# Application of PCA and Geodesic 3D Evolution of Initial Velocity in Assessing Hippocampal Change in Alzheimer's Disease

Lei Wang[1], Laurent Younes[3], M. Faisal Beg[2], J. Tilak Ratnanather[3], John C. Morris[1], John G. Csernansky[1], Michael I. Miller[3]
[1]Washington University, St Louis, MO, [2]Simon Fraser University, Burnaby, BC, Canada, [3]Johns Hopkins University, Baltimore, MD

## Large-Deformation Diffeomorphic Metric Matching

**Anatomic Model:** Given observable anatomical images $I_0, I_1 \in \mathcal{I}$

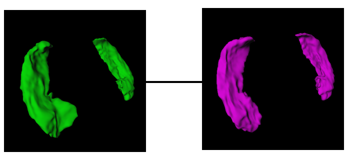
he solution to the variational problem for matching the images

$$\hat{v} = \underset{v:\dot{\phi}_t = v_t(\phi_t)}{\mathrm{argmin}} \left( \int_0^1 \|v_t\|_V^2 dt + \frac{1}{\sigma^2} \|I_0 \circ \phi_1^{-1} - I_1\|_{L^2}^2 \right).$$

under constraints on the set $V = \{u : \Omega \to \mathbb{R}^3 : \|u\|_V = \|Lu\|_{L^2} < \infty\}$

of vector field being sufficiently smooth is in the space of diffeomorphisms

$$\mathcal{G} = \{\phi_1 : \frac{\partial}{\partial t}\phi_t = v_t(\phi_t), t \in [0,1], v_t \in V\}$$



$I_0$, baseline $I_1$, follow-up

Conservation of Momentum: The cost function of the above variational problem gives rise o the gradient:

$$\nabla_v E_t = 2v_t - K\left(\frac{2}{\sigma^2}|D\phi_{t,1}| \nabla I_0(\phi_{t,0}) \left[I_0(\phi_{t,0}) - I_1(\phi_{t,1})\right]\right)$$

where $K$ is a compact self-adjoint operator such that for any smooth vector field

$v \in V$, $K(L^\dagger L)v = v$. The optimizer is a geodesic satisfying, for vanishing gradient,

$$(L^\dagger L)v_t = \frac{2}{\sigma^2}|D\phi_{t,1}| \left[I_0(\phi_{t,0}) - I_1(\phi_{t,1})\right] \nabla I_0(\phi_{t,0})$$

which leads to the conservation of momentum: the momentum at any time $t$ is completely etermined by the momentum at time 0:

$$(L^\dagger L)v_t = |D\phi_{t,0}|(D\phi_{t,0})^t (L^\dagger L)v_0 \circ \phi_{t,0}$$

Geodesic Shooting: The initial velocity, via the conservation of momentum equation, leads o recovering the optimal flow that matches the given images. Hence, the target shape is arameterized by the initial velocity. Since these are a vector space, linear techniques such as CA can be performed and the evolution of the principal components of shape variation can e used to assess the variability observed in target shapes

## Correlation between vector deformation fields and initial velocity fields

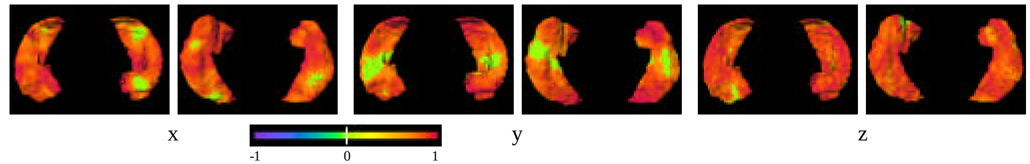
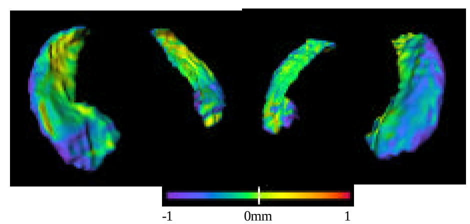



**Table.** Correlations between displacement vector fields and initial velocity vector fields. After adjusting for multiple comparisons, number of surface vertices that show significant correlation is shown as a percentage of total number of surface vertices.

| Axis | Mean (SD) | Median | Range | Percentage of Surface Vertices |
|---|---|---|---|---|
| x | 0.87 (0.08) | 0.89 | [0.64 0.99] | 96.2 |
| y | 0.85 (0.09) | 0.87 | [0.64 0.99] | 89.7 |
| z | 0.87 (0.07) | 0.88 | [0.64 0.99] | 98.3 |




## Shape Analysis on Displacement Vector Fields

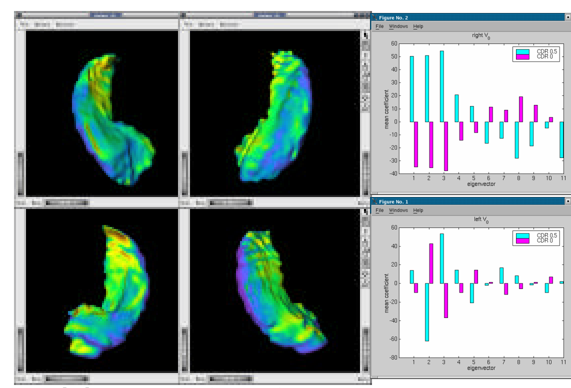
(small-deformation assumption)

1. PCA on displacement vector field for statistical analysis
2. Reconstruct displacement vector field for visualization using selected principal components

## Shape Analysis on Initial Velocity Fields

(large-deformation assumption)

1. PCA on initial velocity vector field for statistical analysis
2. Reconstruct initial velocity fields using selected principal components
3. Compute evolution based on the reconstructed velocity for visualization



Right hippocampus (top row): eigenvectors 3, 8, 11
Left hippocampus (bottom row): eigenvectors 2, 3

## References:

1. M. Miller, A.Trouve, and L. Younes, ``Geodesic shooting for computational anatomy,'' Journal of Mathematical Imaging and Vision, Jan 2006.
2. M. Vaillant, M.I. Miller, L.Younes, and A.Trouv'e, ``Statistics on diffeomorphisms via tangent space representati ons,'' NeuroImage, vol. 23, no. 1, pp. S161--S169, 2004.