



# Dimensionality Reduction and Divergence Estimation for Polarization-Resolution Trade In SAR Images

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## Abstract

# Dimensionality Reduction and Divergence Estimation for Polarization-Resolution Trade In SAR Images

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In the Integrated Sensing and Processing paradigm, agile sensors operating in a setting of limited power, bandwidth, etc. can receive feedback regarding sensor settings for subsequent data collection (sensor scheduling). In the system design phase, trades are often conducted balancing utility of different sensor settings or sensor configurations. Information-theoretic tools are useful for assisting in evaluating information gain in both of these settings. In particular, given fully polarimetric, measured synthetic aperture radar (SAR) images of two targets, we apply two dimensionality reduction techniques and a non-density-based divergence estimation approach to evaluate the relative target divergences over differences in effective spatial resolution and in the number of available polarization states. Divergence at the pre-classifier stage serves as a surrogate for target separability in an Automatic Target Recognition (ATR) setting, avoiding the extra layer of complexity induced by the choice of classifier and classifier parameter settings.

We show that for the two measured targets, having multiple polarizations available at a lower spatial resolution provides Henze-Penrose Divergence similar to that achieved for a single polarization available at a higher spatial resolution. We also find that for achieving denoising in the embedding space, a linear dimensionality reduction approach (Principal Component Analysis / PCA) can sometimes be more effective than a nonlinear dimensionality reduction approach (Isometric Mapping / ISOMAP).



## Data Description

- **Advanced Detection Technology Sensor (ADTS)**
  - **Air-to-ground SAR sensor operated by MIT Lincoln Laboratory**
    - Fully polarimetric: HH, HV, VV, with polarimetric calibration
    - Operates at Ka band (~ 33.6 GHz with 600 MHz bandwidth)
  - **1989/1990 data collection: slant range of 7 km**
    - 22.5° depression angle; 0° – 360° azimuth coverage in 1.5° – 2° steps with some larger gaps
    - Pixel spacing in object space: 9"; Resolution: 12" in both range and cross-range
    - Targets: M48 Tank (228 samples), M55 Self-Propelled Howitzer (227 samples)
    - 128 pixel x 128 pixel chips already cut from extended scenes
  - **Public Release Data: [https://www.sdms.afrl.af.mil/request/data\\_request.php](https://www.sdms.afrl.af.mil/request/data_request.php)**



ADTS Radome on belly of Gulfstream G1



M48 Tank  
Length: hull 21'; 31' with gun forward  
Width: 12'  
Height: 10'



M55 Self-propelled Howitzer  
Length: 26'  
Width: 12'  
Height: 12'

# Combining multiple pols into "PWF" Images

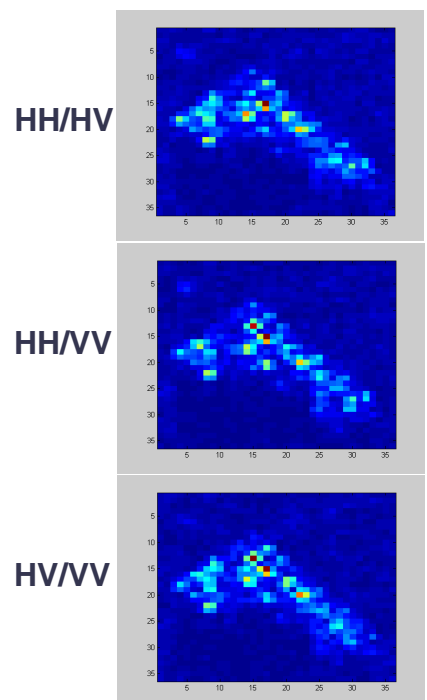
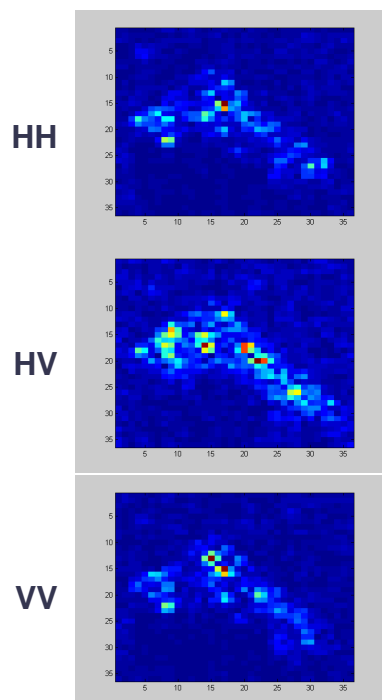
- Polarization Whitening Filter (PWF)**

- MIT Lincoln Laboratory prescription for combining images from different polarizations such that speckle is minimized (M48 example below; central 36 pixel x 36 pixel region shown)
- Novak *et al.* (1990). *Lincoln Laboratory Journal* 3 273.

$$\sigma_{HH} = E(|HH|^2), \quad \epsilon = \frac{E(|HV|^2)}{E(|HH|^2)}, \quad \gamma = \frac{E(|VV|^2)}{E(|HH|^2)}, \quad \rho = \frac{E(HH \cdot VV^*)}{[E(|HH|^2)E(|VV|^2)]^{1/2}}$$

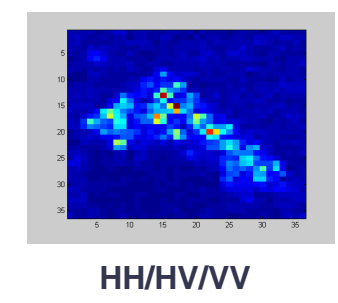
$$\Sigma = \sigma_{HH} \begin{pmatrix} 1 & 0 & \rho\sqrt{\gamma} \\ 0 & \epsilon & 0 \\ \rho^*\sqrt{\gamma} & 0 & \gamma \end{pmatrix} \Rightarrow \mathbf{X} = \begin{bmatrix} HH \\ HV \\ VV \end{bmatrix} \Rightarrow \sum_{\mathbf{X}}^{-1/2} \Rightarrow \mathbf{W} = \begin{bmatrix} HH \\ \frac{HV}{\sqrt{\epsilon}} \\ \frac{VV - \rho^*\sqrt{\gamma}HH}{\sqrt{\gamma(1-|\rho|^2)}} \end{bmatrix}$$

Unrelated Images  
Equal mean power



$$PWF = \sqrt{\frac{|HH|^2 + \left|\frac{HV}{\sqrt{\epsilon}}\right|^2 + \left|\frac{VV - \rho^*\sqrt{\gamma}HH}{\sqrt{\gamma(1-|\rho|^2)}}\right|^2}{3}}$$

Noncoherent mean RSS of three whitened images

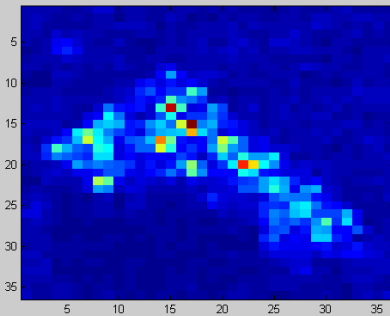




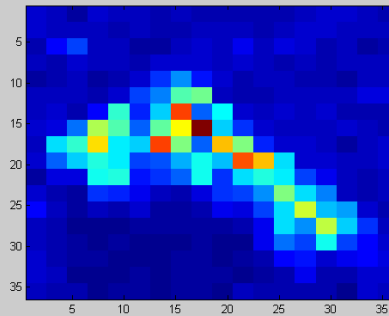
# Resolution Degradation

- Initial Resolution is 12"x12" with 9" pixel spacing
- In spatial domain, degrade resolution by replacing blocks of complex pixel values with average (simulates sensor with lower spatial resolution)
- Extrinsic dimensionality remains  $36 \times 36 = 1296$ , but intrinsic dimensionality decreases as square of blocking factor
- M48 example images shown below, labeled by resolution/intrinsic dimensionality

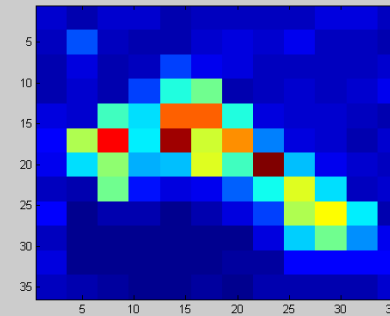
12"/1296



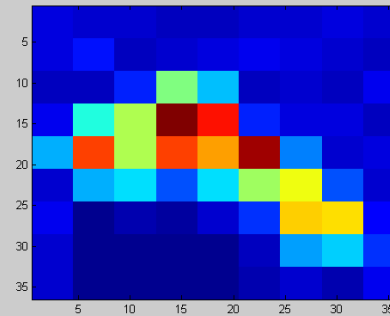
21"/324



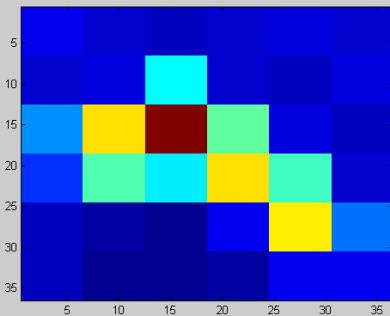
30"/144



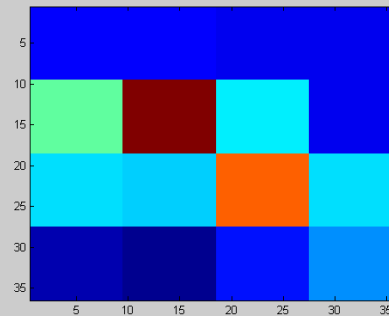
39"/81



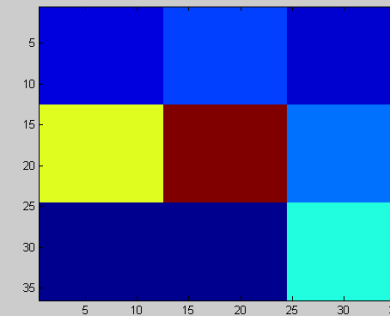
57"/36



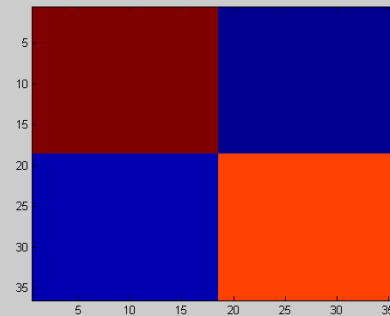
84"/16



111"/9



165"/4





# Information-Theoretic Tools

- **Representations**

- **Data Vectors**

- Reshape 36 pixel x 36 pixel images into 1296-dimensional vectors

- **Isometric Mapping (ISOMAP)**

- Apply ISOMAP ( $k=8$ ) to Data Vectors
- Retain 1<sup>st</sup> 10 embedding coordinates
- Tenenbaum *et al.* (2000). *Science* **290** 2319.

- **Principal Component Analysis (PCA)**

- Apply PCA to Data Vectors
- Retain 1<sup>st</sup> 10 or 1<sup>st</sup> 150 embedding coordinates

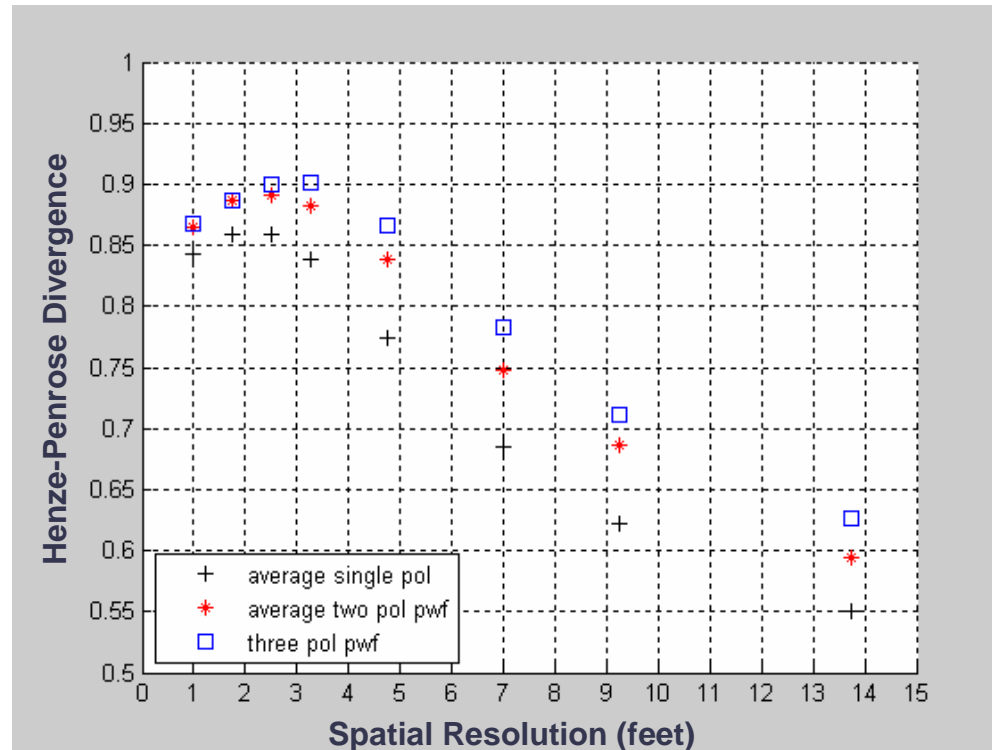
- **Divergence Metric: Henze-Penrose Divergence**  $H(f,g) \equiv \frac{1}{2} \int \frac{f^2 + g^2}{f + g} \mathbf{d}\mathbf{x}$

- **calculated using Friedman-Rafsky Statistic  $S$  with  $p=3$**

- When  $f, g$  are represented with an equal number of samples  $n$ , and the Friedman-Rafsky Statistic  $S$  is calculated using  $p$  orthogonal MSTs spanning the joint sample set, then asymptotically in  $n$ :  
$$H(f, g) \approx 1 - \frac{S - 1}{2np}$$
  - Henze & Penrose (1999). *Annals of Statistics* **27** 290.
  - Friedman & Rafsky (1979). *Annals of Statistics* **7** 697.



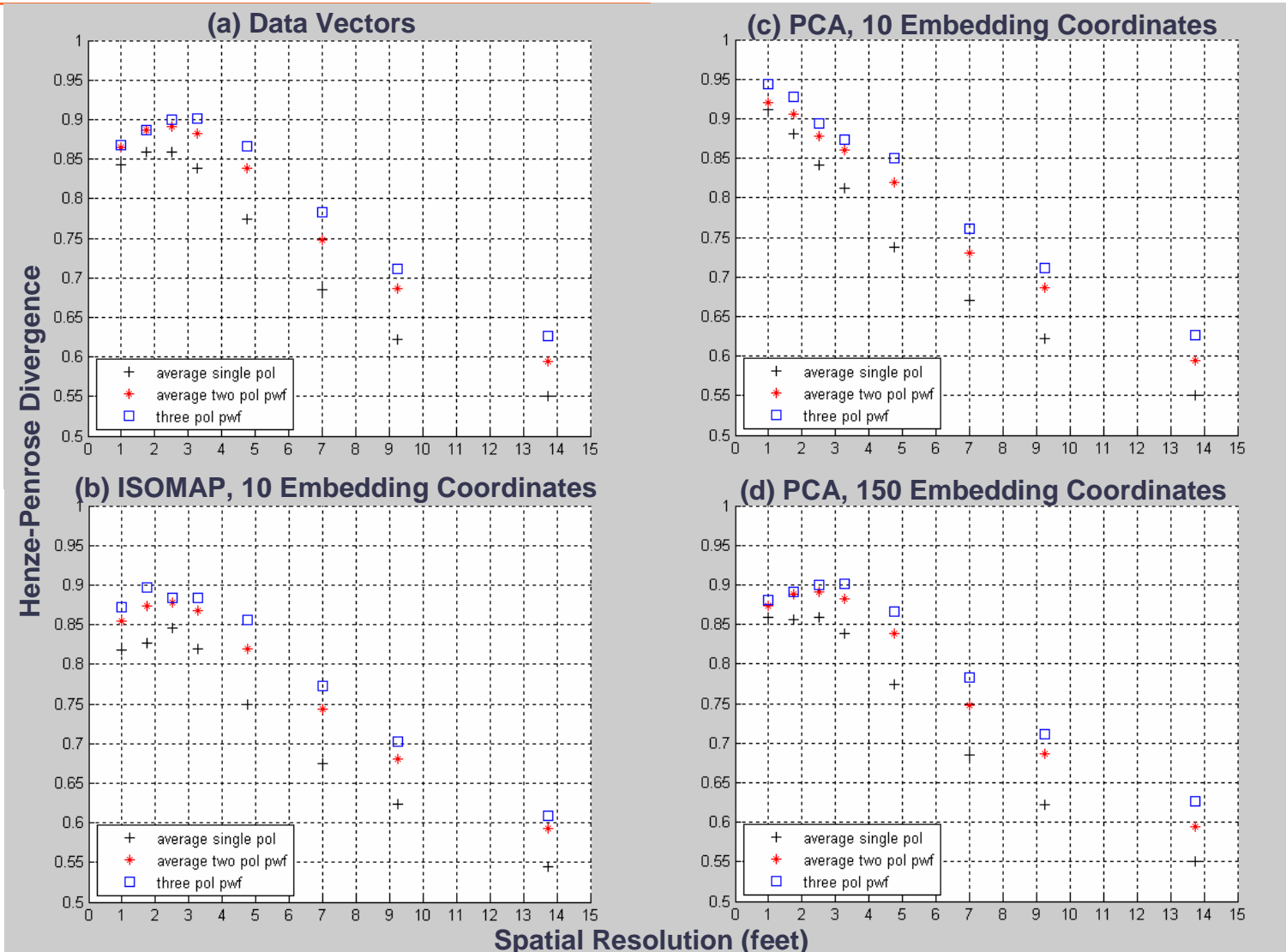
# Approximate Centering



- Generally, 2 pols give improvement over 1; 3 pols give smaller improvement over 2.
- As resolution coarsens from 3', divergence drops (expected: limit is single pixel).
- Why is there a drop in divergence as resolution gets finer than ~ 3' / 4 pixels?
  - Perhaps data contain significant noise at scales < 3' / 4 pixels.
  - Images are not perfectly registered/centered. Reduce jitter by using cross-correlation from image to image to find 36 pixel x 36 pixel region.



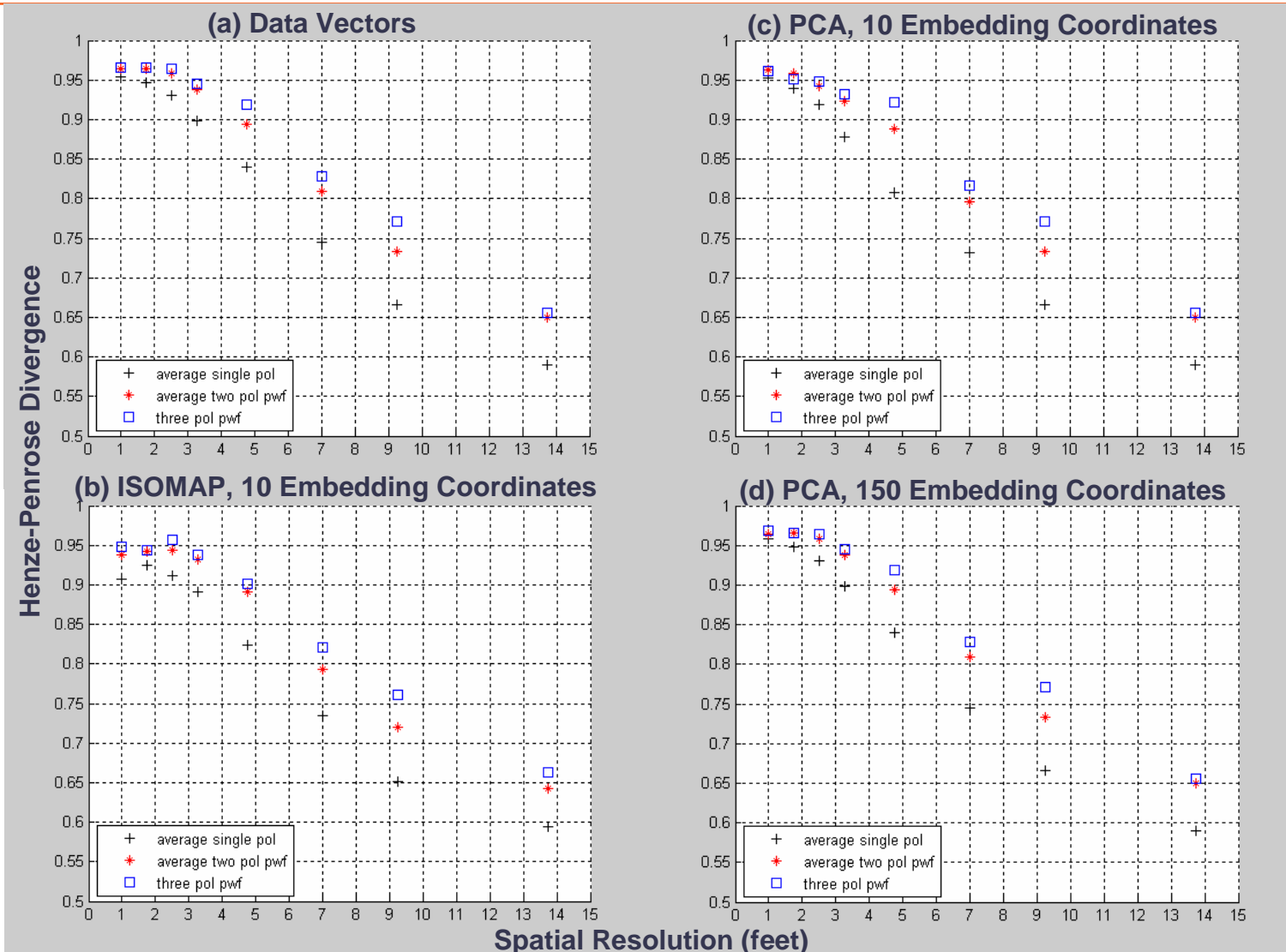
# Approximate Centering



- PCA with 10 embedding coordinates has limited fitting capacity, so it effectively denoises the data.
- PCA with 150 embedding coordinates - no longer effectively denoising; embedding signal + noise.
- ISOMAP with 10 embedding coordinates has sufficient capacity to embed signal + noise reasonable well.



# Improved Centering



- Divergence no longer drops at finer resolutions.
- Highlights point that for registration-based approaches, sharper resolution calls for better registration.
- If registration is improved more, will divergence continue to increase as resolution sharpens?