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Transport and Mixing in Complex and Turbulent Flows

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Rayleigh–Taylor turbulence: a simple model for heat transfer in thermal convection

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G. Boffetta, A. Mazzino, S. Musacchio and L. Vozella *J. Fluid Mech.* **643** 127 (2010)
G. Boffetta, F. De Lillo and S. Musacchio *Phys. Rev. Lett.* **104** 034505 (2010)
G. Boffetta, A. Mazzino, S. Musacchio, and L. Vozella *Phys. Fluids* **22** 035109 (2010)

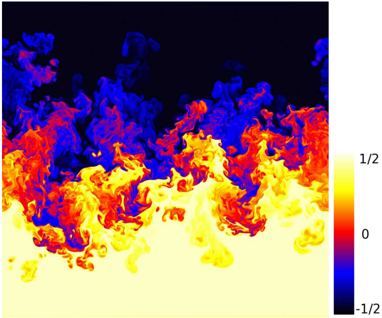
Numerical simulations

High resolution direct numerical simulations of Boussinesq equations

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v} - \beta \mathbf{g} T \quad (1)$$

$$\partial_t T + \mathbf{v} \cdot T = \kappa \nabla^2 T \quad (2)$$

with $\nabla \cdot \mathbf{v} = 0$ and initial condition $\mathbf{v}(\mathbf{x}, 0) = 0$, $T(\mathbf{x}, 0) = -(\theta_0/2) sgn(z)$.



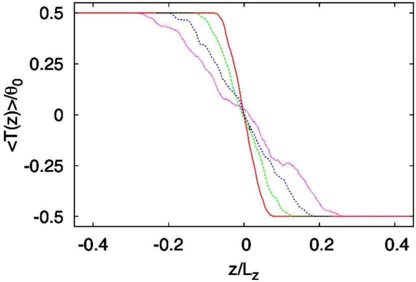
Snapshot of a (x, z) section of the temperature field obtained from the integration of Boussinesq equations by a standard pseudo-spectral code at resolution 1024^3. Parameters: $\beta g = 0.5$, $\theta_0 = 1$ ($Ag = 0.25$), $Pr = \nu/\kappa = 1$.

Evolution of mixing layer

RT instability develops in a turbulent mixing zone of width $h(t)$ which is expected to grow in time according to

$$h(t) = \alpha A g t^2 \qquad A = 1/2\beta\theta_0 \tag{3}$$

The dimensionless parameter α which represents the efficiency of mixing, can be computed as $\alpha = \dot{h}^2/(4Agh)$ which is an inverse drag coefficient.



Evolution of the mean temperature profile $\overline{T}(z,t)$ with the development of RT turbulence. $h(t)$ is defined from $\overline{T}(z,t)$ by means of a mixing function $h(t) = \int M(\overline{T})dz$ with $M(x) = 4x(1-x)$

Nonlinear diffusion model

The equation for the normalized temperature profile $\overline{c}(z) \equiv 1/2 - \overline{T}/\theta_0$

$$\partial_t \overline{c} + \partial_z \overline{wc} = \kappa \partial_z^2 \overline{c} \tag{4}$$

is closed in terms of an eddy diffusivity $K(z,t)$ which is taken, within the framework of Prandtl mixing length theory as $K(z,t) \simeq H^2 \partial_z V \simeq (Ag)^3 t^5 \partial_z \overline{c}$ to obtain the nonlinear model

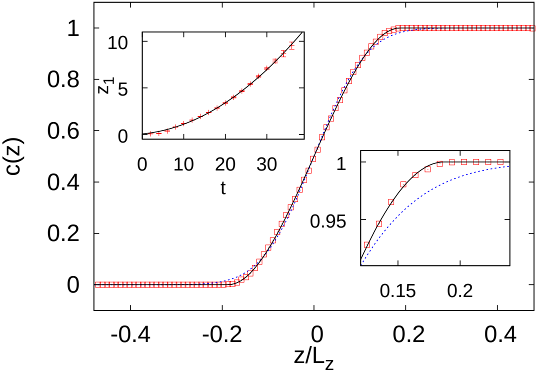
$$\partial_t \overline{c} = a(Ag)^3 t^5 \partial_z (\partial_z \overline{c})^2 \tag{5}$$

The solution is a simple cubic function

$$\overline{c}(z) = \frac{z}{4z_1}\left[3 - \left(\frac{z}{z_1}\right)^2\right] + \frac{1}{2} \tag{6}$$

with $z_1(t) = \gamma A g t^2$.

Mean temperature profile



Comparison between the result from numerical simulations (squares), **nonlinear diffusion model** (line) and linear diffusion model with constant diffusivity $K(t)$ (dotted line). Lower inset: Enlargement of the profile at the edge of the mixing region. Upper inset: Evolution of $z_1(t)$ fitted with a quadratic law which gives $\alpha = 0.019$

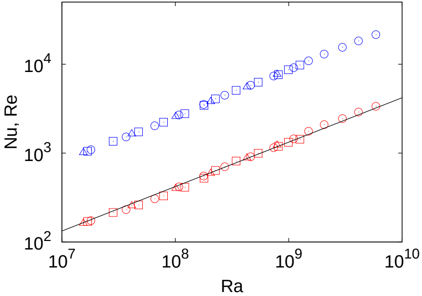
From geometry to dynamics

The diffusion model expresses the heat flux in terms of the mean temperature profile $\overline{wT} \simeq (Ag)^3 t^5 (\partial_z \overline{T})^2 / \theta_0$.

Integration over z gives a prediction for the Nusselt number $Nu \equiv \langle wT \rangle h/(\kappa \theta_0)$ as a function of the Rayleigh number $Ra \equiv Agh^3/(\nu\kappa)$ as

$$Nu = \frac{\gamma^{1/2}}{5\sqrt{2}} Pr^{1/2} Ra^{1/2} \tag{7}$$

Kraichnan's ultimate state of thermal convection.



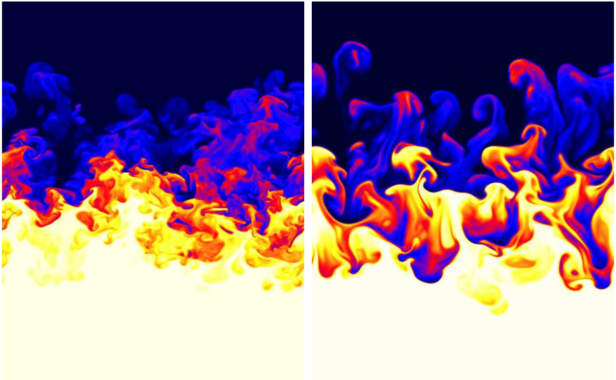
Nusselt number Nu and Reynolds number $Re = v_{rms}h/\nu$ versus Rayleigh number Ra from three different set of simulations at resolutions $256 \times 256 \times 1024$ (squares), $512 \times 512 \times 2048$ (circles) and $1024 \times 1024 \times 1024$ (triangles) at $Pr = 1$. The line is the prediction (7) with $\gamma = 0.025$.

Polymer effects on thermal convection

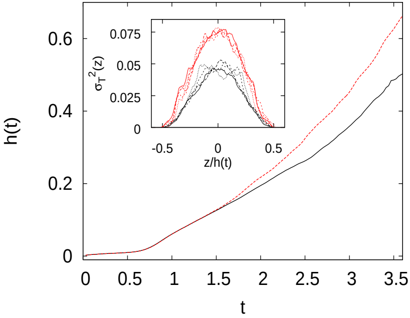
The addition of long-chain polymers induces a strong drag reduction in a turbulent flow.

Is there an analogous effect on the trasport of heat?

Direct numerical simulations of Boussinesq-Oldroyd-B model for a viscoleastic flow



Newtonian - viscoelastic



Polymers accelerate the growth of the mixing layer and reduce the drag coefficient.

Growth of the mixing layer thickness for a Newtonian run and a viscoelastic run with $\tau = 2$ starting from the same initial condition.

Polymers accelerate heat transfer. Evolution of Nusselt number Nu (blue lines), Reynolds number Re (red lines) and Rayleigh number Ra (black lines) for the Newtonian run (continuous lines) and the viscoelastic run (dotted lines). Inset: Nu vs Ra.