Lecture 13

Multidimensional Persistence

June 23, 2009
Image Patch Data: \( M[t, k] \)

- \( T \) - percentage
  - \( k \) - smoothing parameter for density

\( T < T' \)

\( M[T, k] \subseteq M[T', k] \)

\( M[T, k, \varepsilon] \subseteq M[T', k, \varepsilon'] \)

\( T < T', \varepsilon < \varepsilon' \)
$f : \mathbb{R}^n \to \mathbb{R}$

Excursion sets (topologically) give qualitatively more about $f$.

If $f$ is given by lists of pot values, then we need 5 scale parameters again:

$E(f, R, \varepsilon) = VR(E(f, P, R), \varepsilon)$

Two parameter system $(R, \varepsilon)$. 
Curvature example: c = curvature on it

If ellipse is given as point cloud, then need curvature + scale

Need 2 params to study this problem.
An equivalence analogous to the

Persistence v. Spaces $\iff$ graded $k[t]$-modules

Multipersistence v. Spaces $\iff$ multi-graded $k[t_1, \ldots, t_n]$-modules
\( \mathbb{N}^k \) - non-negative integers

\( \mathbb{N}^k \) - poset

\((n_1, \ldots, n_k) \leq (n'_1, \ldots, n'_k) \)

\( \Leftrightarrow \ n_i \leq n'_i \quad \forall i \)

\( \{ V_{\tilde{n}} \} \) \( \tilde{n} \in \mathbb{N}^k \)

\( L(\tilde{m}, \tilde{n}) : V_{\tilde{m}} \to V_{\tilde{n}} \)

where \( \tilde{m} \leq \tilde{n} \)
Satisfying $L(\vec{m}, \vec{n}) \cdot L(\vec{p}, \vec{n}) = L(\vec{p}, \vec{n})$ whenever applicable.

Morphism $\{ V_n \} \rightarrow \{ W_n \}$

family $\phi_n : V_n \rightarrow W_n \forall n$.

\[ \begin{array}{ccc}
V_n & \xrightarrow{\phi_n} & W_n \\
\downarrow L(\vec{m}, \vec{n}) & & \downarrow L(\vec{m}, \vec{n}) \\
V_\vec{m} & \xrightarrow{\phi_\vec{m}} & W_\vec{m} \\
\end{array} \]

Commutative
\[ M = \bigoplus_{n \in \mathbb{N}} M^n \quad \text{if} \quad M \in \mathbb{N} \]
If you know this, then

\[ f(n, m) = \text{count}(L(n, m)) \]

and each node in \( V \) and \( E \) is parsable.
Then one can construct a generalized fan code of dimension $R$. (Think about $R=2$)

Partition of $N^2$
If given a path $D$, then $D$ has a path of $N$.

Let $\sim$ be the other reduction predicate.

$m \sim n$ if $L([m, n])$ is an instance of $\sum_n$.
Possible Picnic

NO8

Second Natural Philosophy

Summing
Observation: Rank invariant is complete.

In the case $r=1$, $I$ is a

dim (V) = rank (L)

Sketch Proof:

Re construct bar code $I$ run
Can theชายสกุล seek invariant form here.

\[ \text{In this case, form } \text{m} \text{to } \text{m}. \]

<table>
<thead>
<tr>
<th>( \text{m} )</th>
<th>( \text{m} )</th>
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</thead>
<tbody>
<tr>
<td>( \text{n} )</td>
<td>( \text{n} )</td>
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\[ \text{Rank}(L(m,n)) \leq 4 \]
(First, the operator union involving)

\[ (n \cup m) > l \]

\[ (n \cup m) + 0 \]

Finish the first in set

\[ \text{invariant} \]

\[ \text{new record} \]

Beginning of draw

Stack invariant (shadow sun)

Mark places A and B

Decompose borders

\[ \text{in the places A and B} \]
\[ F \overset{\sim}{\longrightarrow} F^0 \overset{\sim}{\longrightarrow} M \overset{\sim}{\longrightarrow} 0 \]

Continued:

Let \( M \) be an \( n \)-graded \( \text{A}^n \)-module.

Use the definition of \( n \)-graded \( \text{A}^n \)-modules.

How to more broadly generalized balanced?
Unique up to iso.

Minimality exists at an age.

And such that P ranks (F/F)' is as smooth, M/F, M/I, as a graded space, R.

By (x_1, x_2, \ldots, x_n)

I = "my mathematical idea", ideal in A.

If F/F_0 \cong M/I is isomorphic

S \to F_0 \to M \to 0 \text{ is minimal}
\( \exists f, \text{ together with } M \equiv f/K \quad \iff \quad K \subseteq f, \text{ K neat and } f - K \)

By M, the subgroup generated a Sylow normalizer is contained in a Sylow subgroup of a centralizer of \( f \) which is maximal, up to isomorphism.

This means that there is a unique Sylow
An extension of submodules of \( F \) to \( F / o(K) \sim F_0 / K \cong M \) is a field isomorphic to \( K \). An extension of \( F \) to \( \text{Gal}(K \mid F) \).
"Geometric Invariant Theory"

on an affine algebraic variety (Grassmannians)

The action of an affine algebraic group graded modules are the graded under W/Work, follows that is o classes of
INFO M. USING GEOPAR PABLES

A WAY OF ENCODING ALL THE
ON A

2 \geq 2 \rightarrow \text{CLASSIFICATION OR PREDICTION}

\text{DISCRETE CLASSIFICATION}
\[ L^T(f) = \alpha \text{ when } n \text{ is the largest } \]


For a - \epsilon \pi [\theta, \theta], \exists \alpha \in \text{ such that }
Given a set \( \{T_1, T_2, \ldots, T_n\} \), we define \( \text{ideal generated term} \) \( I(T) \) as the \( \text{ideal generated term} \) under \( T \) and \( \text{ideal generated term} \) under \( \{T_1, T_2, \ldots, T_n\} \).
Given $f(x) \neq 0$ for all $x \neq a$, the function $g(x)$ is continuous when $a$ is not a point of discontinuity.

Hence $g(x)$ is a differentiable function if $g(x)$ is continuous and $f(x)$ is differentiable.

Furthermore, if $f(x) = 2 \sqrt{x} + 2$, then $g(x) = \sqrt{2 \sqrt{x} + 2}$, where $u = 2 \sqrt{x} + 2$.

And $g(x) = e^{u}$, thus $g(x) = e^{2 \sqrt{x} + 2}$.

Puichbarg: Every I need is a problem of

...
A student's approach or a student's degradation
can result in college, affecting the scholar to
come back, and it can be curriculum
where curriculum is too thorough, too
innovative. If A = A_1 + A_2 + AE

If this were reasoning for my whole

Schrörer Alg. $\Rightarrow$ Suppose $m_{11}, \ldots, m_s \in \mathbb{P}(\text{free})$

and suppose \{ $m_{11}, \ldots, m_s$ \} is a Gröbner basis.

Then there is an explicit formula

for the set of relations $(a_{11}, \ldots, a_s)$ s.t.

$\sum a_i m_i = 0$. Gröbner basis theory

permits the evaluation of kernels of

homomorphisms between free modules.
Local work of A. Semenovskaya, 1818.  

Algorithm is performed in a setting where the ideal program is finding. We are dealing with the space between the middle and another. We are interested in the navigation interface. After the - graded reading, the human and I decided on the next phase of grammar.