

# Prospects for fission of rotating stars

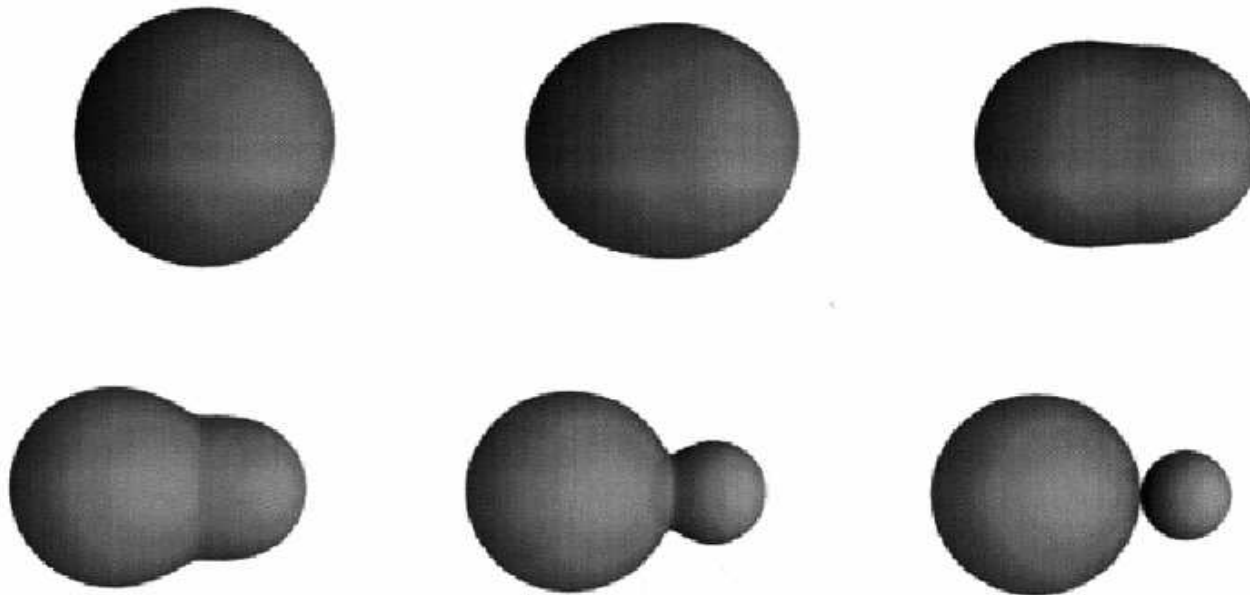
Norman Lebovitz

Department of Mathematics

The University of Chicago

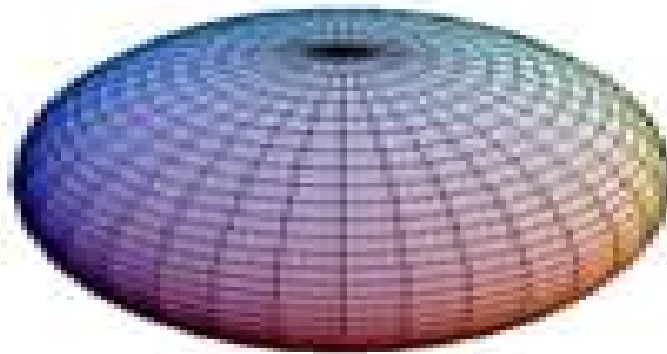
# Introduction

The sky is full of multiple objects: planet-moon systems, stellar systems, binary stars, . . . . There are multiple explanations for their origin as well. I want to tell you the unfinished story of one of these explanations, the fission theory of binary stars – attributed to Kelvin.



# Maclaurin Spheroids

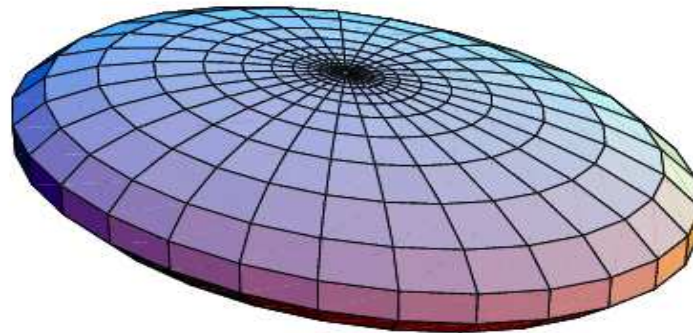
Kelvin knew of certain figures that served as steady solutions of the equations of fluid dynamics for a homogeneous, rotating mass: Maclaurin spheroids and Jacobi ellipsoids.



The Maclaurin spheroid is symmetric about the axis of rotation, with an axial radius smaller than its equatorial radius. For given density it's shape is determined by its angular momentum  $J$ , progressively flattening as  $J$  increases.

# Jacobi Ellipsoids

The Jacobi ellipsoid has three unequal axes except for a limiting case when it coincides with certain Maclaurin spheroid. It exists only when the angular momentum exceeds a critical value.



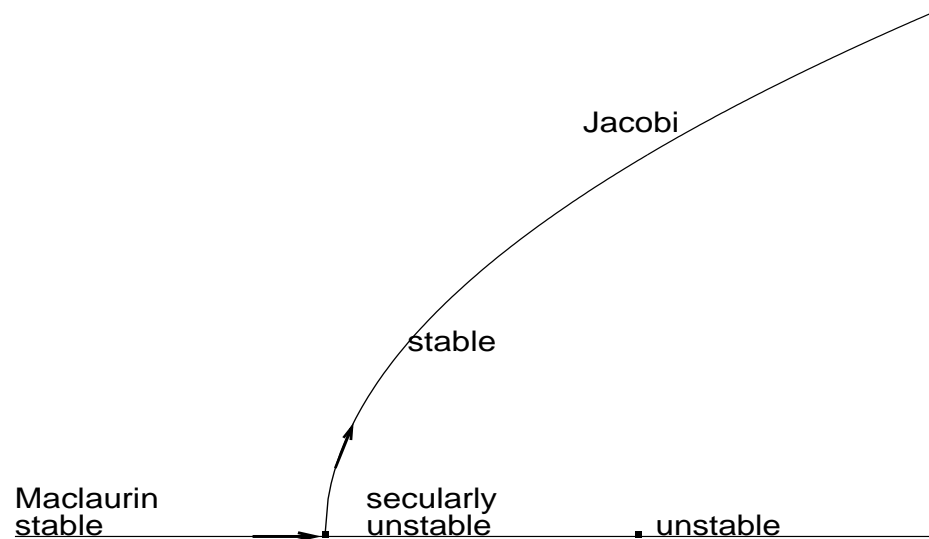
Along either of the Maclaurin or Jacobi families, the shape is determined by the parameter

$$\mu = J\rho^{1/6};$$

the size is then determined by the mass.

# Slow evolution through bifurcation

Schematic of Kelvin's Original Picture



- Timescales: dynamical ( $t_D$ ) and contraction ( $t_C$ ),  $t_D/t_C \ll 1$ .
- Evolution begins: Maclaurin spheroid, angular momentum  $J=\text{constant}$ .
- Energy decreases slowly, so  $\rho$  increases: thus  $\mu = J\rho^{1/6}$  increases.
- Evolution proceeds along the stable branches.
- Assumption: Jacobi branch becomes unstable, fission occurs.
- Note implicit assumption:  $t_D \ll t_\nu \ll t_C$ .

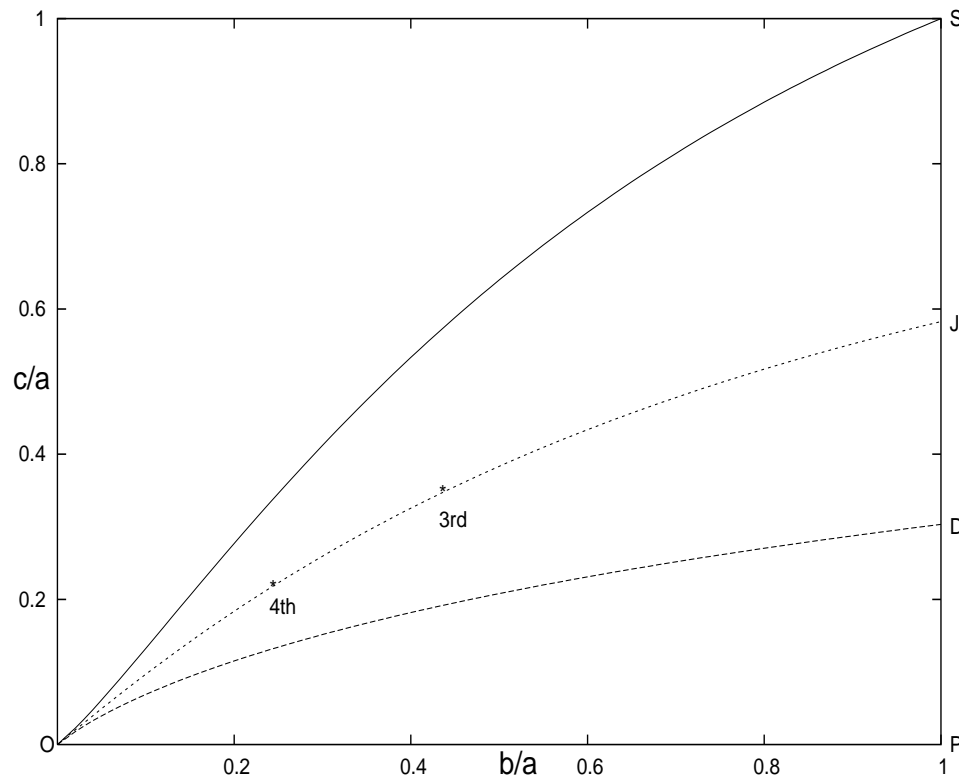




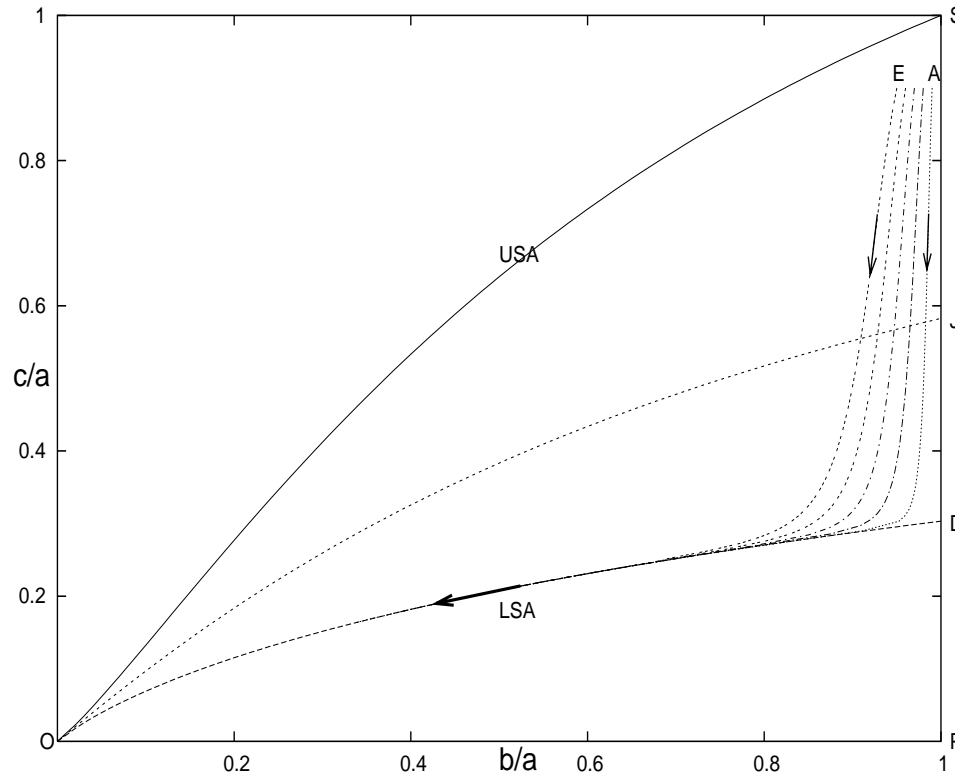
# Frictionless route recovery

Assume instead  $t_\nu > t_C$ . Then there are steady states (Riemann ellipsoids) with relative velocity

$$\mathbf{v} = \lambda \left( -\frac{b}{a}y, \frac{a}{b}x, 0 \right).$$

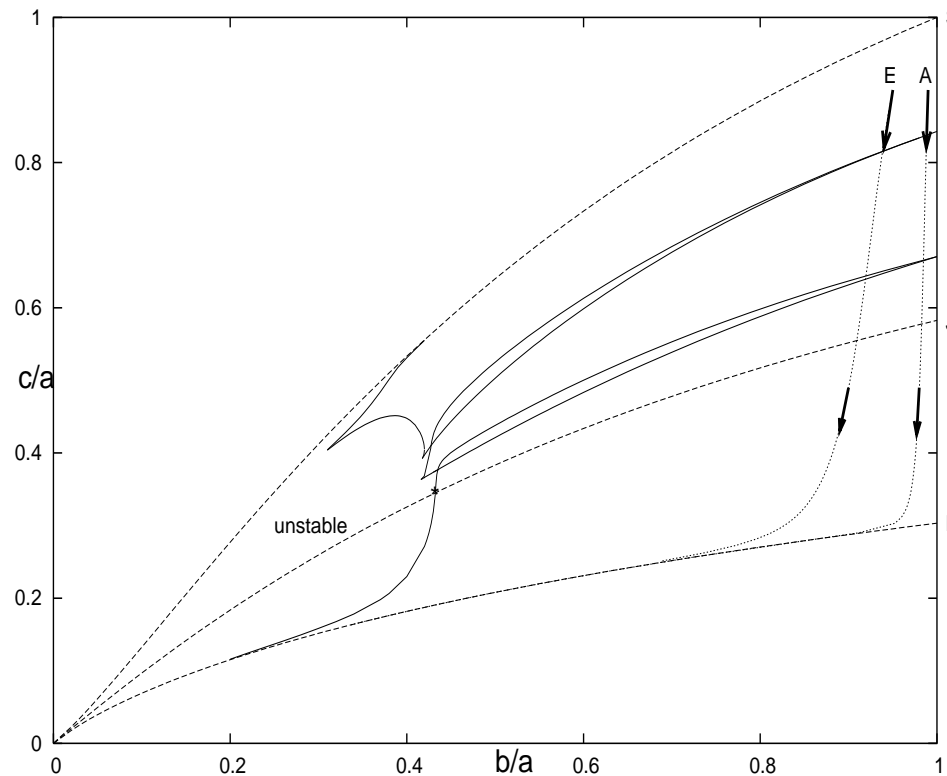


# Inviscid trajectories



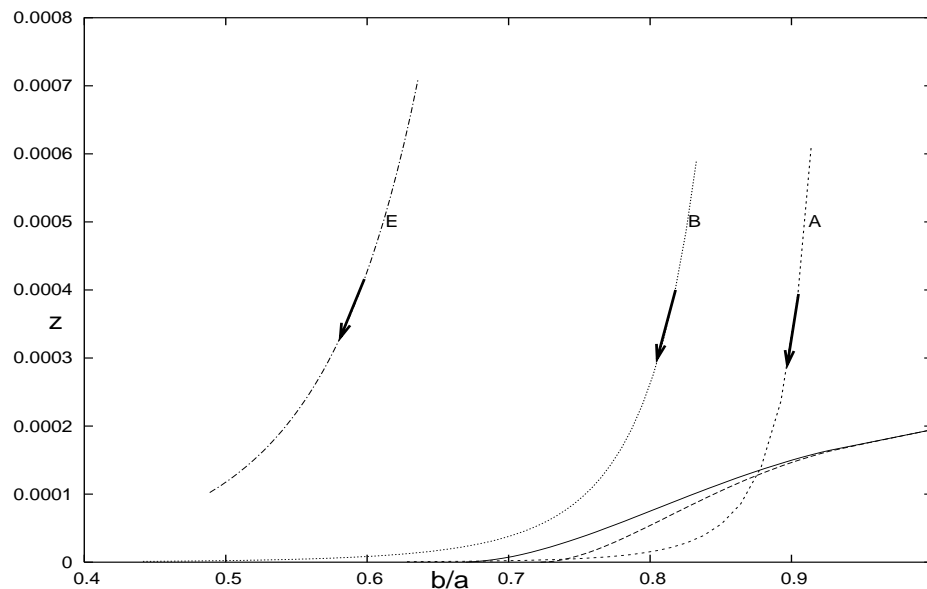
Five inviscid, quasi-steady paths are shown. The roles played by the arcs SJ and JO in the classical fission theory are played instead by the arcs SD and DO in the inviscid (modified) theory.

# Third ellipsoidal harmonics



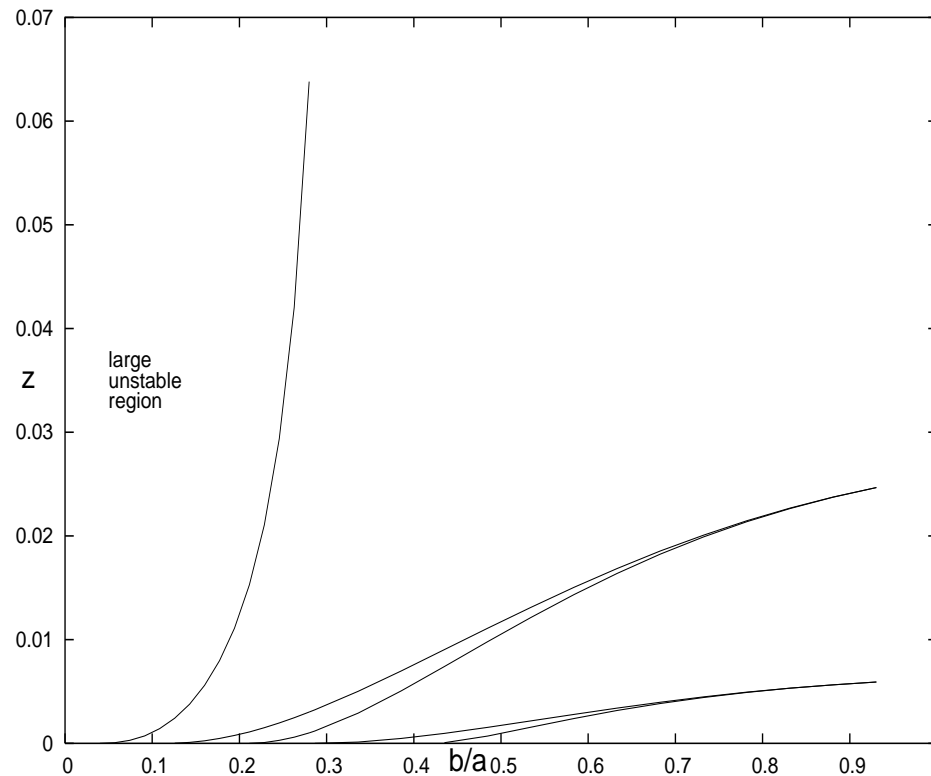
Regions of instability to third harmonics are shown. Only paths  $A$  and  $E$  are shown. The narrow tongues reflect the *elliptical instability*, first identified in 1970's.

# Continuation



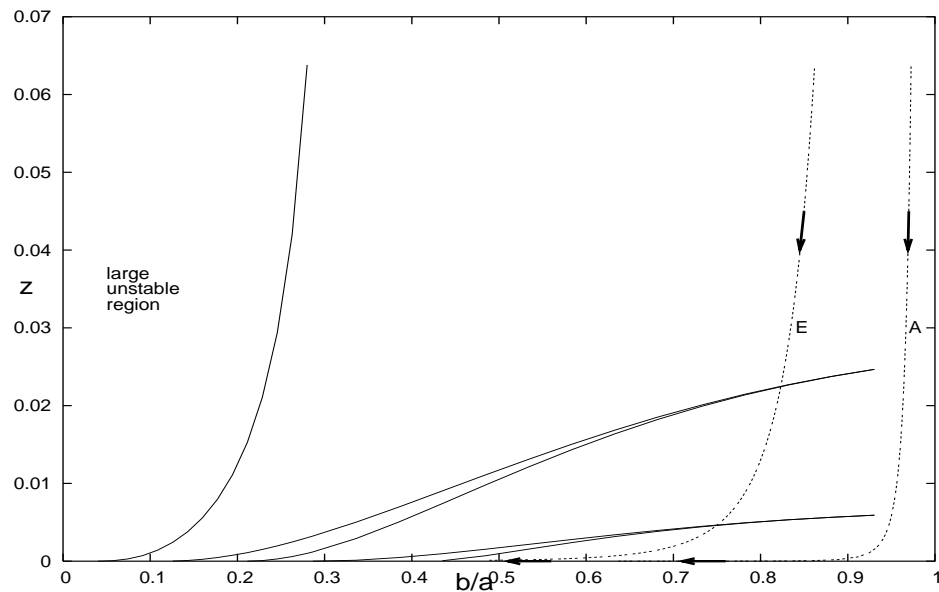
Third-harmonics instabilities. In this diagram, the LSA has been straightened out, i.e., the variable  $z$  is not  $c/a$  but rather the difference  $c/a - (c/a)_{lsa}$ , the height above the LSA sequence. This shows an instability tongue connecting a band along the LSA with a point on the Maclaurin line less than  $2 \times 10^{-4}$  above the LSA. Of the five trajectories considered, only that marked  $A$  is influenced by this tongue.

# Fourth ellipsoidal harmonics



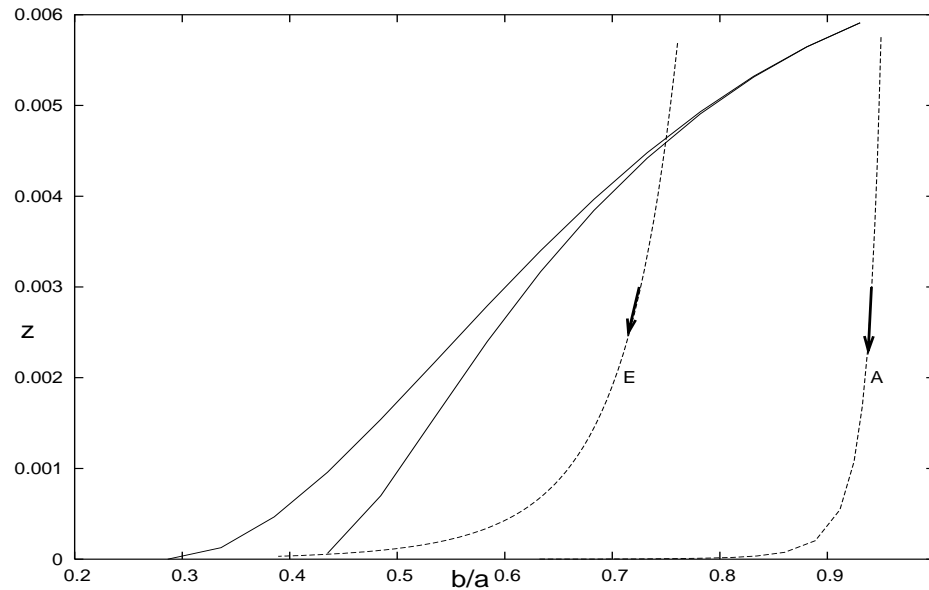
Fourth-harmonics instabilities. Vertical scale amplified by 100. Narrow tongues emanating from LSA extend to the Maclaurin line ( $b/a = 1$ ) but become extremely narrow near that line: at  $b/a = 0.93$ , thickness of lower tongue is less than  $2 \times 10^{-7}$ .

# Continuation



Here the vertical axis has been amplified by a factor of 100. Two of the five evolutionary paths are indicated. They cross the narrow tongues at points insignificant for the stability of the paths, but then recross the lower of these essentially on the LSA. Here the instability gap is not at all narrow (it lies roughly in the interval  $(0.28, 0.42)$ ), and serious consequences for the paths must ensue.

# A “dangerous” region

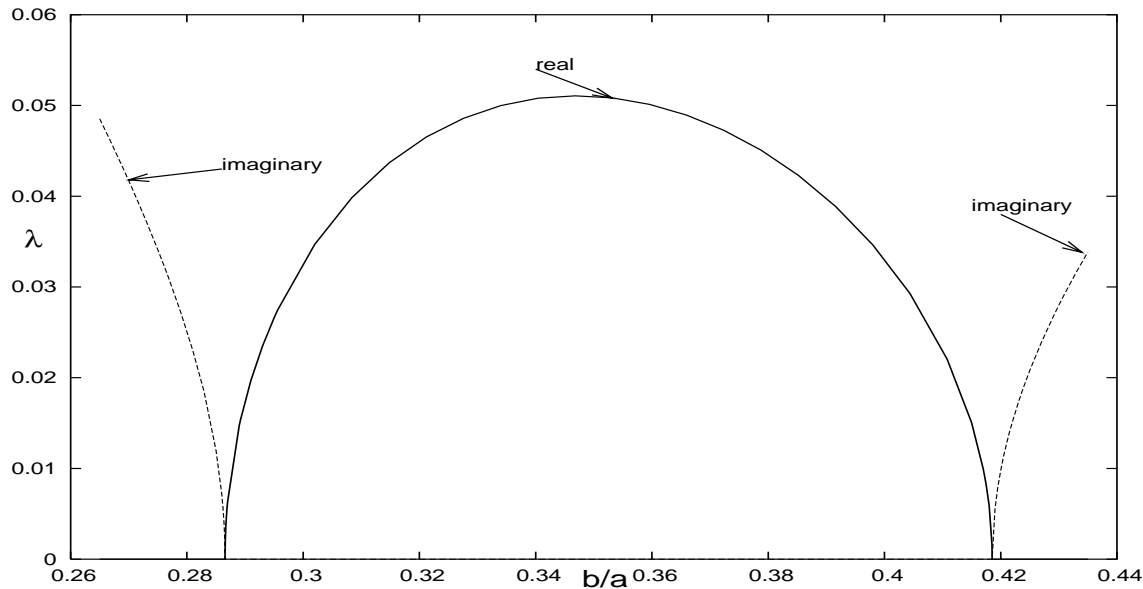


Blown-up version of preceding figure (amplified by 1000) showing only the lower of the two instability tongues associated with fourth-harmonic perturbations. This is the more “dangerous” of the two. .

# Conclusion

1. Nonlinear problem for the inviscid version has not been carried out.
2. The explicit pattern for fission laid out by Kelvin and Poincaré was found wanting, but
3. The underlying idea that binary stars originate via the fission of a single star remains a possibility.

# Extra slide 1



This is an example of the graph of a growthrate across a band of instability. In this case, the abscissa ( $b/a$ ) represents a portion of the LSA, that part of it near which most evolutionary trajectories must pass. The ordinate ( $\lambda$ ) measures the size of the eigenvalue in the unit  $\sqrt{\pi G \rho}$ . The eigenvalue is purely imaginary to the right and to the left of the instability band, and purely real inside the band. The width of the band is about 0.14, and its maximum height is about 0.05, in rough agreement with heuristic predictions.