Asymmetries in a disconnecting air bubble: Up/down & around

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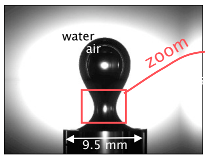
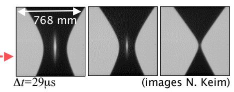
Familiar phenomena: bubble breakup





inertial implosion

singularity (interface velocity diverges) at time $t=t_b$ as radius $h_{min}\to 0$

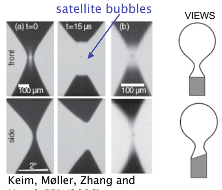
Many studies on how minimum radius evolves: Longuet-Higgins et al, JFM (1991); Oguz et al, JFM (1993); Burton et al, PRL (2005); Keim et al, PRL (2006); Bergmann et al, PRL (2006); Gordillo et al, PRL (2005); Eggers et al, PRL (2007); Thoroddsen et al, Phys. Fluids (2007); Duclaux et al, JFM (2007)

This system has rich *memory* of initial state

slight tilt of nozzle → visible asymmetry at breakup

initial state remembered at breakup

conflicts with idea that dynamics near singularity is *universal*



Keim, Møller, Zhang and Nagel, PRL (2006)

more extreme distortions: bursts of air through slot nozzle

low speed

high speed

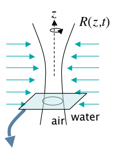
What is the memory mechanism?

consider ideal system: slender, axisymmetric neck

flow in water: irrotational $\vec{u} = \vec{\nabla}\phi \rightarrow \nabla^2\phi = 0$

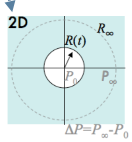
→ cross-sections decouple

Longuet-Higgins et al, JFM (1991); Oguz et al, JFM (1993)



poltential flow: $\phi(r,t) = Q(t)\ln r \longrightarrow u_r(r,t) = \frac{Q(t)}{r}$

kinematic & dynamic boundary conditions determine evolution of void radius $R(t)$.



interface moves with normal velocity $\nabla\phi\cdot\mathbf{n}|_R = \dot{R} \rightarrow u_r(r,t) = \frac{R\dot{R}}{r}$

unsteady Bernoulli eqn. energy/volume conserved $P_\infty - P_0 = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}(\vec{\nabla}\phi)^2\right)\Big|_{R(t)}^{R_\infty}$

$\Delta P = P_\infty - P_0$

Dynamics of symmetric implosion, $R(t)$, is Hamiltonian and integrable, thus conserving energy with perfect *memory*.

Implosion converts potential to kinetic energy

Hamiltonian, $H=E_0$: $H(R,P_R) = \left[\frac{P_R^2}{2M(R)}\right] + \Delta p\pi R^2 + \gamma\, 2\pi R$

KINETIC + POTENTIAL

Hamilton's equations of motion: $\dot{R} = \partial H/\partial P_R,\ \dot{P}_R = -\partial H/\partial R$

yield: $P = M(R)\,\dot{R}$, with $M = 2\pi\rho\, R^2 \ln(R_\infty/R)$, and an ODE for $R(t)$

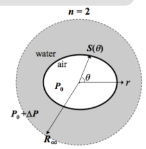
equivalent to the 2D Rayleigh-Plesset equation.

As $R\to 0$, kinetic energy dominates and must be constant, equal to E_0. Since $M(R)\to 0$ the interface velocity dR/dt must then diverge

Perturbing the system to destroy the symmetry excites shape vibrations

Apply general distortion to imploding circle:

$$R(t) \rightarrow S(\theta,t) = R(t) + \sum_{k=1}^{\infty} a_k \cos k\theta$$



Linearize about leading order collapse $R(t)$

Calculate corrections to flow (u_r, u_θ)

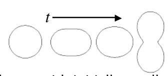
ODE for $a_n(t)$: $\ddot{a}_n + \left(\frac{2\dot{R}}{R}\right)\dot{a}_n + \left(\frac{\ddot{R}}{R}(1-n)\right)a_n = 0$

a_n oscillates with increasing frequency as $R\to 0$ ($t\to t_b$)

$$a_n(\tau) = a_0 \cos\left(\frac{\sqrt{n-1}}{2}\ln\tau + \phi_0\right)$$

$\tau = t_b - t$ time till breakup

Mode distribution a_k is *preserved* *memory*



shapes with initially small a_2, (rescaled by R)

Inevitably, the distortion takes over as mean radius R thins, but a_k=const.

More careful analysis: very weak growth in a_n

Use Hamiltonian form to rewrite $a_n(R)$ eqn:

$$R^2a'' + R\left(1 + \frac{1}{2\ln(R_\infty/R)}\right)a' + (n-1)\left(1 - \frac{1}{2\ln(R_\infty/R)}\right)a = 0$$

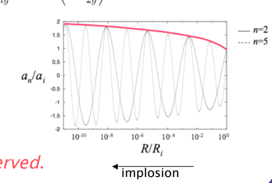
change variable ($y=\ln(R_\infty/R)$) $\frac{d^2a}{dy^2} - \frac{1}{2y}\frac{da}{dy} + (n-1)\left(1-\frac{1}{2y}\right)a = 0.$

introduce long-scale $Y=\ln(y)$

reduces to Kummer's eqn, solns

$a_0(\ln(R_\infty/R))^{1/4}$

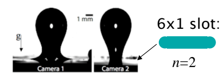
envelope (–) independent of n

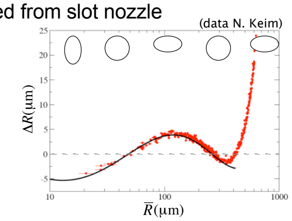


Mode distribution a_k remains *preserved*.

Vibrational mode excited in experiment!

bubble released from slot nozzle

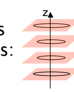




period & amplitude consistent with prediction

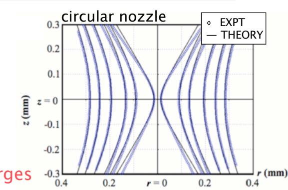
Axial direction: Evolution of 3D surfaces

build 3D surfaces by stacking layers:

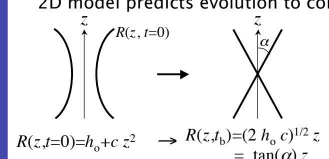


evolve surface using $H(z)=E_0(z)$, conserved

slight up/down asymmetry emerges

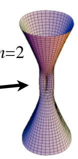


2D model predicts evolution to cones.



α related to initial curvatures. Bubble gently released from nozzle always has an axial curvature $c\approx 1/h_0$, → large (~45°) cones

3D shapes with non-circular cross sections have creases at minimum & form n lobes.



Unequal cone angles → motion of minimum

Work in progress:
In slender-body limit, using boundary integral formulation to calculate vertical flow near minimum.

Vertical motion of minimum is coupled to radial collapse, and depends on the difference of angles $\Delta\alpha$.

Conclusions

The disconnecting air bubble exhibits exceptional *non-universality*.

A perturbation to its initial state is remembered at breakup and alters the nature of the singularity.