Chaos in a 1-dim'l cardiac model

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$$\partial_t a = \partial_{xx} a - \partial_x a - \frac{1}{\Lambda}\int_0^x a(x',t)\,dx'$$

$$+ \sigma a - a^3$$

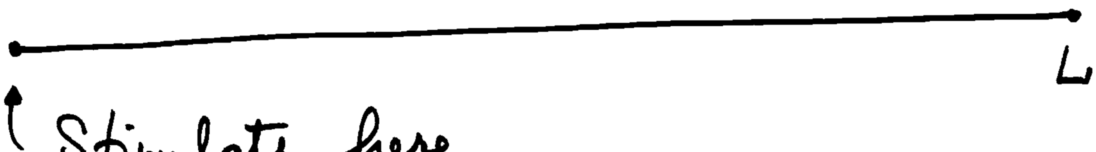
drives instab. as σ increases

nonlinear term that controls exp. growth of lin'z$^{\underline{n}}$

Echebarria-Karma PRE 2007
and PRL 2002

Electrical rhythms of heart

Periodic stimul^{n} at one end of cardiac fiber



Stimulate here

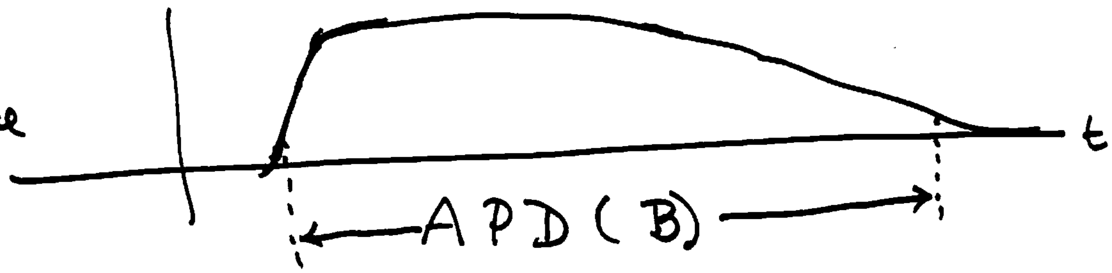
Period = B (for basic cycle length)

<u>Slow pacing</u> (large B):

Each stimulus generates an action pot'l (elevated voltage), a pulse that propagates down fiber.

Fixed time

Fixed space

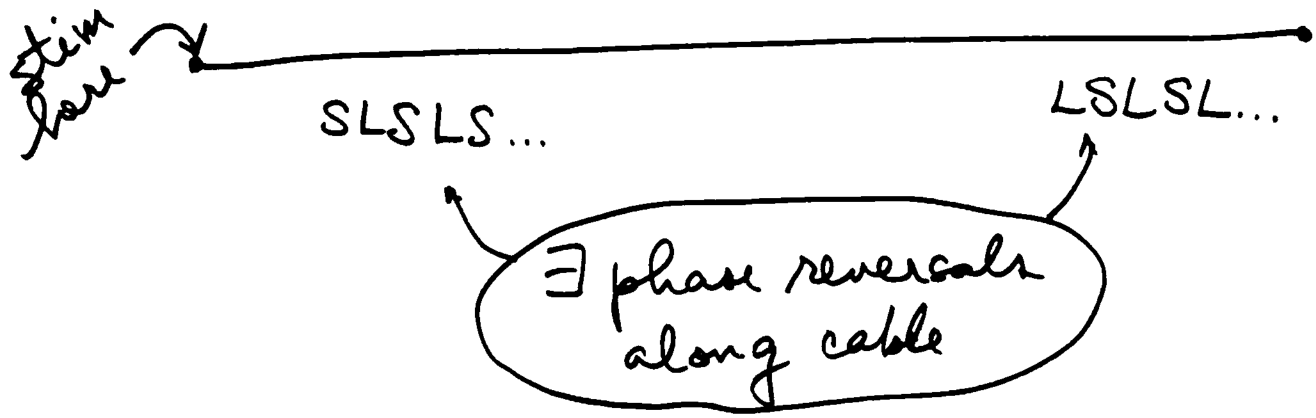


Rapid pacing (small B): alternans

- 0-dim ODE model: simple period-doubl. bif$^{\underline{n}}$ (no prop'g$^{\underline{n}}$)

  ➡ Even though pacing unif, APD's alternate: short, long, short, ...

- 1-dim PDE model: discordant alternans (w/ prop'g$^{\underline{n}}$)



Patterns may either

be stationary

move toward stim. site

Weakly nonlin. modulation eq^n:

Ansatz: $B = B_{crit} - \sigma$

$$APD_n(x) = A_{crit} - \text{Const}\cdot\sigma + (-1)^n a(x, nB)$$

t cont. time

$$\boxed{\partial_t a = \partial_{xx} a - \partial_x a - \frac{1}{\Lambda}\int_0^x a(x', t)\,dx' + \sigma a - a^3}$$

reflects fact that speed depends on local APD

$$\partial_t a = \partial_{xx} a - \partial_x a - \frac{1}{\Lambda} \int_0^x a(x', t)\, dx' + \sigma a - a^3$$

Bif$^{\underline{n}}$ analysis: $a \equiv 0$ is sln for all σ, it's stable for $\sigma < \sigma_{crit}$.

Unstable modes

Stationary (1)
Traveling (many)

Which mode first?

large Λ
small Λ

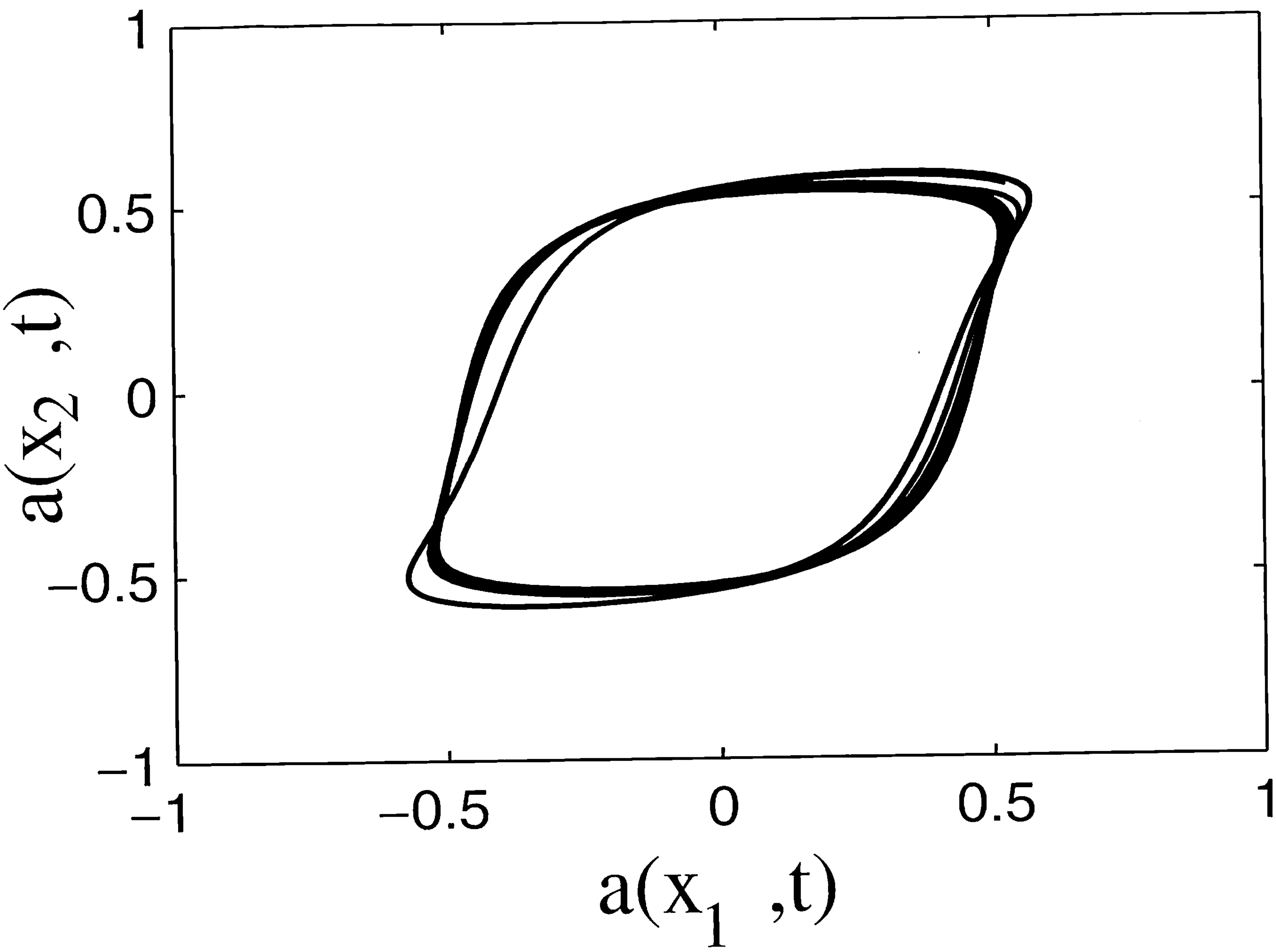
which mode bif's for smaller σ?

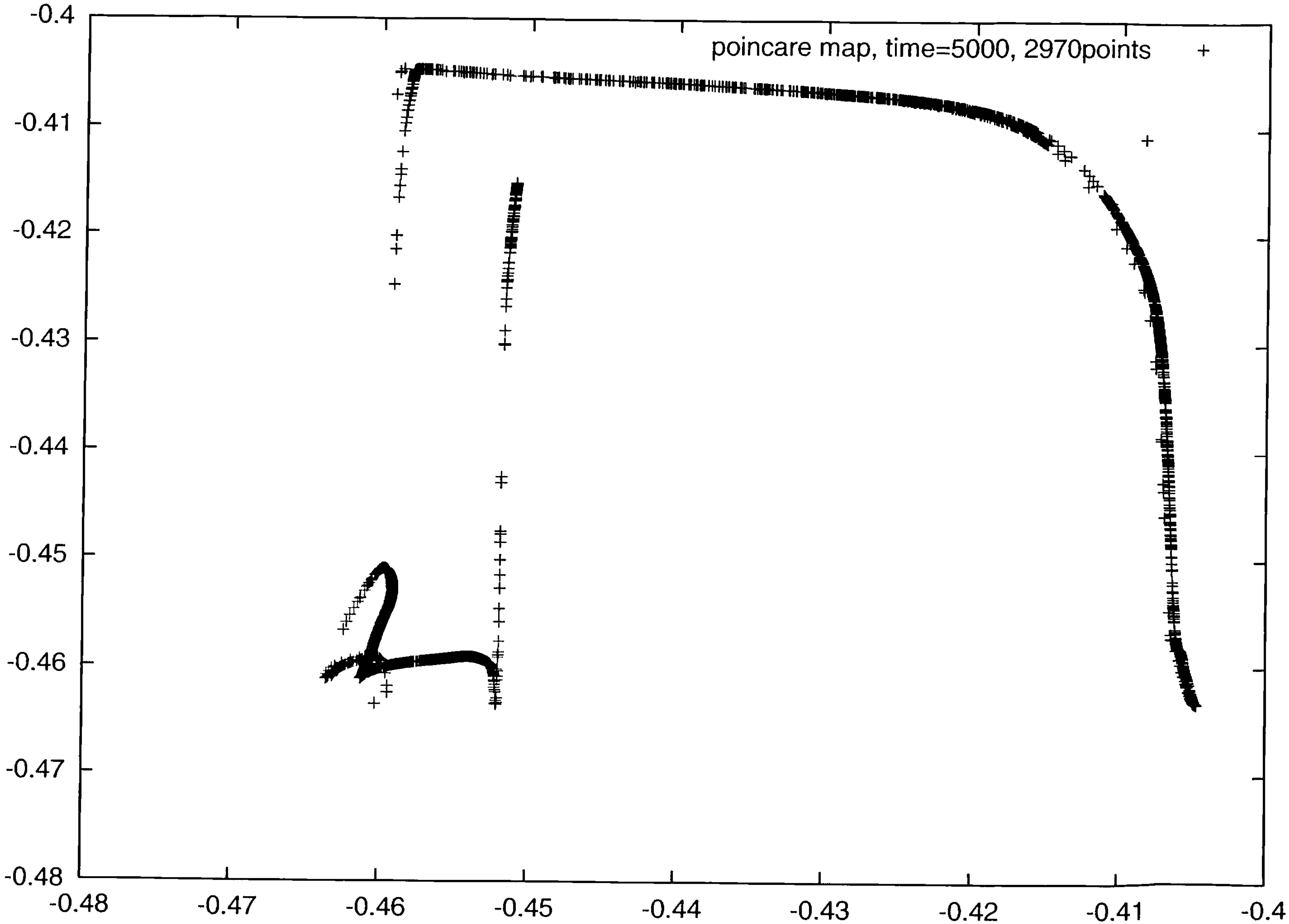
Intermediate $\Lambda = \Lambda_{crit}$:

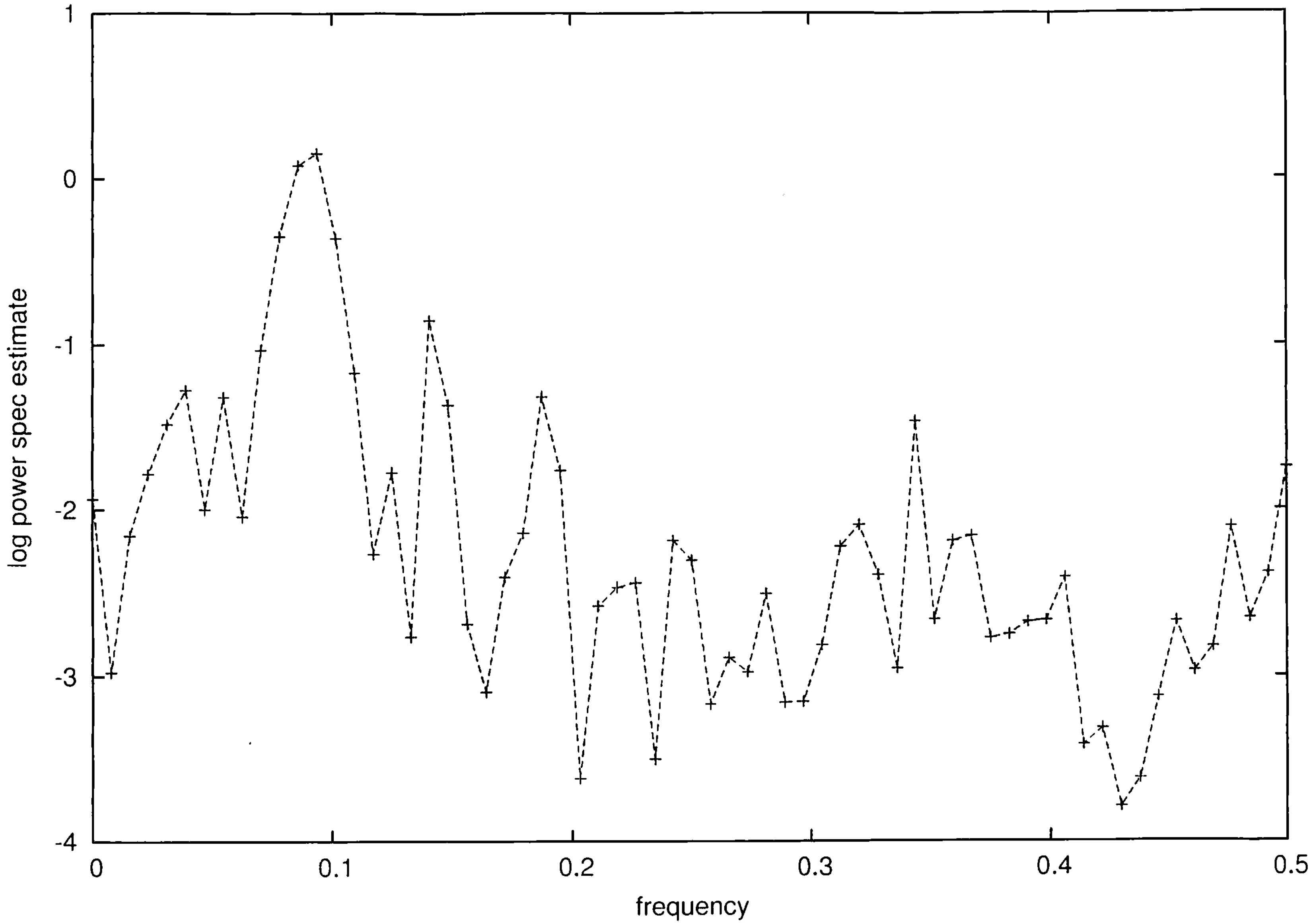
codim 2 bif$^{\underline{n}}$
Hopf + s.s. bif$^{\underline{n}}$ occur for same σ

Guckenheimer + Holmes, Ch. 7:

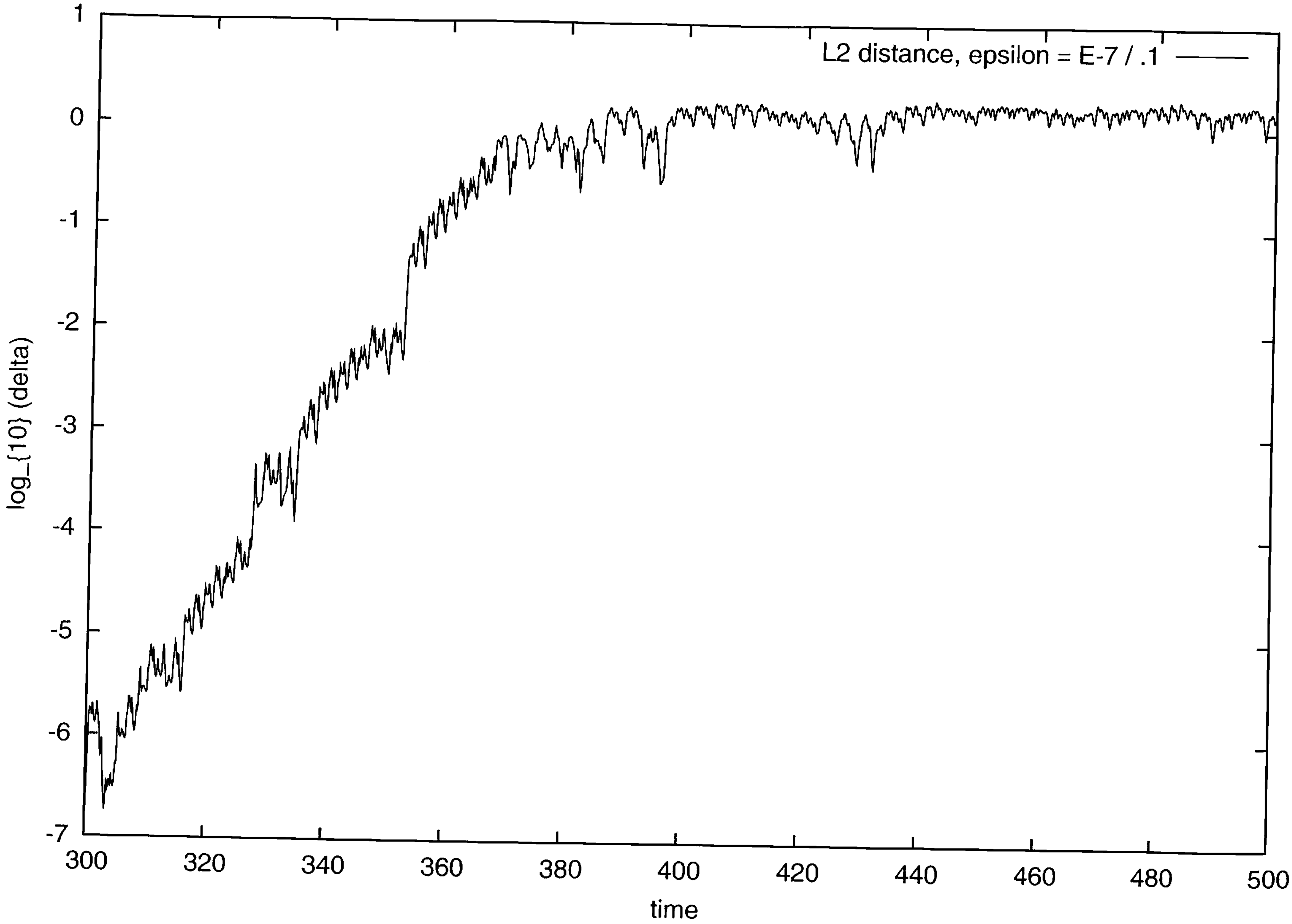
Generically, expect chaotic sln for certain param values close to degenerate bif$^{\underline{n}}$.

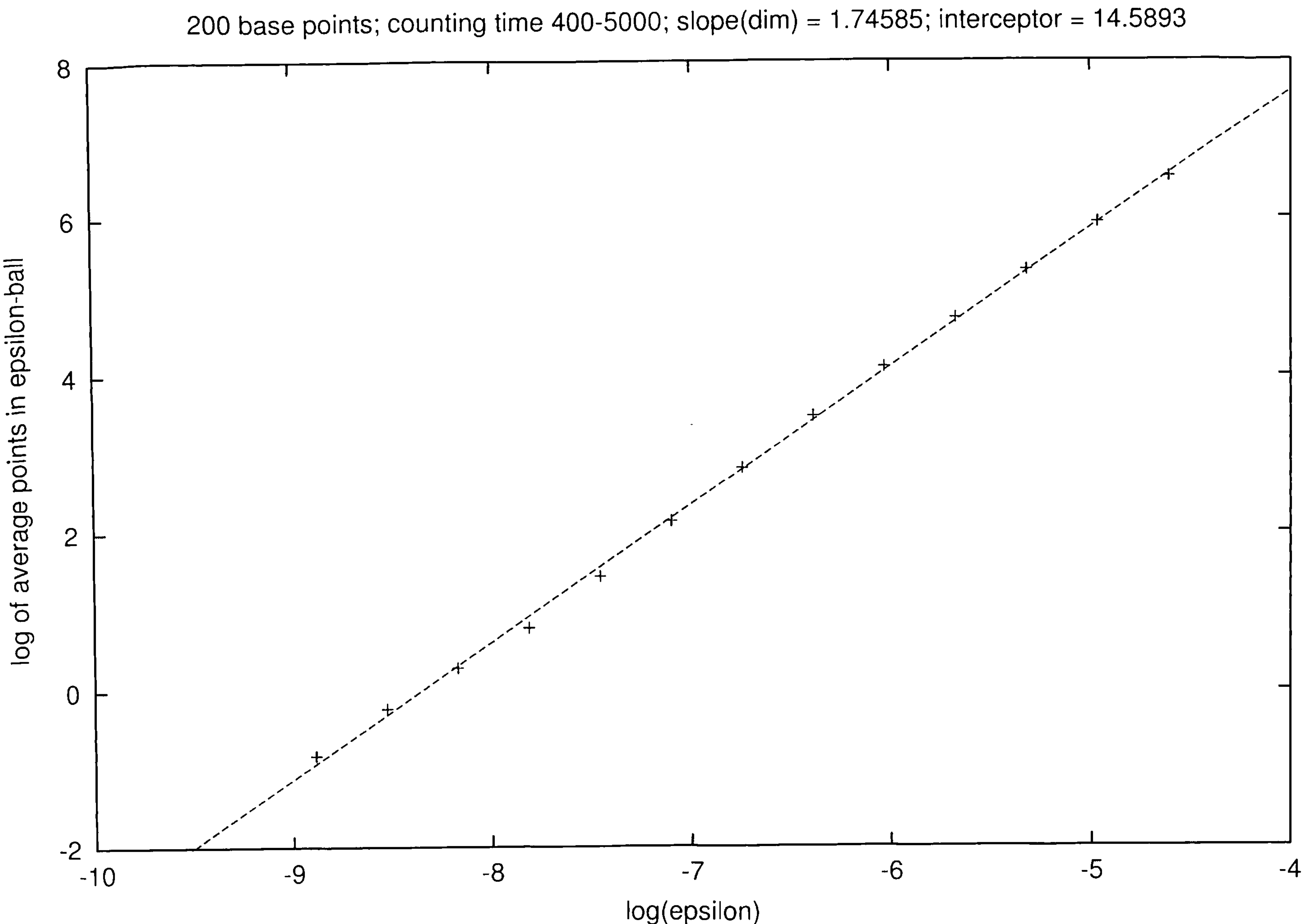

1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
a(x_2 ,t)
a(x_1 ,t)


poincare map, time=5000, 2970points
-0.4
-0.41
-0.42
-0.43
-0.44
-0.45
-0.46
-0.47
-0.48
-0.48
-0.47
-0.46
-0.45
-0.44
-0.43
-0.42
-0.41
-0.4



log power spec estimate
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
frequency


L2 distance, epsilon = E-7 / .1
log_{10} (delta)
1
0
-1
-2
-3
-4
-5
-6
-7
300
320
340
360
380
400
420
440
460
480
500
time


200 base points; counting time 400-5000; slope(dim) = 1.74585; interceptor = 14.5893
log of average points in epsilon-ball
log(epsilon)
8
6
4
2
0
-2
-10
-9
-8
-7
-6
-5
-4

1C