Method for determination of Hubbard model phase diagram from optical lattice experiments by two parameter scaling

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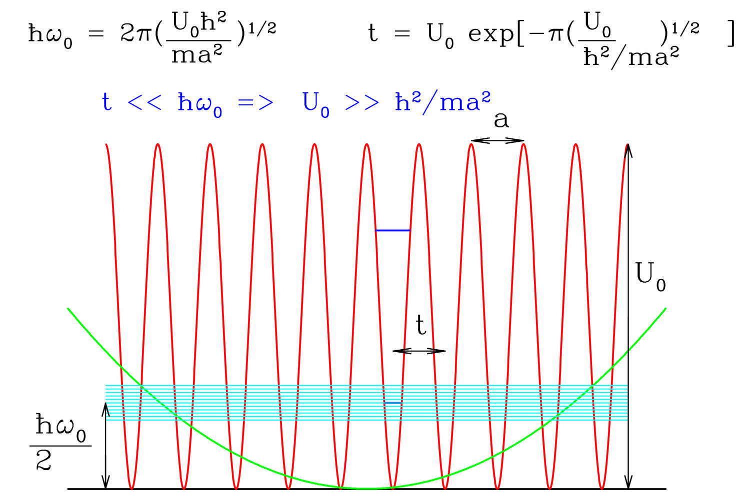
Abstract

We propose[1] an experimental scheme to obtain the phase diagram of the Hubbard model using cold atoms in optical lattices.[2] The scheme is based on measuring the total energy for a sequence of trapping potentials with different profile and is independent of dimensionality. Its essential ingredient is a two-parameter scaling procedure that combines a variant of the familiar finite-size scaling with a nontrivial additional 'finite-curvature scaling' necessary to reach the homogeneous limit. We illustrate the viability of the scheme in the one-dimensional case, using simulations based on the Bethe Ansatz local-density approximation as a substitute for experimental data, and show that the filling corresponding to the Mott transition can be determined with better than 3% accuracy.

Physical System



Inhomogeneous Hubbard Model

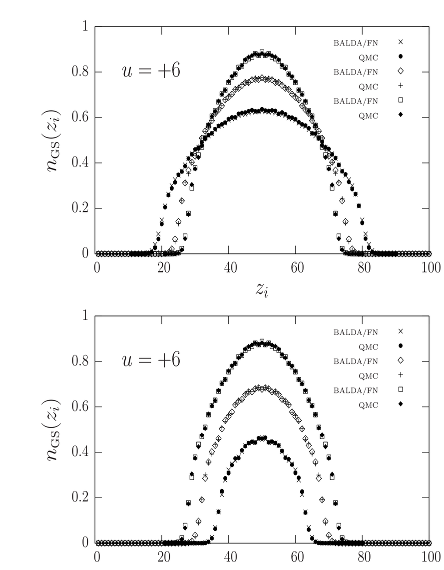
$$H = -t \sum_{<i,j>,\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}) + U \sum_i \hat{n}_\uparrow \hat{n}_\downarrow$$

$$+ \sum_i V_{i,\sigma} \hat{n}_{i,\sigma}$$

DFT approach to 1D model

Bethe-Ansatz LDA[3]: from the exact solution[4] of the homogeneous model, we get the correlation energy and use it in a LDA:

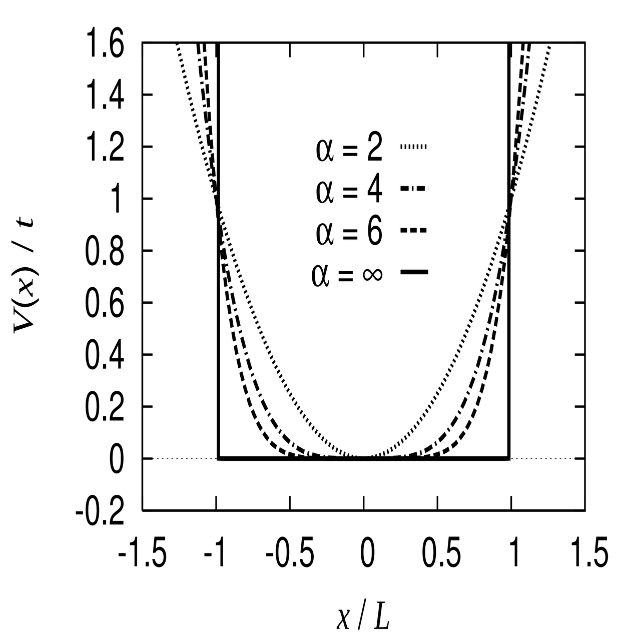
$$e_c(n, U/t) = e(n, U/t) + \frac{4t}{\pi} \sin(\frac{\pi n}{2}) - \frac{U}{4} n^2$$



BA-LDA × QMC

Top panel: site occupation as a function of z_i for a repulsive Fermi gas with $N_f = 30$ atoms, trapped in a harmonic potential ($V = V_2(z-50)^2$) with strength $V_2/t = 2 \times 10^{-3}, 4 \times 10^{-3}, 6 \times 10^{-3}$ in a lattice with $N_s = 100$ sites. Bottom panel: now the number of fermions is varied, $N_f = 10, 20$ and 30, trapped in a harmonic potential of strength. $V_2 = 6 \times 10^{-3}$.(Ref.[5])

Finite-size & Finite-curvature scalings

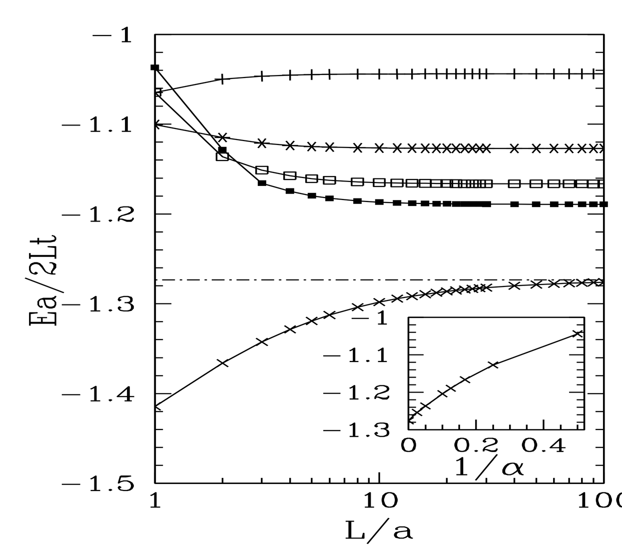


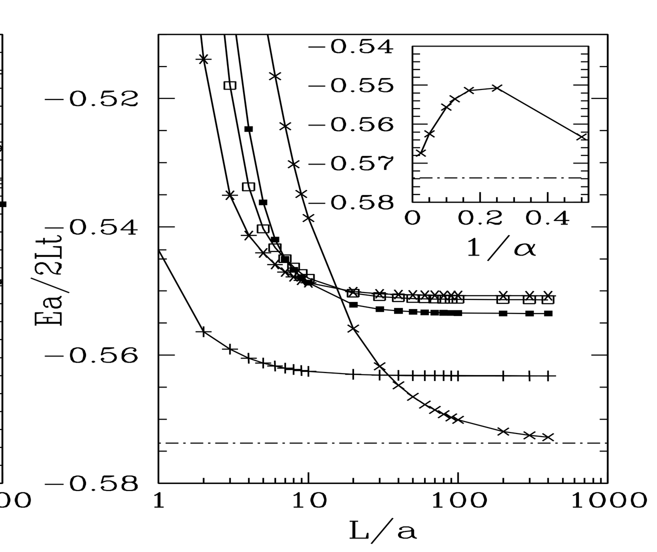
$$V(x) = t \left(\frac{x}{L}\right)^\alpha$$

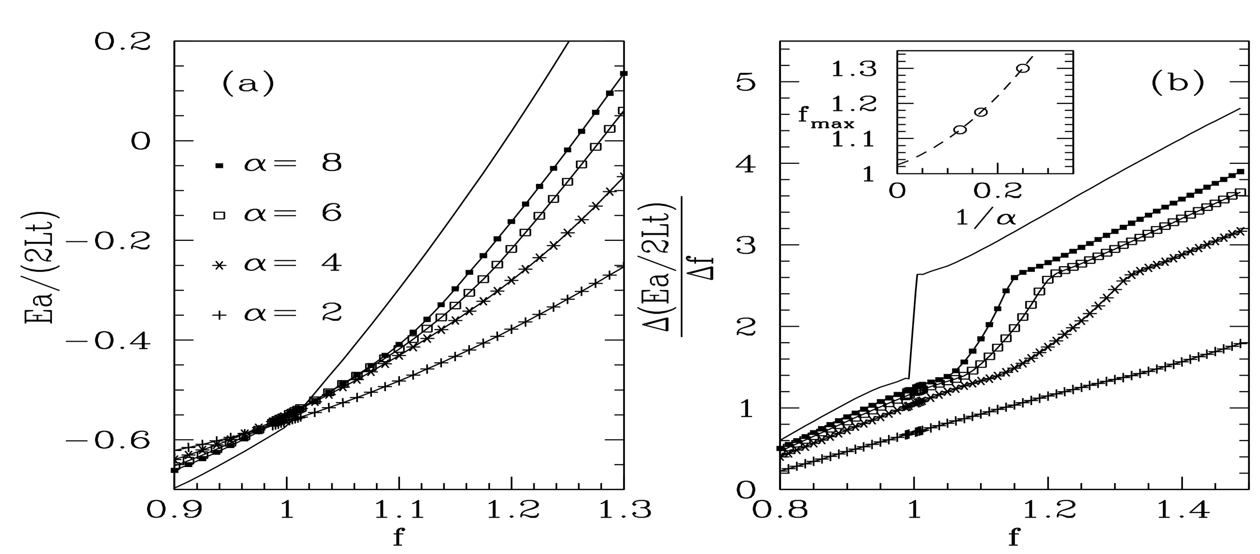
The limit $L \to \infty$ gives results that depend on α.[6]

For 1D, we simulate experimental data using BA-LDA and illustrate the procedure showing how we could detect the insulator behavior at $f = 1$.

At each changing of phase, we would have a discontinuity in the derivative dE/df. Finding these discontinuities for each value of the interaction U and then running the experiment for different U's, one could extract the phase diagram (space U X f, T = 0)







References

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