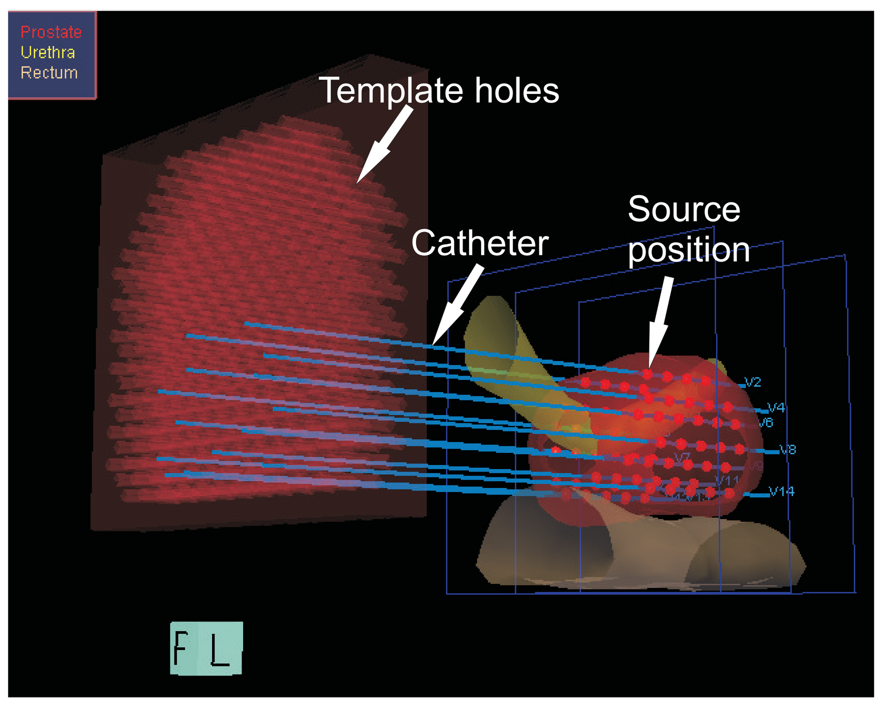
HIPO: A non-linear mixed integer constrained optimization algorithm for treatment planning in brachytherapy

A. Karabis[1], S. Giannouli[1], D. Baltas[2]

[1]Pi Medical Ltd, 40-42 Karneadou Str., 10676 Athens, Greece

[2]Dept. of Medical Phys. & Eng., Klinikum Offenbach, 63069 Offenbach, Germany

The Problem



Definition:

Find the optimum positions on the template for a predefined number of catheters as well as the optimum irradiation times for the stepping positions within them, in a way that the tumour will get the prescribed dose while the organs will be protected.

Variables:

The array of the holes of the template are the binary variables of the problem

$$\mathbf{b} = \{b_k\}_{k=1}^{N_{FCP}} \quad , \quad b_k = \begin{cases} 0, & \text{Catheter inserted in } k\text{-th hole} \\ 1, & \text{Catheter not inserted in } k\text{-th hole} \end{cases}$$

The irradiation times of the sources (i-th position in j-th catheter) are the continuous variables of the problem

$$\mathbf{t} = \{t_{jk}, j = 1,\dots,N_{SDP}^{k}\}_{k=1}^{N_{FCP}} \quad , \quad t_{jk} = x_{jk}^2$$

Constrains:

The constrains are concerning the number of catheters, the maximum time and other more complicated clinical requirements

$$\sum_{k=1}^{N_{FCP}} b_k - N_c = 0 \quad , -\sqrt{t_{\max}} \cdot b_k \le x_{jk} \le b_k \cdot \sqrt{t_{\max}} \quad , \quad \tilde{\mathbf{g}}(\mathbf{x},\mathbf{b}) \le 0$$

Objectives:

The objectives are penalizing overdose of organs and underdose of the target (tumour) using a transition function (Θ)

$$f^H(\mathbf{x},\mathbf{b}) = \sum_{j=1}^{N_{SDP}} \Theta\big(d_i(\mathbf{x},\mathbf{b}) - D^H\big)\big(d_i(\mathbf{x},\mathbf{b}) - D^H\big)$$

$$f^L(\mathbf{x},\mathbf{b}) = \sum_{j=1}^{N_{SDP}} \Theta\big(D^L - d_i(\mathbf{x},\mathbf{b})\big)\big(D^L - d_i(\mathbf{x},\mathbf{b})\big)$$

Where the dose at i-th point $d_i(\mathbf{x},\mathbf{b}) = \sum_{k=1}^{N_{FCP}} \sum_{j=1}^{N_{SDP}^k} b_k x_{jk}^2 \tilde{d}_{ij}$

It is a multi-objective optimization problem given that there are more than one organs to protect or even more than one targets Thus an aggregate objective function is used

$$F(\mathbf{x},\mathbf{b}) = \sum_{i=1}^{N_{OBJ}^L} w_i^L f_i^L + \sum_{j=1}^{N_{OBJ}^H} w_j^H f_j^H$$

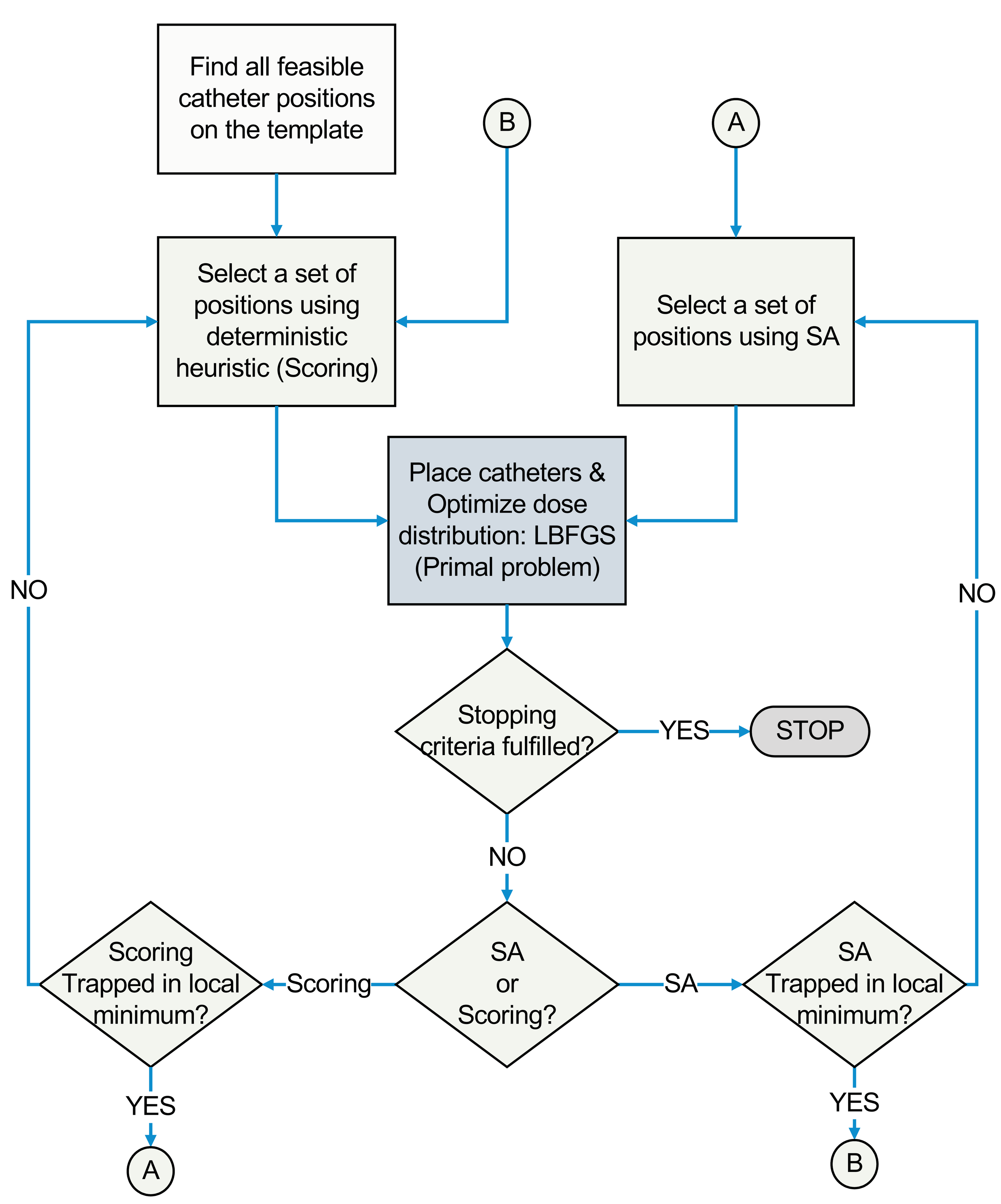
MINLP Problem:

The optimization problem can be formulated as a MINLP problem

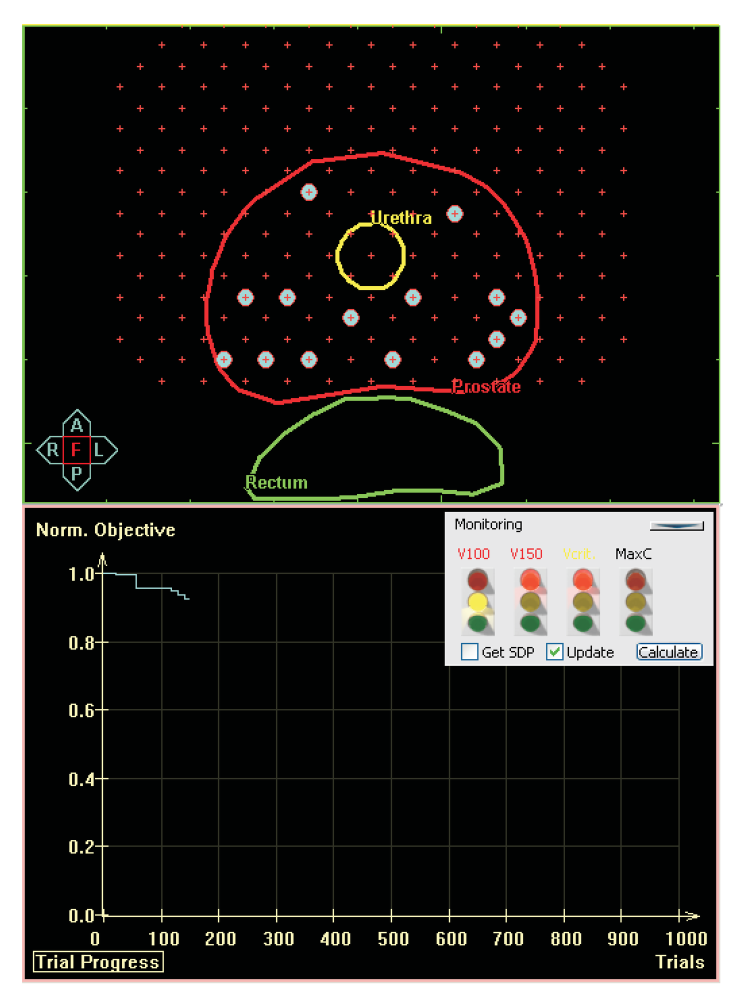
$$\min_{\mathbf{x},\mathbf{b}} F(\mathbf{x},\mathbf{b})$$
$$h(\mathbf{b}) = 0, \ \mathbf{g}(\mathbf{x},\mathbf{b}) \le 0$$
$$\mathbf{b} \in \{0,1\}^{N_{FCP}}, \ \mathbf{x} \in \mathbf{X} \subset \Re^{N_{SDP}}$$

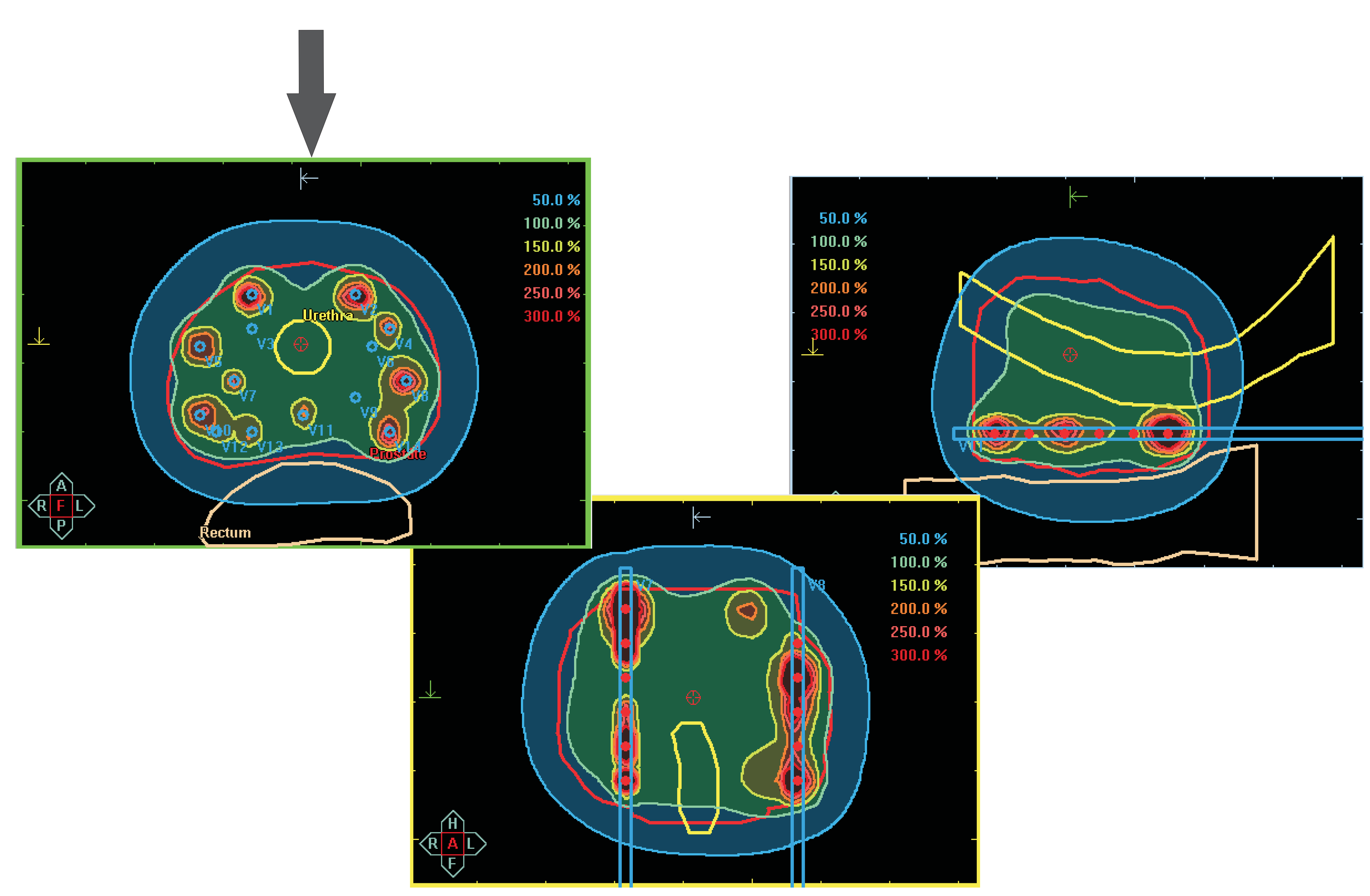
The Algorithm

HIPO is giving a near optimal solution in relatively short time (typically 0.5-5min), convenient for real-time planning in the operation room. A hybrid algorithm, based on Simulated Annealing and a problem-specific, deterministic heuristic, is used for the binary optimization problem of catheter positioning. The problem-specific heuristic is incorporating expert knowledge. The continuous dose optimization for each set up of the catheters (primal problem) is solved by the standard LBFGS algorithm.



Results





An algorithm for the solution of a real-life MINLP within a short time (0.5-5min) is proposed. The algorithm can handle clinical requirements as hard and soft constrains and deliver a clinically acceptable plan, adapted to the specific 3D volume reconstruction data for each patient.

This way, a minimum quality for all the plans of a clinic is ensured, which does not depended on the experience of the planner.

The first version of HIPO is commercially available since 2005 within the products Oncentra Prostate™ and Oncentra GYN™ and is used for brachytherapy treatment planning of prostate and gynaecological cancers.

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References

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[3.] Lahanas M., Baltas D., Zamboglou N., "A hybrid evolutionary algorithm for multi-objective anatomy-based dose optimization in high-dose-rate brachytherapy", Phys. Med. Biol., 48, (2003) 399-415.

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