Recovering Sparsity in High Dimensions

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Abstract

We assume that we are in $I\!R^N$ with N large. The first problem we consider is that there is a function f defined on $\Omega := [0,1]^N$ which is a function of just k of the coordinate variables: $f(x_1,\dots,x_N) = g(x_{j_1},\dots,x_{j_k})$ where $j_1,\dots,j_k$ are not known to us. We want to approximate f from some of its point values. We first assume that we are allowed to choose a collection of points in Ω and ask for the values of f at these points. We are interested in what error we can achieve in terms of the number of points when we assume some smoothness of g in the form of Lipschitz or higher smoothness conditions.

We shall consider two settings: adaptive and non-adaptive. In the adaptive setting, we are allowed to ask for a value of f and then on the basis of the answer we get we can ask for another value. In the non-adaptive setting, we must prescribe the m points in advance.

A second problem we shall consider is when f is not necessarily only a function of k variables but it can be approximated to some tolerance ϵ by such a function. We seek again sets of points where the knowledge of the values of f at such points will allow us to approximate f well.

Our main consideration is to derive results which are not severely effected by the size of N, i.e. are not victim of the curse of dimensionality. We shall see that this is possible.