Speed Analysis of Traveling Wave Solutions of Some Nonlocal Evolutionary Equations

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Abstract

We will be concerned with the speeds of the traveling waves of the following nonlocal evolutionary equations arising from synaptically coupled neuronal networks:
(I)

$$u_t + u = \alpha \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) dy.$$

(II)

$$u_t + u = (\alpha - \beta u) \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) dy.$$

In these model equations, K is a kernel function representing synaptic coupling between neurons, H stands for the Heaviside step function. The parameters c, α, β, and θ are positive, each representing some biological mechanism.

Our main purpose is to study the speeds of the waves of these model equations. In particular, we will investigate the influence of synaptic couplings on the speeds by deriving lower and upper bounds for the speeds and by comparing the speeds of the waves of these models with different kernel functions.

Key words and phrases: traveling waves, speed index function, speed, influence of synaptic coupling function on speeds.

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