Movie and Video Scale Time Equalization
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Motivation

Image flicker is especially known for its presence in old movies. It consists of fast variations of the frame contrast and brightness. Old movies suffer at the same time from age related degradations and from the primitive technology used at the early age of cinema. In their case, flicker can be caused by physical degradations of the film, by the use of unstable chemical products, or by a non constant exposure time for each frame, etc. But intensity flicker can also be observed in many amateur videos, whose luminosity has not been controlled during the shooting, or more strongly in the frequent case of low time sampling, as in video surveillance.

Flicker can have many aspects, and is not well removed by simple affine transformations on the intensity of each frame. Most of the time, the problem is considered as a global degradation of the image. In a sense, this strong assumption is a simplification of the problem, but global methods (dependent only on the images histograms) are a priori much more robust to shaking, to motion and to noise presence, than local ones.

Under basic assumptions, we will see what kind of “time smoothing” generic method can be considered for removing flicker. Such a method should involve a scale of correction, representing the size of the time neighborhood used to change each image. This size should correspond to the limit between the flicker variations and the “natural” intensity variations in the movie. The role of time scale however must be formalized and can lead to involve scale-space theory ([Witkin], [Marr],[Koenderink],[Lindeberg]). This scale-time dimension ensures the stability of the method.

Some notations

Let \(u\) be a function of the real line \(\mathbb{R}\) and \(T\) be a time frame at time \(t\). \(H_T:\{0,\ldots,255\}\rightarrow\{0,\ldots,1\}\) is the normalized cumulative histogram of \(u\) (the repartition function of its gray levels), and \(H_T^{-1}\) the inverse of \(H_T\). \(T\) denotes a delphering operator. When \(T\) is applied to \(u\), the resulting film is noted \(T(u)\) = \(u\).

Some basic assumptions

**Globality** A local treatment of flicker involves motion detectors, as sensible to global shaking, noise and blitaches \(\rightarrow\) it may mix several phenomena and depends on the size of the chosen locality. A global definition of flicker relies only on the histograms of the frames, which do not change much in case of shaking, object motion or in presence of jitter noise \(\rightarrow\) this definition is kept.

**Hyp. 0, Globality:**

\[
\forall t \in \mathbb{R}, \forall (x,y) \in \Omega, \quad (u(x) = u(y) \Rightarrow H_T(x) = H_T(y)),
\]

\[
\forall t \in \mathbb{R}, \forall (x,y) \in \Omega, \quad (u(x) < u(y) \Rightarrow H_T(x) > H_T(y)).
\]

This means that \(T\) affects each frame \(u_t\) of the film by a contrast change \(\gamma_T\) (\(T\) leaves the topographic map of each \(u_t\) unchanged). Delphering then becomes mostly independent from denoising and from motion detectors.

**Figure/Background Independence**

The flicker that affects the object should be treated independently of the flicker that affects the background. This strong assumption localizes the action of the operator \(T\) separately on the object and on the background. It is equivalent to say that \(T\) acts independently on each “level set of rank \(\lambda\)” sequence.

**Hyp. 1, Figure/Background Independence:** for \(\lambda \in [0,1]\), let \(\Gamma_\lambda(t)\) be the level set of rank \(\lambda\) of \(u_t\)

\[
\Gamma_\lambda(t) = \{ \lambda \in \Omega, \quad H_T(u_t(x)) = \lambda \}
\]

\[
\forall \lambda \neq 0, \quad \text{the action of } T \text{ on the sequence } t \rightarrow \Gamma_\lambda(t) \text{ is independent of the action of } T \text{ on the sequence } t \rightarrow \Gamma_\lambda(t).
\]

Level sets of rank \(\lambda = \frac{k}{l}\) for each of the four images of the Chaplin’s film The Cure. These level sets correspond respectively to the grayscale \(\{(x,y): \frac{k}{l} \leq H_T(u_t(x)) \}\) (left image), \(\{(x,y): \frac{k}{l} > H_T(u_t(x)) \}\) (right image) and \((0,1)\).

It follows from hypotheses 0 and 1 that the action of \(T\) is completely defined by its separate actions on the functions \(t \rightarrow H_T^{-1}(\lambda)\) (such a function gives the evolution during the time of the gray levels of rank \(\lambda\) in the images of the film). Thus, defining \(T\) boils down to define a “smoothing” operator \(R\) on functions of the real line \(t\).

**Time Shift Invariance** The action of \(T\) should naturally not depend on the choice of the time origin for the film, and should also give the same result when the time direction is reversed.

**Hyp. 2, Time Shift Invariance:** \(T\) commutes with translations and symmetries in time :

\[
\forall u, \forall t \in \mathbb{R}, \quad T(p(u,t)) = p(T(u)),
\]

\[
\forall u, \forall t \in \mathbb{R}, \quad T(u(x,-t)) = T(u(-x,-t)).
\]

**Preservation of constants** \(T\) should also leave constant (in time and space) films unchanged.

**Hyp. 3, Preservation of constants:** if \(u_t\), \(\forall t \in \mathbb{R}, u_t = \mu\), then \(T(u) = \mu\).

Scale-Time Equalization

If \(T\) satisfies hyp. 0 and hyp. 1, its action is entirely defined by the action of an operator \(R\) on the set \(\mathcal{F}_R\) of functions of the real line. Now, if \(T\) also follows hypotheses 2 and 3, it means that \(R\) has to commute with translation and symmetries, and leaves constant functions unchanged.

In order to involve a scale of correction \(s\), the operator \(R\) is replaced by a family \(\{R_s\}\) of operators on \(\mathcal{F}_R\).

Under the classical assumptions of scale-space theory (regularity, causality, linearity), the only choice left for \(\{R_s\}\) is a linear gaussian scale-time.

**Proposition 1** Let \(\{G_t\}_{t \in \mathbb{R}}\) be a family of operators acting on films and satisfying hypotheses 0 to 3. We call \(\{G_t\}\), the corresponding family acting on real functions : the action of \(\{G_t\}\) on any film \(u\) is described by the action of \(R\) on every function \(\{G_t(u(x))\}\). We suppose that \(\{R_s\}_{s \in \mathbb{R}}\) satisfies the axioms of linear scale-space. Then, there exists a rescaling \(s' = f(s)\), such that

\[
\forall t \in \mathcal{F}_R, \quad R_s(f) = G_{s'} \ast f, \text{ where } G_{s'}(t) = \frac{1}{(\sqrt{2\pi} s')^T} e^{-t^2 / (2s'^2)}.
\]

⇒ Other non-linear solutions can be considered for \(\{R_s\}\). For example, a median filter on a neighborhood is a good alternative choice, well adapted in the case of scene changes.

Stability results

**Mean function** If \(u\) is corrected by a Scale-Time Equalization, then its mean function \(m_u(t)\) is smoothed by the heat equation.

\[
\forall t, m_{u(t)} = G_t \ast m_u(t).
\]

In the same way, every value \(u_{tk}\) which can be written linearly from the values \(H_T^{-1}(\lambda)\) is also smoothed by the heat equation, as for instance the median of the image which is equal to \(H_T^{-1}(\lambda)\).

**Asymptotic result** Let \(H_s\) denote the repartition function of the image \(u_s\) after a correction at scale \(s\). Then, under basic hypotheses, it can be shown that all the functions \(H_s\) converge toward the same limit \(H_{\infty}\) when \(s \rightarrow \infty\).

**Sensitivity to noise** If \(u\) is added to an image \(v\), the cumulative histogram of \(u\) becomes \(H_{u+v}\), where \(H_v\) is the noise distribution. If the noise is Gaussian, \(H_v\) is smoothed by this operation, which is not disturbing for the correction. If \(v\) is an impulse noise added to \(p\)% of pixels, then \(H_{x+(1-p)H_u+p\delta}\). This does not disturb the correction either if \(p\) is not too large.

Experiments

**Synthetic flicker**

*Top:* Three images of a film to which a global and highly non-affine artificial oscillating flicker was added, and their intensity histograms. *Bottom:* Same images after Scale-Time Equalization. The artificial flicker has completely vanished. The intensity histograms of the restored images are very close to each other.

**With Gaussian noise**

(a) Same images as above, with a gaussian noise of standard deviation \(\sigma = 0\) added before adding the flicker (b) Same images after Scale-Time Equalization.

**Real flicker**

*Top:* Three images of Chaplin’s film His New Job, taken at equal intervals of time. *Bottom:* Same images after Scale-Time Equalization at scale \(s = 100\). The flicker observed before has globally decreased. Left: Evolution of the mean of the current frame in time and at three different scales. The most oscillating line is the mean of the original sequence. The second one is the mean at scale \(s = 3\). The last one, almost constant, corresponds to the large scale \(s = 100\). As expected, the mean function is smoothed by the heat equation.

Images of Chaplin’s film The Cure after a Scale-Time Equalization at a large scale.