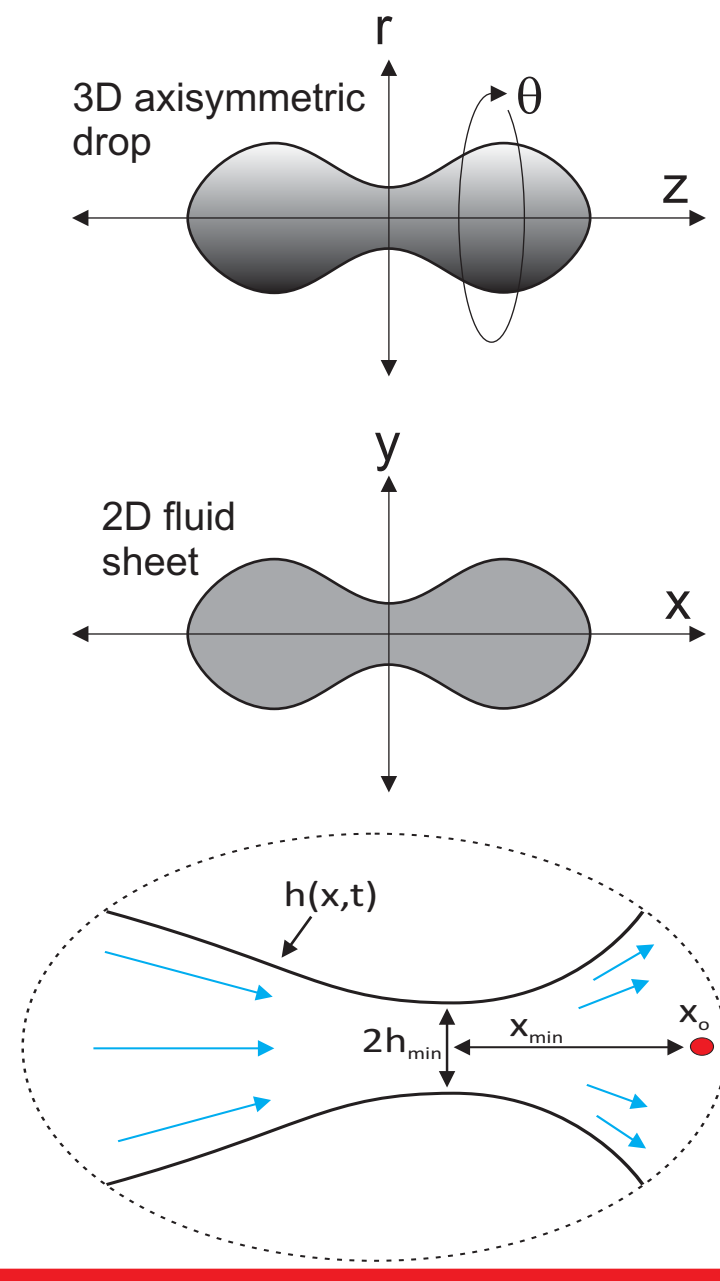


Dimensionality in Fluid Pinch-off: 2D vs. 3D



We are interested in the effects of dimensionality on fluid pinch-off singularities*. On the left is a cartoon of a generic axisymmetric 3D drop and a 2D fluid sheet. For free-surface flows, the most obvious difference between 2D and 3D is that the curvature of the interface (Laplace pressure) has two terms in 3D (axial and rotational), where in 2D there is only one curvature (axial).

The local profile of a fluid filament during pinch-off can be described by a function $h(x,t)=A(t_0-t)^\beta H(\eta)$, where h is the similarity variable $(x_0-x)/(\gamma(t_0-t)^\alpha)$. The exponents α and β are determined by an asymptotic balance of the relevant forces near the singularity. If either α or β are irrational numbers, this is known as "self-similarity of the second kind."

*J.C. Burton and P. Taborek. "2D Inviscid Pinch-off: An Example of Self-Similarity of the Second Kind." *Physics of Fluids* 19, 102109 (2007)

Calculation of Self-Similar Exponent

Inviscid pinch-off is dominated by an axial jet of velocity dx_{min}/dt . In 2D and 3D the Laplace pressure (curvature) should balance with the kinetic energy (velocity²). In 3D, there are 2 curvatures and 2 constraints giving $\alpha=\beta=2/3$, while in 2D there is only one curvature (κ_{min}). The other constraint is found by solving a nonlinear eigenvalue problem.

$$h(x,t) = A(t_0-t)^\beta H(\eta) \quad \left(\frac{d}{dt}x_{min}\right) \propto \kappa_{min} \rightarrow \beta = 4\alpha - 2$$

$$\eta = \frac{x_0-x}{\gamma(t_0-t)^\alpha}$$

$$\kappa_{min} = \frac{d^2h}{dx^2} \Big|_{min} \quad \kappa_{min} = \frac{1}{h_{min}} \quad \left(\frac{d}{dt}x_{min}\right) \propto \kappa_{min} \rightarrow \beta = 2 - 2\alpha$$

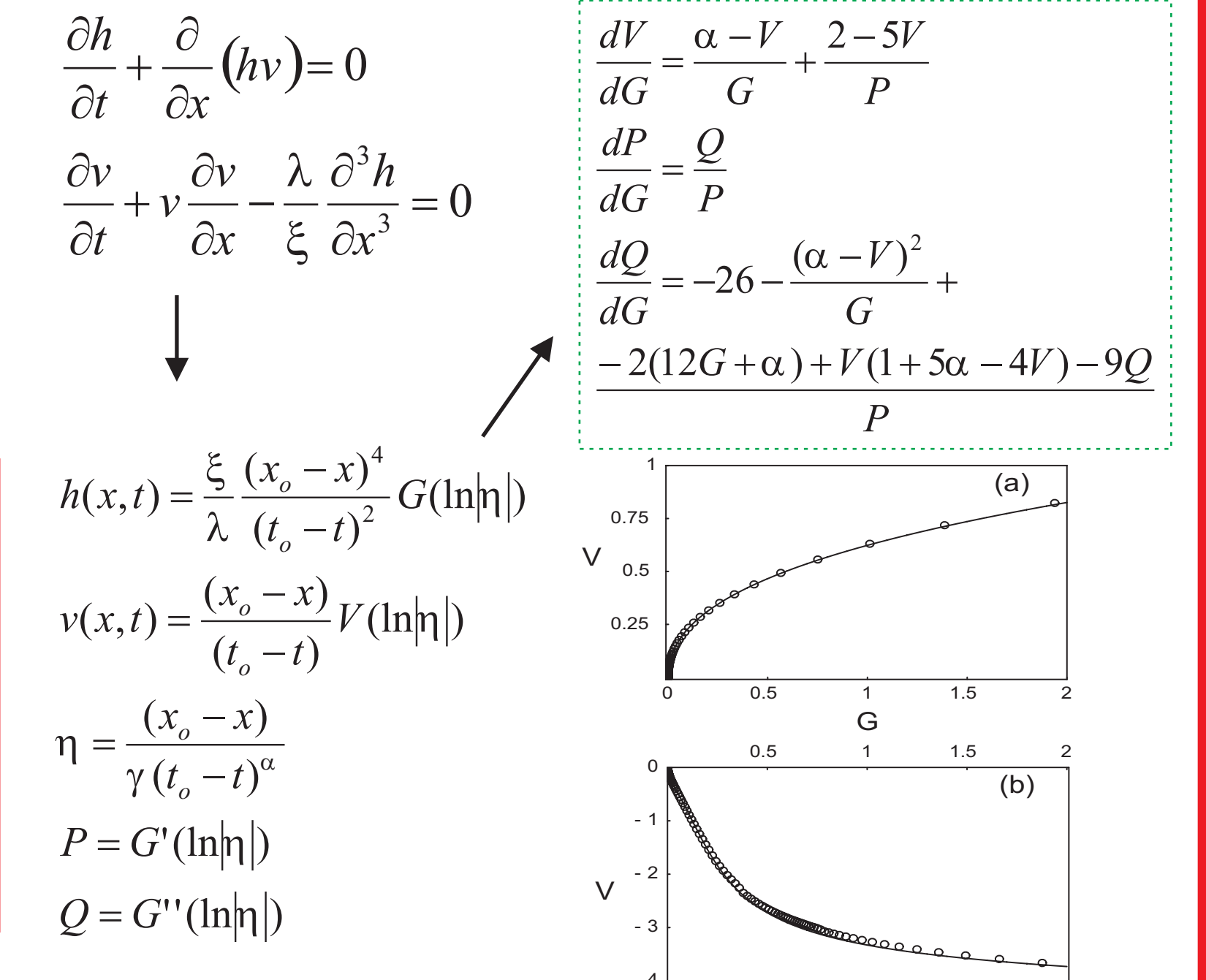
Quantity	β	Relation to α
h_{min}	0.7477 ± 0.0012	$4\alpha - 2$
$x_{min} - x_0$	0.6869 ± 0.0003	α
v_{min}	-0.3131 ± 0.0003	$\alpha - 1$
κ_{min}	-0.6261 ± 0.0006	$2\alpha - 2$

We solve the equations in the green box by "shooting" from $G=\infty$ and $G=0$ and matching the solutions. This happens at only one value of the similarity exponent α .

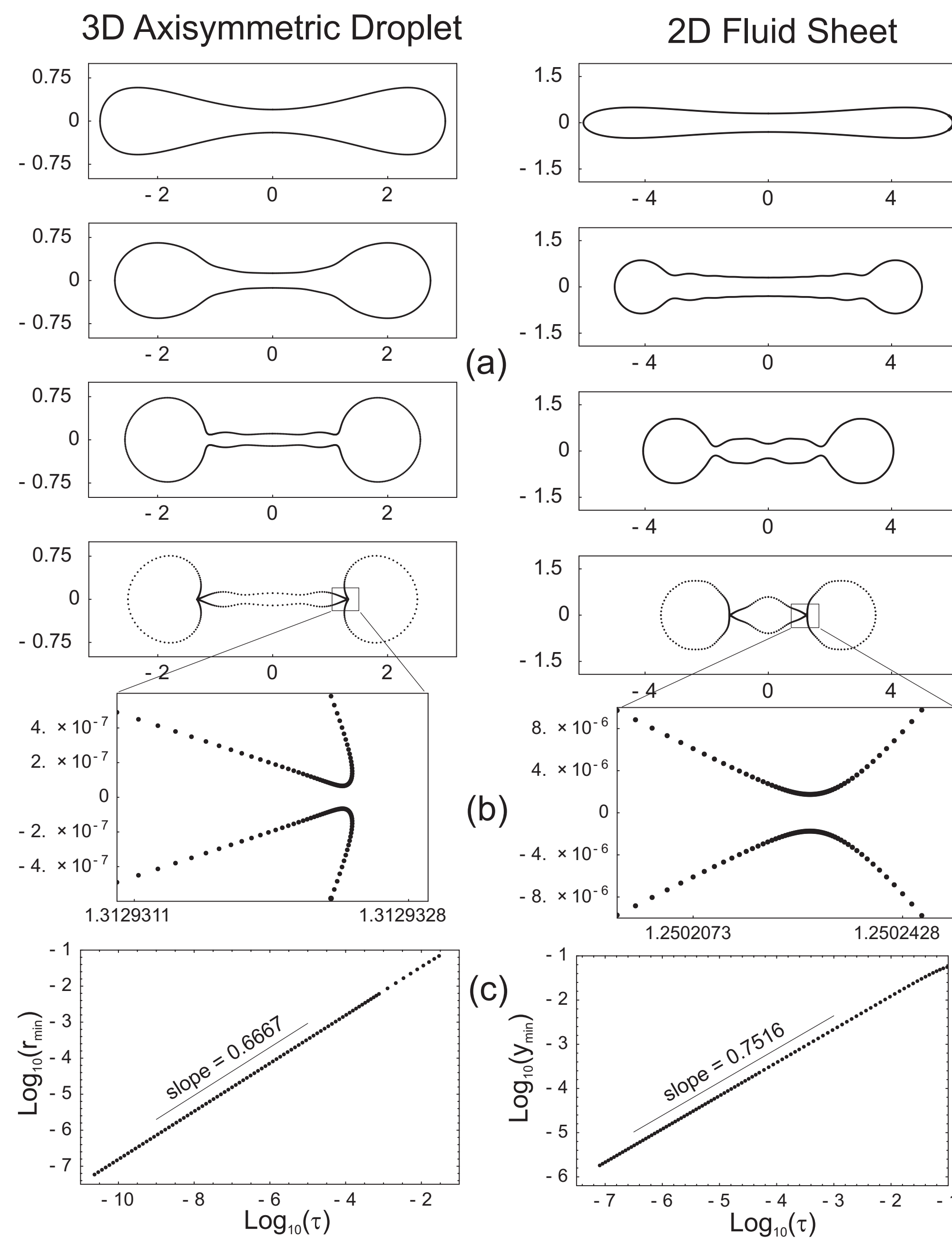
The graphs in the lower right consist of the data from a singularity formed in the original PDE's (top left), and the solid line is the solution calculated from the shooting method.

On the left are the power-law exponents of various quantities associated with the minimum neck width (h_{min}) of the pinch-off, such as velocity (v_{min}) and curvature (κ_{min}).

All exponents are analytically related to the similarity exponent α . The fact that they are irrational and not predicted by dimensional analysis indicates self-similarity of the second kind.

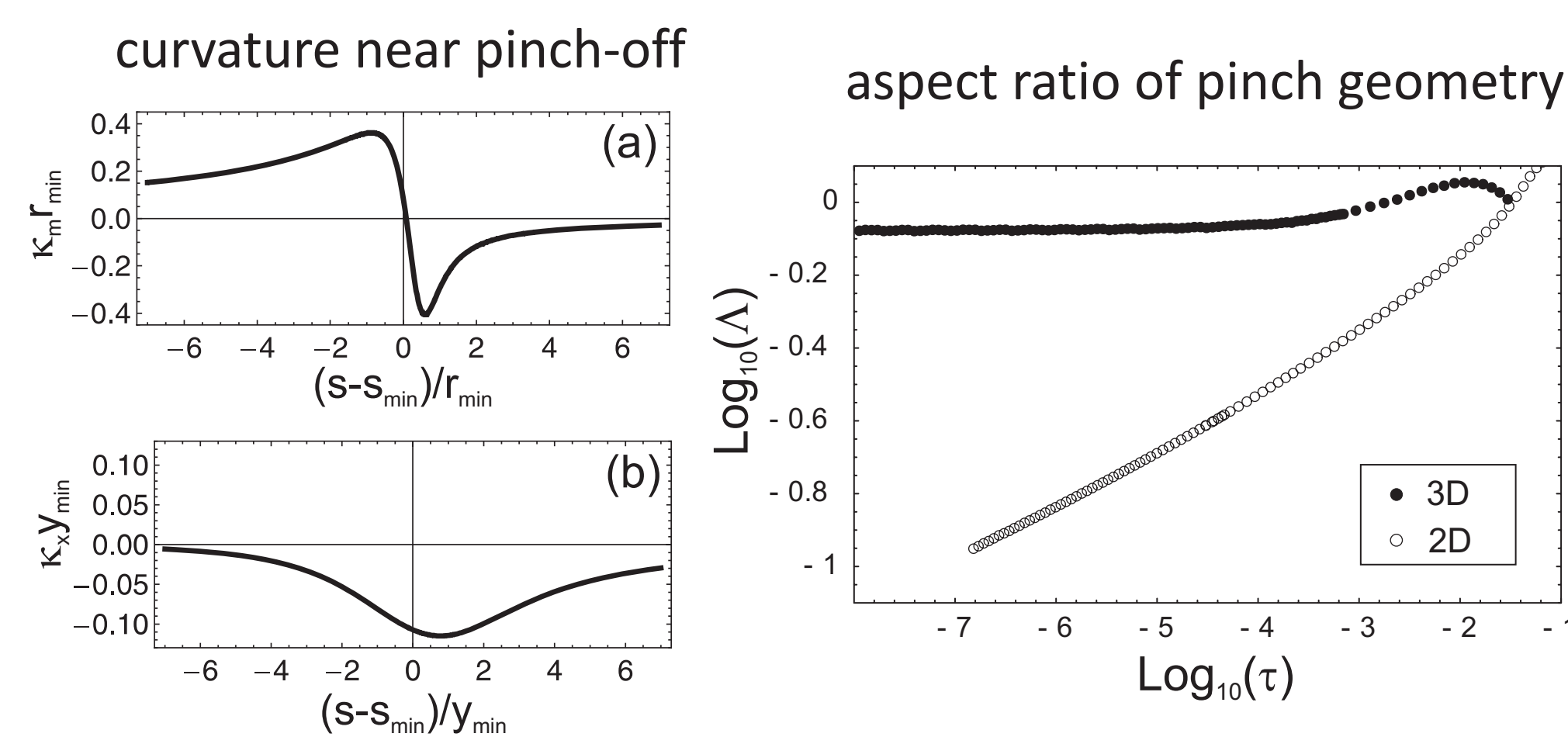


Inviscid Boundary Integral Simulations



The sequence of images on the left are from numerical simulations showing the evolution of an inviscid, 3D axisymmetric drop and an inviscid, 2D fluid sheet. Both simulations are started from rest and are only acted upon by their individual surface tension (3D) and line tension (2D). The size of the minimum neck width (r_{min} in 3D, y_{min} in 2D) is shown in the $\text{Log}_{10}\text{-Log}_{10}$ plots. The variable $\tau=t_0-t$ is the time remaining until the singularity. In 3D the scaling exponent is predicted by dimensional analysis $r_{min} \sim (\sigma/\rho)^{1/3} \tau^{2/3}$. Although the dimensional analysis predicts a $2/3$ exponent for 2D as well, the simulations show that this is not the case.

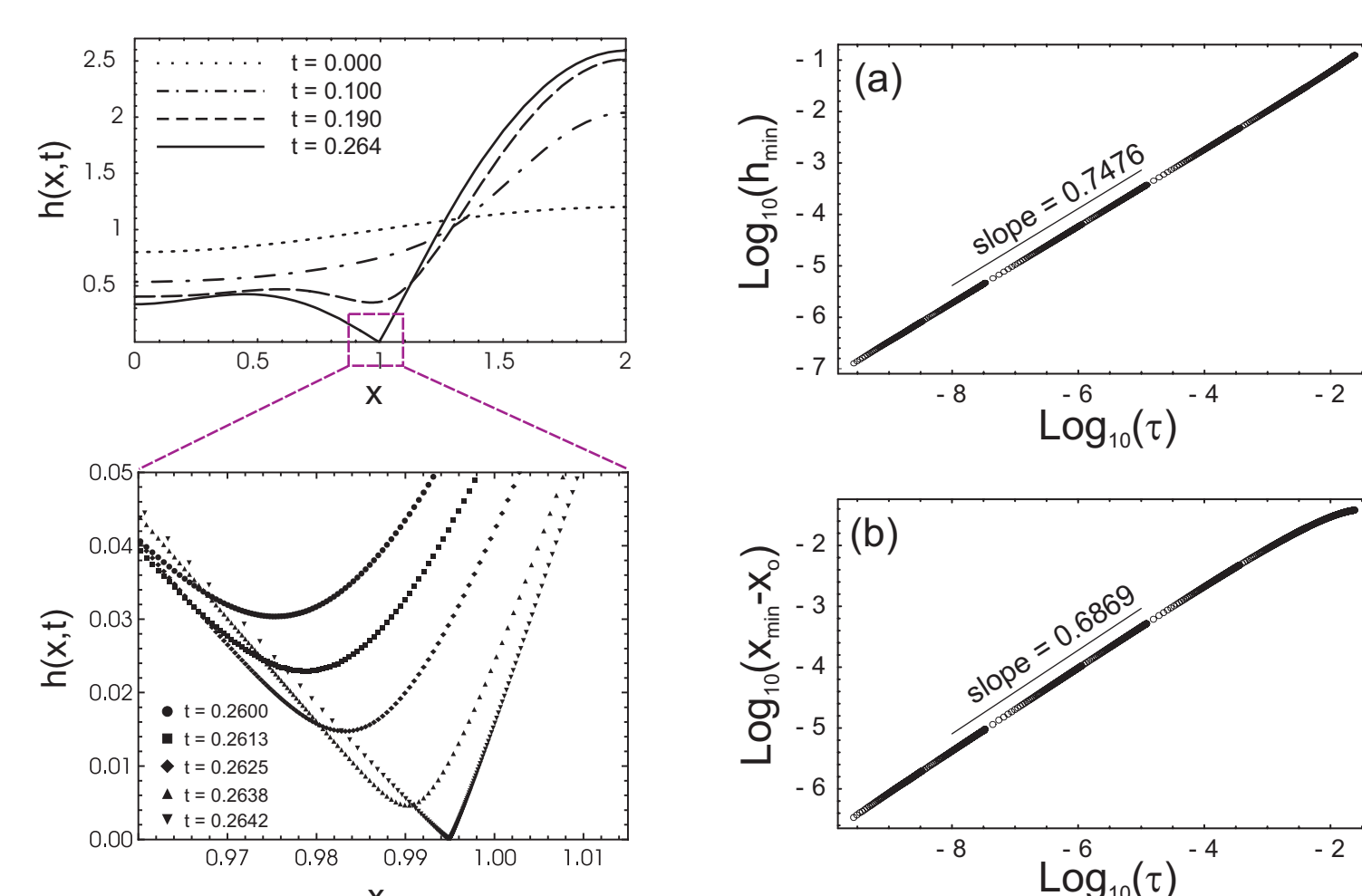
The graphs below to the left show the curvature (2D) and mean curvature (3D) near the singularity for each simulation. In 3D, the mean curvature changes sign near the pinching region, so that the Laplace pressure in the drop changes sign. A plot of the aspect ratio Λ of each pinching geometry shows that the Λ remains constant in 3D, but that the pinch-off becomes asymptotically "slender" in 2D.



slender description of 2D inviscid pinch-off

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hv) = 0$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} - \frac{\lambda}{\xi} \frac{\partial^3 h}{\partial x^3} = 0$$



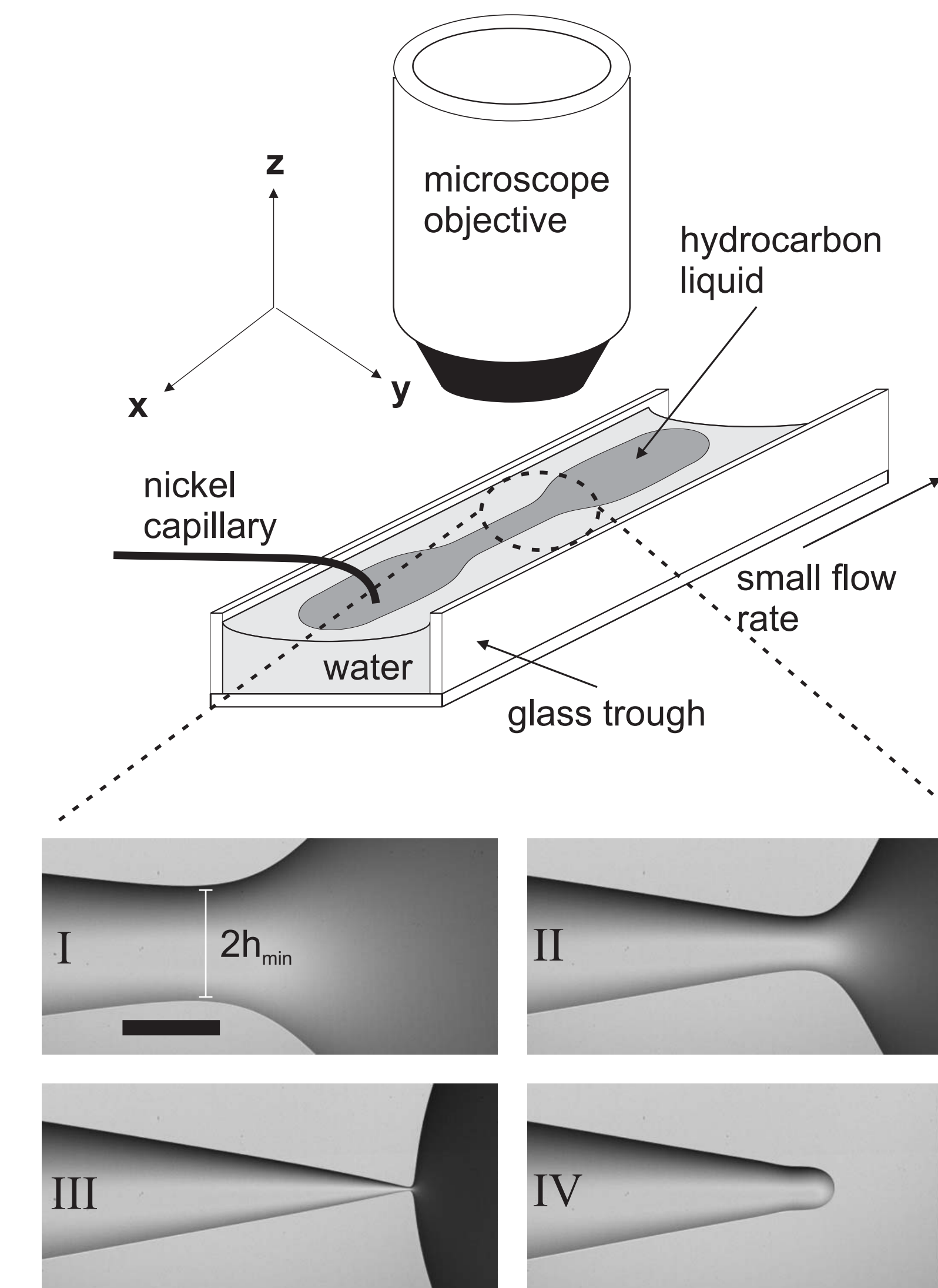
Because the slender asymptotic nature of the 2D pinch-off, the full equations can be simplified to a coupled set of partial differential equations where $2h(x,t)$ is the width of the fluid filament and $v(x,t)$ is the axial velocity.

The graphs to the right of the equations show a typical solution where a singularity is formed. The graphs on the immediate left show the position and width of the neck minimum, which both scale with different exponents.

Quasi-2D Experiment: Liquid Alkane Lenses

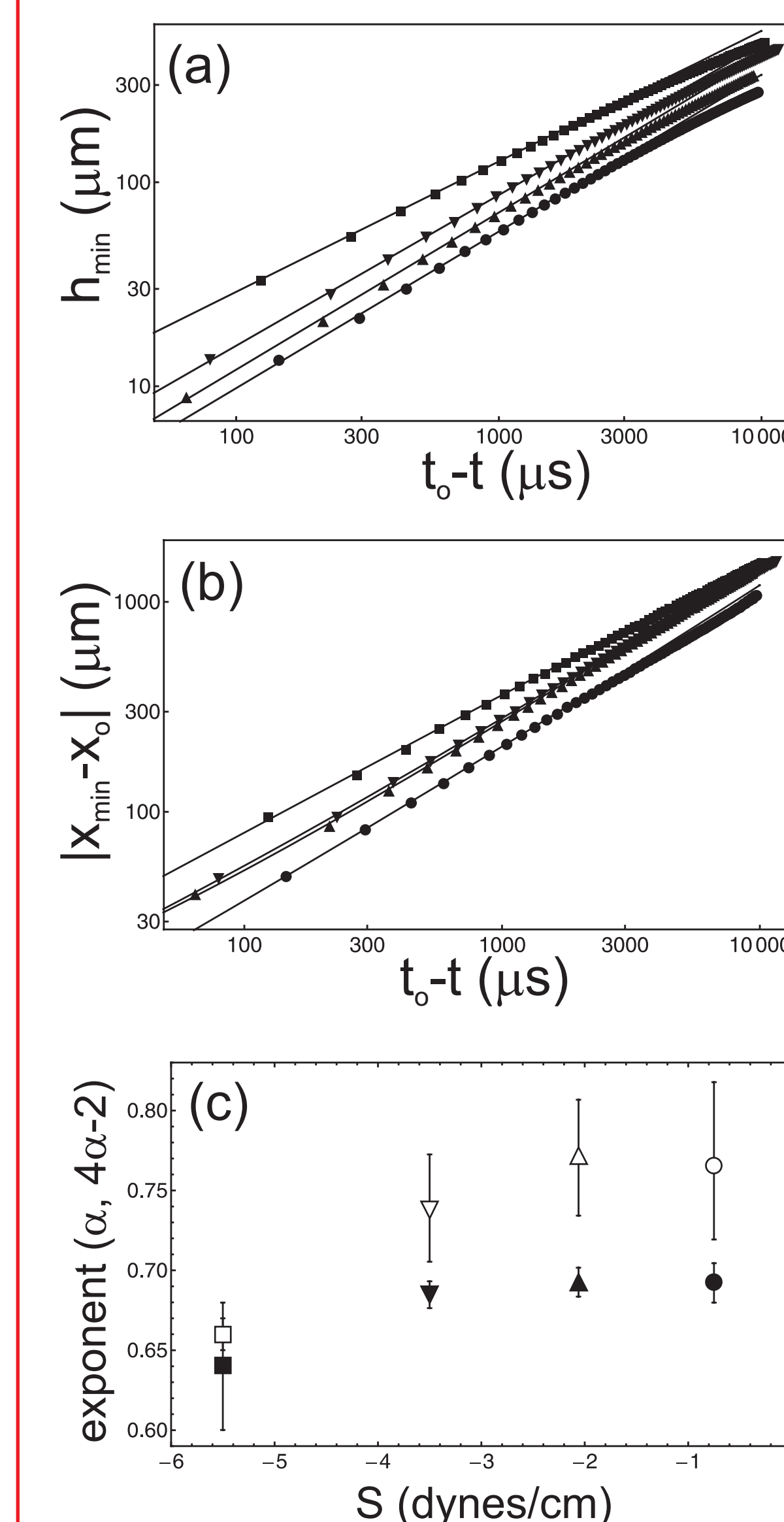
We have found a simple, quasi-2D experimental system composed of floating liquid lenses which displays the properties of a 2D inviscid fluid.

The cartoon below shows the experimental setup, and the images are taken from a high-speed video showing the pinch-off of a large nonane lens. The scale bar is 0.5mm, and the time between frame I and IV is 8.9ms.



The graphs on the left below show the position and width of the minimum neck for a dodecane (squares), decane (down triangle), nonane (up triangle), and octane (circles).

The lowest graph shows the power-law exponent as a function of the spreading coefficient. For thicker lenses (lower S), the exponents are closer to the 3D value of $2/3$, while for thin lenses the exponents are closer to the expected 2D values.



Below are successive profiles extracted from the pinch-off of an octane lens. If we scale these profiles with the theoretical exponents, we see a collapse onto a single curve.

