, 0.09]$.

Simulated data sans noise $(N_\phi, N_\theta, N_r) = (800,400,512)$

Machine precision, as it should be.
Thermoacoustic Tomography – Inherently 3D Reconstruction

Low Contrast Detectability

$\sigma_{abs} = 0.006$
Low Contrast Detectability
Partial Scan Reconstructions

\( (N_\phi, N_\theta, N_r) = (400, 200, 200) \)

- FBP with \( \frac{1}{2} \) data in \( \theta \)
- \( \rho \)-filtered with \( \frac{1}{2} \) data in \( \phi \)
Consistency Conditions – Necessary, but maybe not sufficient

\[ R_{TCT} f(p, r) = R_{TCT} f(p, -r) \quad \text{even wrt } r \]

\[ Mom_k(p) = \int_{R^n} r^k R_{TCT} f(p, r) \, dr = \int_{R^n} |x-p|^k f(x) \, dx \]

\[ Mom_{2k}(p) = \int_{R^n} \left( |x|^2 - 2x \cdot p + |p|^2 \right)^k f(x) \, dx \]

\[ Mom_{2k}(p) \bigg|_{p \in S^{n-1}} = Q_k(p) \]
Consistency Conditions – Implications

\[ R_{TCT} f(p, r) = \sum_{n=0}^{\infty} c_k(p) \tilde{P}_{2k}(r) \]

\[ c_k(p) \equiv \int_{r=-1}^{1} \tilde{P}_{2k}(r) R_{TCT} f(p, r) \, dr \]

\[ = \int_{r=-1}^{1} \left( \sum_{l=0}^{k} c_l r^{2l} \right) R_{TCT} f(p, r) \, dr \]

\[ c_k(p) = \sum_{l=0}^{k} c_l^k Q_l(p) \text{ poly of degree } k \text{ in } p!! \text{ measure } c_k \text{ on } S^-; \text{ evaluate } c_k \text{ on } S^+ \]
Polynomial Expansion

Accuracy

\[ R_{TCT} f(p_1, p_2, p_3, r) = R_{TCT} f(p_3, r) = \sum_{n=0}^{\infty} c_k(p) \tilde{P}_{2k}(r) \]

High-order expansions required!!!
Polynomial Extrapolation – Stability

Measure over $p_3 \in [-1,0)$
- rescale so $q_3 \in [-1,1)$
- fit measurements to another set of Leg. polys

Evaluate for $p_3 \in [0, 1)$ i.e., $q_3 \in [1,3)$
Thermoacoustic Tomography – Inherently 3D Reconstruction

extrapolated data

measured data

radius
additive white noise
\( \sigma = 0.01 \)

\( \frac{1}{2} \) scan only  \( \frac{1}{2} \) scan + deg-10 extrapolation
Thermoacoustic Tomography – Inherently 3D Reconstruction

(\(N_f, N_q, N_r\)) = (800, 400, 512)

\(\frac{1}{2}\) scan reconstruction – zero-filling vs. data extension with respect to \(z\)-only

Window width = 0.6

Window width = 0.2
Thermoacoustic Tomography – Inherently 3D Reconstruction

½ scan FBP reconstruction – 0.2% “absolute” additive white noise

zero-filling vs. data extension

window width = 1.2

window width = 1.2 deg 4

window width = 0.6 deg 8

window width = 0.6 deg 12

window width = 0.6 deg 16
Attenuation Blurs

\[ e^{-0.1c|\tau|t} \]

where

- \( \tau \sim t \) are dual Fourier variables
- \( b \sim 1 \)
- \( \alpha \sim 0.1 \text{ MHz}^{-1} \text{ cm}^{-1} \)
- soundspeed \( c = 1500 \text{ m/s} \)

PNT Wells, *Biomedical Ultrasonics*
Heuristic Image Quality Impact

use 2D xray transform & exploit projection-slice

Ideal Object/Full Scan                 Attenuation-Partial Scan
non-Math Conclusions

• Positives
  – cheap ??
  – non-ionizing
  – high-res (exploits hyperbolic physics)
  – 2x depth penetration of ultrasound, *sans* speckle
  – detect masses

• Issues
  – will not detect microcalcifications
  – contrast mechanism not understood
  – fundamental physics (attenuation, etc) and HW constraints will impact IQ

GOAL : biannual screening

TCT for small low-contrast masses

xrays miss.

Xrays for precursors (microcalcs)
Math Conclusions

- FBP type inversion formulae
- Partial scan - unstable outside of "audible zone"
  - Palamodov
  - Davison & Grunbaum
  - Anastasio et al, Xu et al – OK inside
- Attenuation – expect blurring
- \( \cos \theta \) transducer response – Finch
Simulated data *sans* noise
\((N_\phi,N_\theta,N_r) = (800,400,512)\)

FBP and \(\rho\)-filtered images windowed to [-0.09 0.09]

FBP vs. \(\rho\)-filtered with 1% absolute noise

grayscale is [-2.55, 2.34]
½ scan reconstruction –
0.2% “absolute” noise
zero-filling vs. data extension

window width = 0.8
Wave Fronts in 2-D Standard Radon

\[ Rf(s, o) = \int f(x) \, d^1 x \quad x \cdot o = s \]

\[ Rf^\wedge(\sigma o) = (Rf)^\wedge(\sigma , o) \]

projection-slice theorem

Recover image edges tangent to measurement surface edges

w/resolution comparable to that of surface parameter s
Wave Fronts in TCT

“direct” information about horizontal edges;

“indirect” information about vertical edges.
Recon Background

Xray CT – line integrals
- 2D
- 3D
  - Grangeat
    - line $\int \rightarrow$ plane $\int$
    - plane $\int \rightarrow$ recon’d image
  - Katsevich
    - line $\int \rightarrow$ recon’d image

Spherical Transforms
- 2D
  - Circles centered on lines
  - Circles through a point
- 3D
  - Spheres centered on plane
  - Spheres through a point
Recon Background

Spherical Transforms

- **2D**
  - Circles centered on circles (Norton)

- **3D**
  - Spheres centered on sphere (Norton & Linzer, approximate inversion for complete data)

- **2D**
  - Circles centered on lines
  - Circles through a point

- **3D**
  - Spheres centered on plane
  - Spheres through a point