Capillary Pinch-off of a Film on a Cylinder

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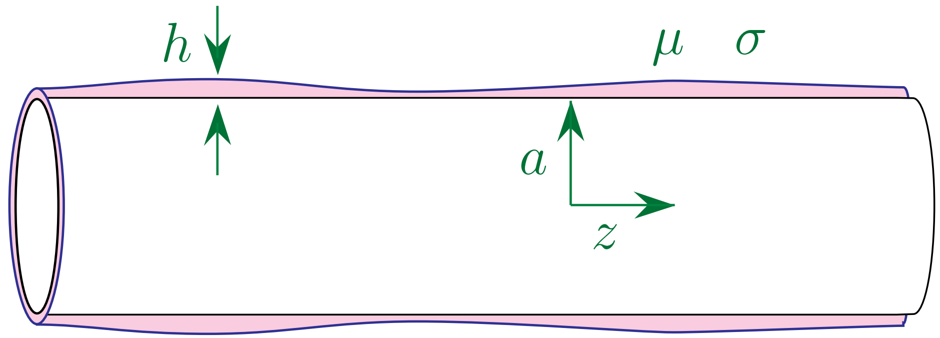
Introduction



- Thin film on/inside cylinder

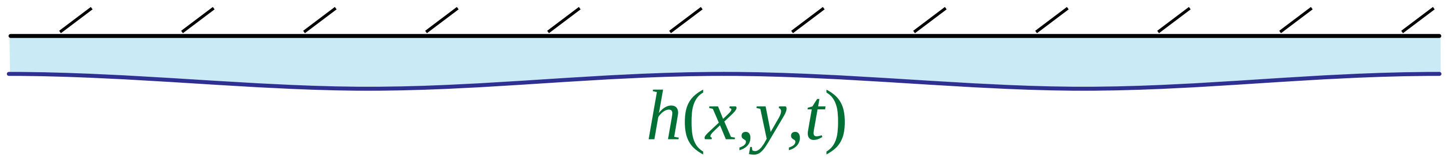
e.g. Hammond 1983, Gauglitz & Radke 1988

- Neglect gravity
- Rayleigh instability in the thin-film (lubrication) limit

$$h_t + \frac{\sigma}{3\mu}\left(h^3(a^{-2}h + h_{zz})_z\right)_z = 0$$

- Linearly unstable for $k^{-2} > a^2$, most unstable with $k^{-2} = 2a^2$
- If initial $h \ll a$ get thin rings of fluid around cylinder

Alternatively: Thin film of fluid under a ceiling e.g. Yiantsios & Higgins (1989)



$$\frac{\partial h}{\partial t} + \frac{1}{3\mu}\nabla \cdot \left(h^3 \nabla(\rho g h + \sigma \nabla^2 h)\right) = 0 \qquad \nabla = (\partial_x, \partial_y)$$

Model Problem

- Evolution from an almost uniform film, no flux at ends

$$h_t + \frac{1}{3}\Big(h^3(h_{zz}+h)_z\Big)_z = 0$$

$$h' = h''' = 0 \quad \text{at} \quad z = 0, L$$

$$h(z,0) = 1 + 0.1\cos(\pi z/L)$$

- Solve numerically:

  Semi-implicit, adaptive grid-spacing and time-step

- Energy $E = \frac{1}{2}\int_0^L (h_z^2 - h^2)\,dz$ decreases monotonically

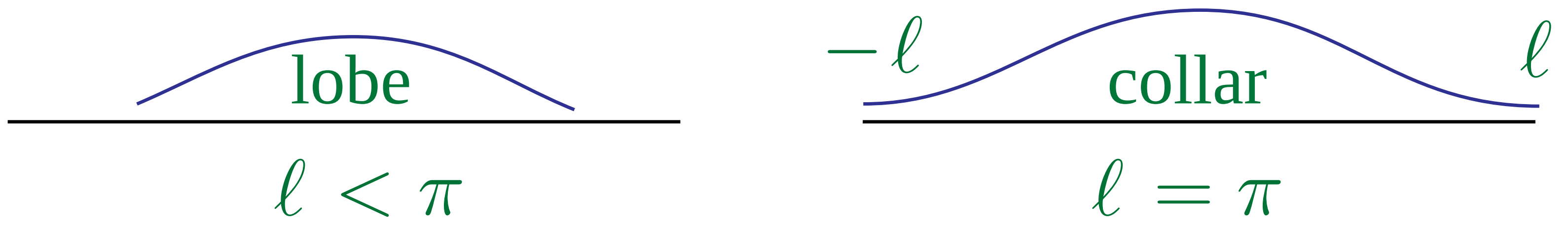
  at rate $dE/dt = -\frac{1}{3}\int_0^L h^3(h_z + h_{zzz})^2\,dz$

- Long-wave instability drives into a series of ‘collars’ and ‘lobes’

Collars & Lobes

- Equilibrium shapes have a constant pressure: $P_z = -(h_{zz} + h)_z = 0$

  e.g. $h = A[\cos z - \cos \ell]$ for $|z| \leq \ell$



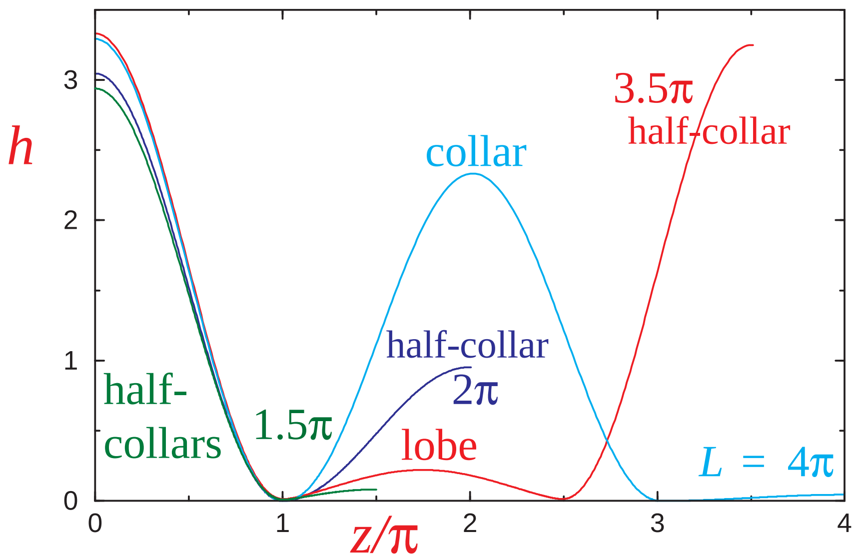
- 'Lobes' have $\ell < \pi$ and h approaches zero linearly at ends
- 'Collars' have $\ell = \pi$: $\quad h = A(1 + \cos z)$ and

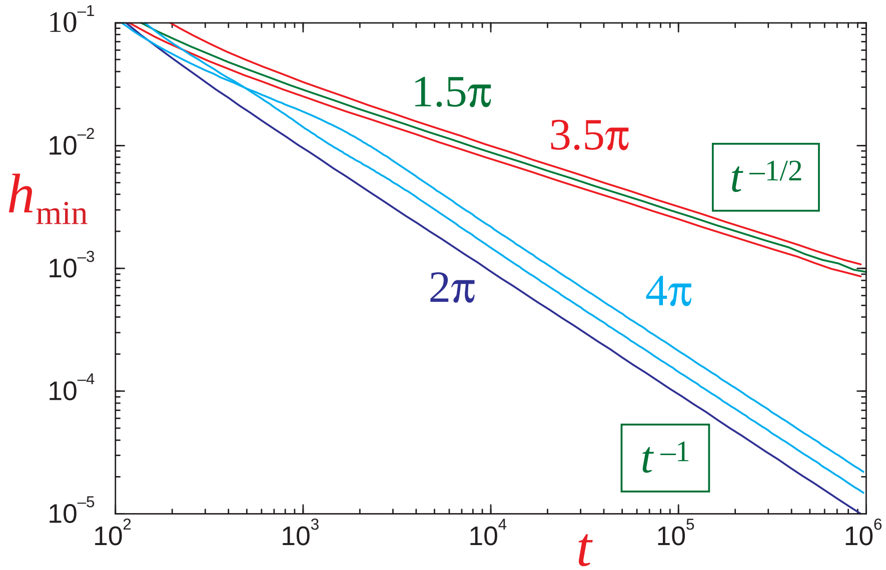
  h approaches zero quadratically at ends

  $\Rightarrow$ a collar can be matched to a uniform film on either side

Pinch-off on a Short Domain

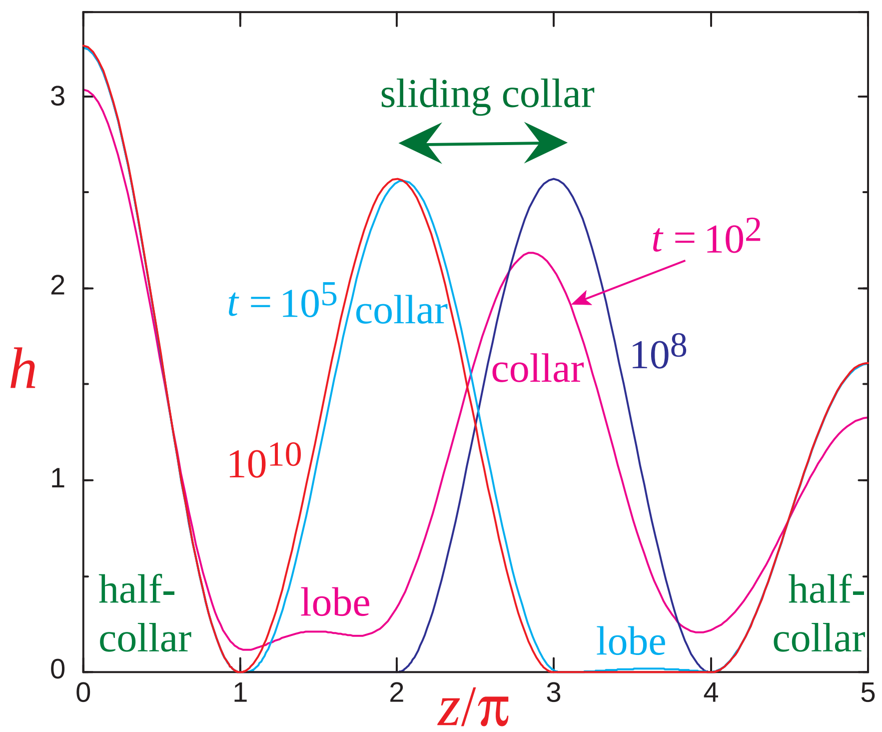
- How many collars will fit?

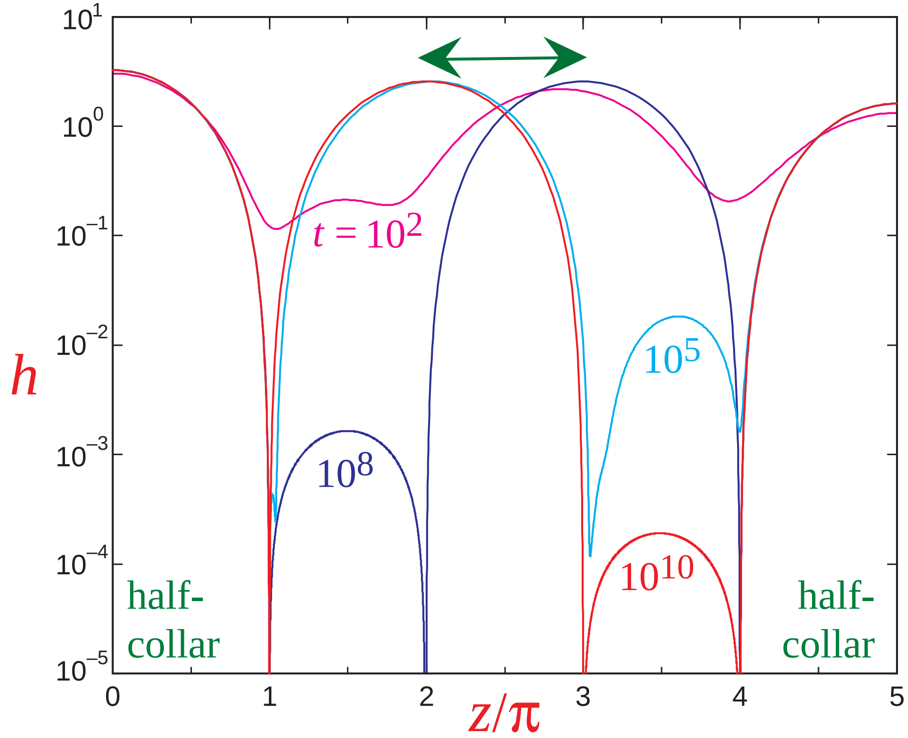




- Generic $t^{-1/2}$ behaviour for a lobe next to a collar (or half-collar)
  Flux $q \sim t^{-5/4}$ from lobe into neighbouring collar
  Described by similarity solution of Jones & Wilson (1978)
- Special t^{-1} behaviour for a collar next to a collar ($L = 2\pi, 4\pi$)
  since the flux $q \sim t^{-5/2}$ is out of, not through, the nip.
  New similarity solution

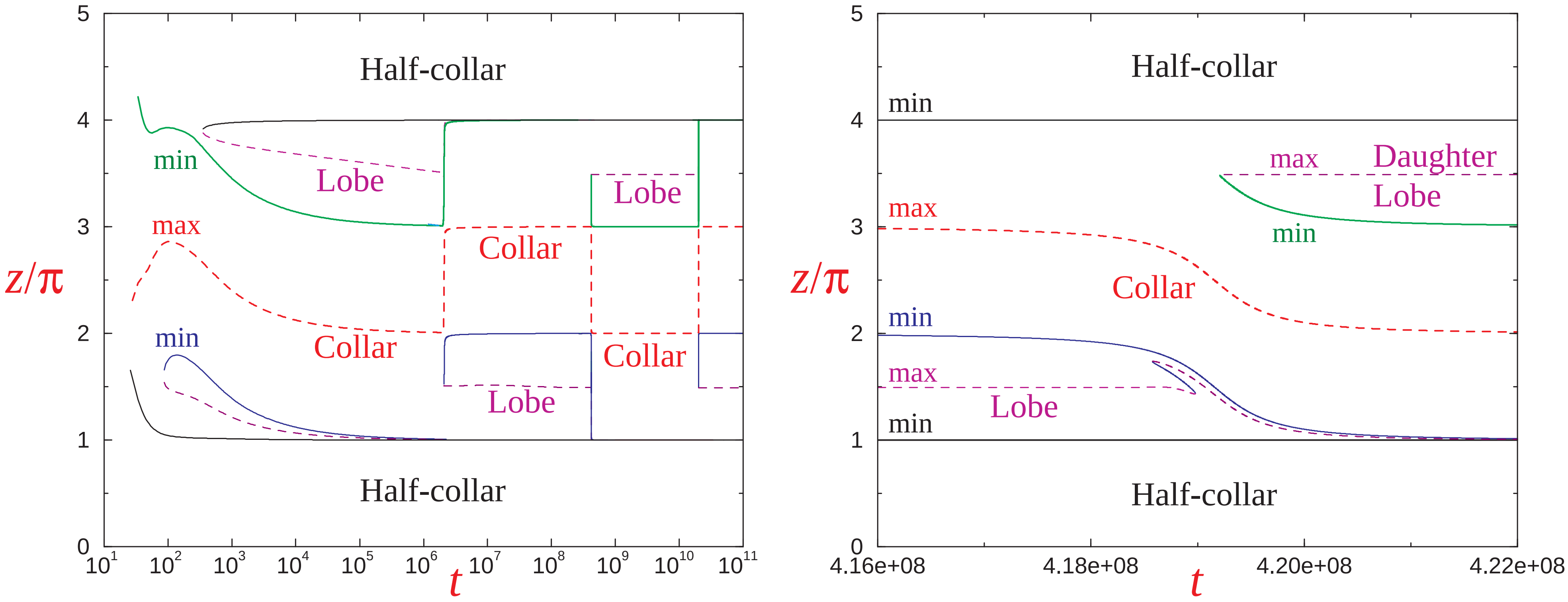
Sliding Collar on a Longer Domain $L = 5\pi$





- Central collar slides back and forth over much thinner lobe
- Lobe decreases in thickness by factor 10 each time
- No coalescence with end half-collars

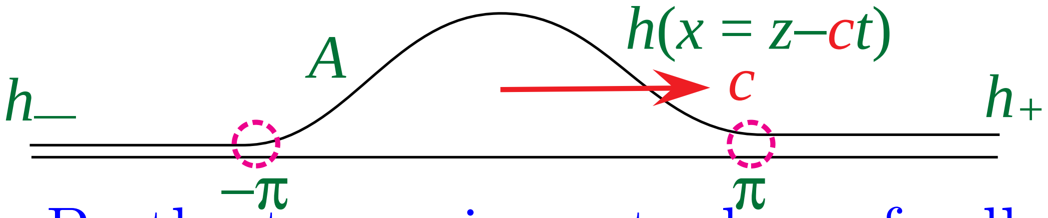
Kinematics of Motion – Maxima & Minima



- Relatively rapid sliding of collar
- Long periods where collar (almost) stopped
- Three regimes to understand
  - Sliding • Stopping • Starting (peeling)

Steady sliding over thin layer

- Steady translation over thin uniform layer



- Bretherton regions at edges of collar

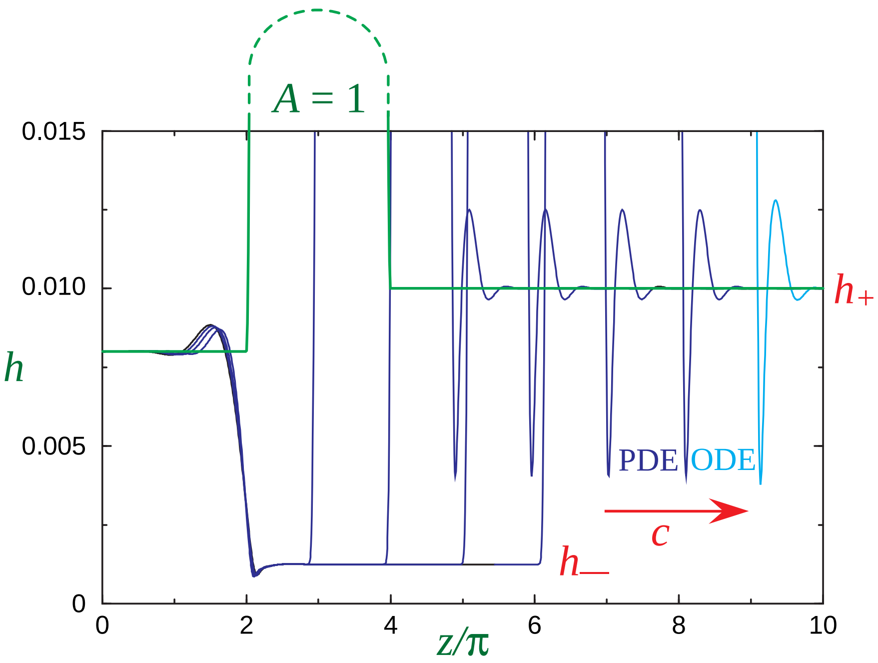
$$h = h_\pm H_\pm(\xi), \qquad H^3 H''' = H - 1$$

- Match Bretherton regions to collar
- ⇒ translation speed

$$c = \tfrac{1}{3}(Ah_+/5.59)^{3/2}$$

- New layer thickness $h_- = 0.115 h_+$
- Energy from growth of collar

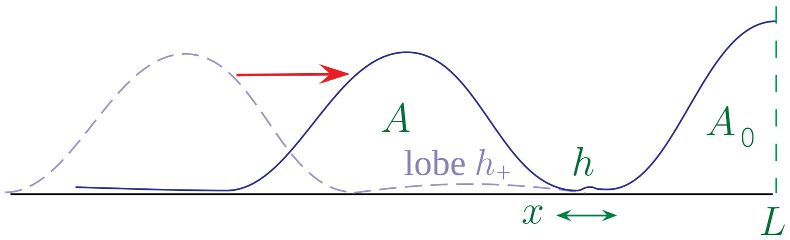
- Numerical confirmation



- Applies to quasi-steady sliding over lobe $h_+(z)$
- Explains factor of 10 decrease in daughter lobe

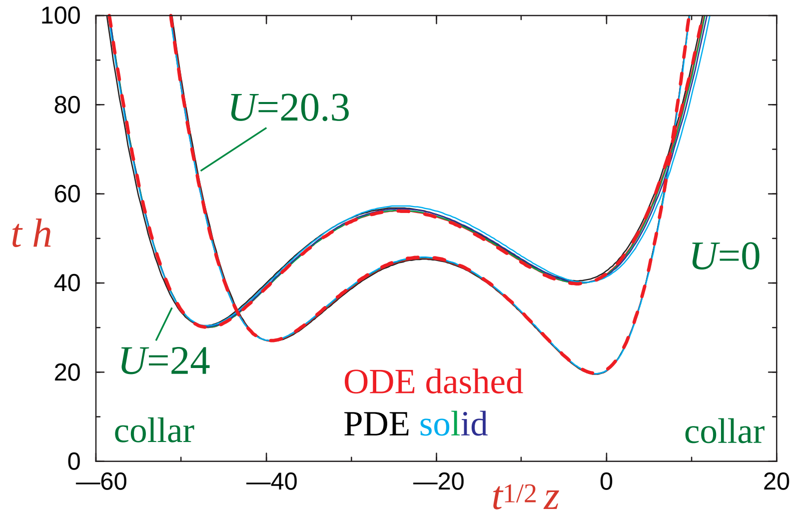
Stopping

- Stopping begins when leading Bretherton hits stationary collar



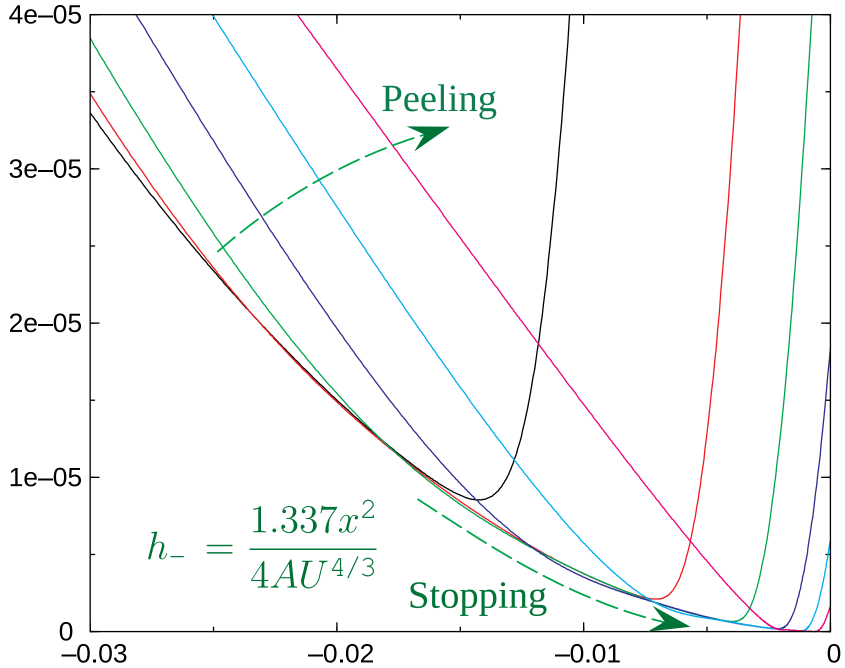
- Stopping continues with collar moving at velocity $\sim Ut^{-3/2}$ &

  Similarity solution with $h \sim t^{-1}$ between colliding collars



- Landau–Levich approx at rear: $h\_A = 1.337(Ut^{-3/2})^{2/3}$

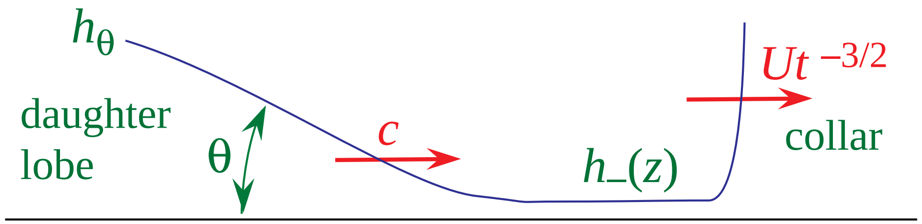
  $\Rightarrow$ moving collar lays down quadratic $h_- = 1.337x^2/4AU^{4/3}$



- Matching front and back determines U

Peeling/Starting

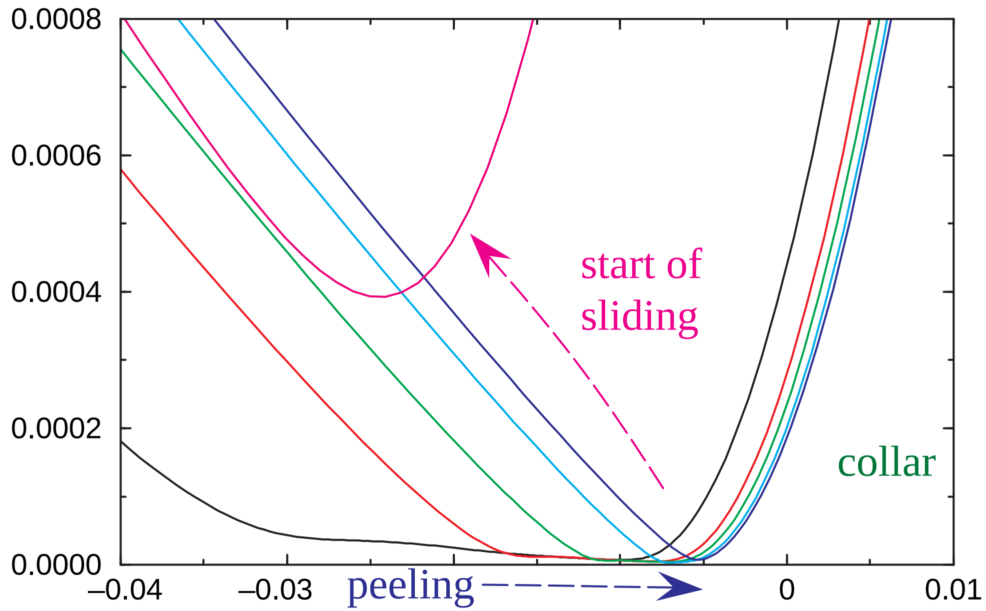
Daughter lobe relaxes toward equilibrium by peeling the quadratic



Similar to spread over a uniform layer (Tanner 1979) with velocity

$$c = \frac{\theta^3}{9 \ln(h_\theta / h_0)}$$

Peeling velocity c eventually exceeds stopping velocity $Ut^{-3/2}$ and sliding starts again



Decrease of Thickness Minima

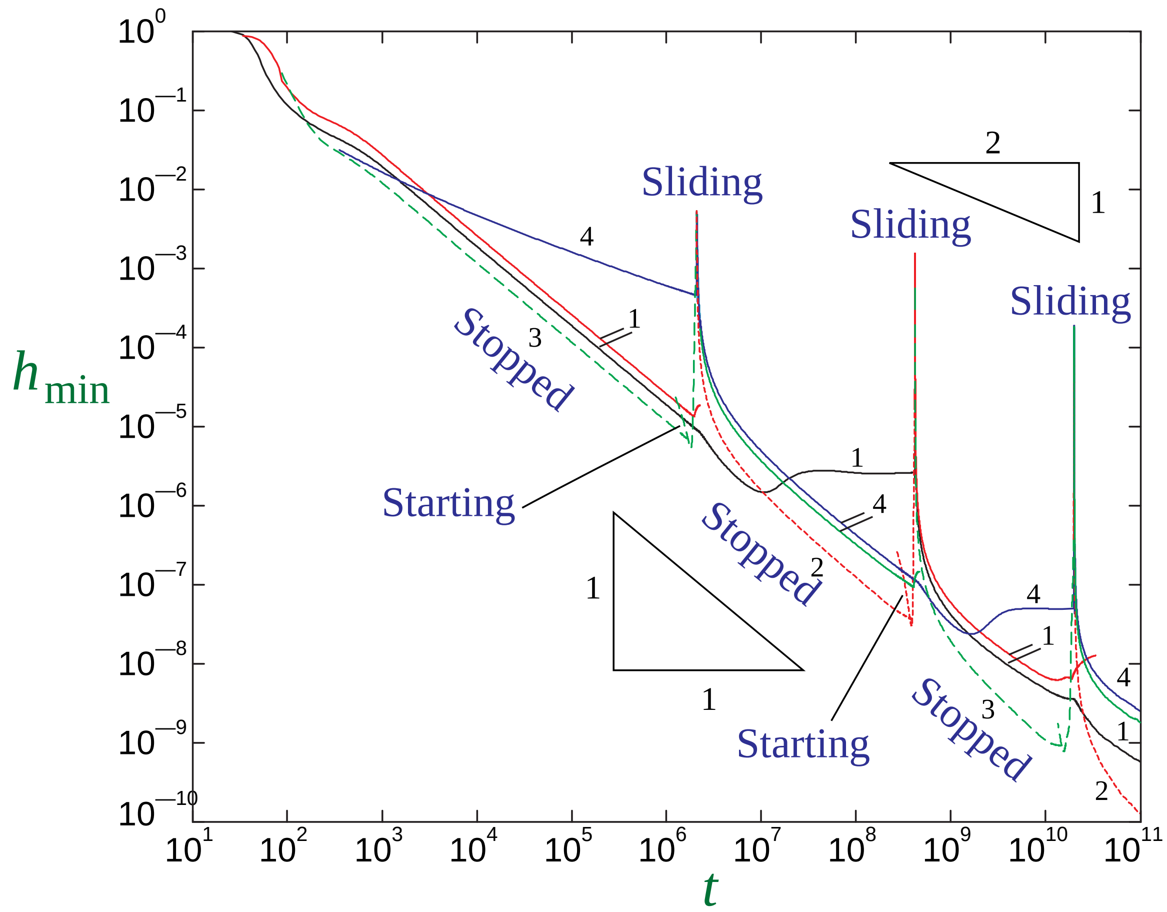
With lobe thickness $h_{\rm lobe}$:

sliding speed $\sim h_{\rm lobe}^{-3/2}$

sliding time $\sim h_{\rm lobe}^{-3/2}$

peeling speed $\sim h_{\rm lobe}^{-3}$

peeling time $\sim h_{\rm lobe}^{-2}$



Dominated by peeling time

$h_{\rm lobe}$ decreases by ≈ 10

when t increases by ≈ 100

$\Rightarrow h_{\rm lobe} \sim t^{-1/2}, \; h_{\rm min} \sim t^{-1}$

- **Oscillatory approach to pinch-off in geometric progression**

Full details in Lister et al., *J. Fluid Mech.*, **552** (2006)