IMA Annual Program Year Workshop
Transport and Mixing in Complex and Turbulent Flows
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Miscible and immiscible Rayleigh–Taylor turbulence

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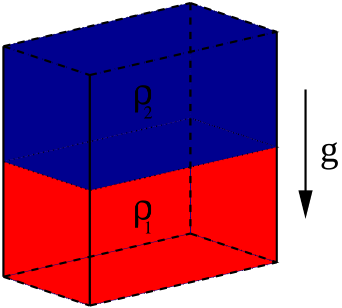
The Rayleigh–Taylor phenomenology

at initial time:



with $\rho_2 > \rho_1$

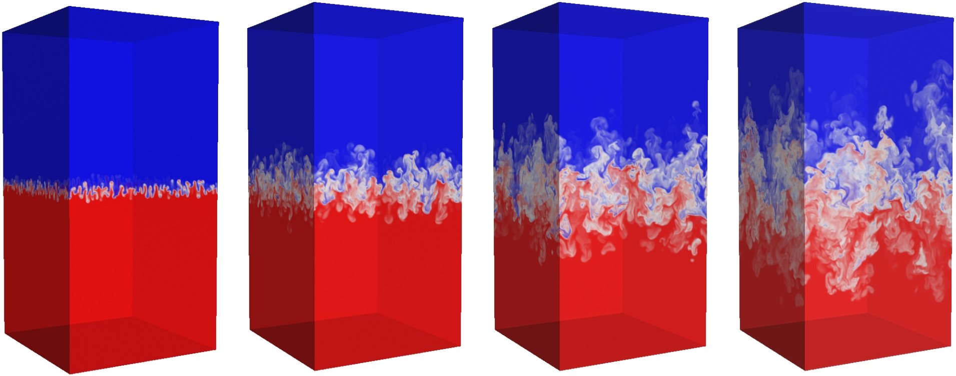
The phenomenology of mixing can be summarized as follows:

- at initial time the system is at rest the heavier fluid placed above the lighter fluid
- a small interfacial disturbance grows in time and it gets deformed in the shape of mushroom-like shape structures (the thermal plumes). The fluids thus penetrate into each other leaving behind them a region of mixed fluid. At sufficiently long time the fluid motions in the mixing layer are turbulent

3D miscible configuration

- the aim is the complete characterization of the statistical properties of mixing

Time evolution



⇒ full details: G. Boffetta, A. Mazzino, S. Musacchio & L. Vozella, *Phys. Rev. E* **79** (2009)
G. Boffetta, A. Mazzino, S. Musacchio & L. Vozella, *Phys. Fluids* **22** (2010)

Dimensional predictions [M. Chertkov Phys. Rev. Lett. 91 (2003)]

$$
\begin{aligned}
\partial_t \mathbf{v} + \mathbf{v}\cdot\boldsymbol{\partial}\mathbf{v} &= -\boldsymbol{\partial} p + \nu\,\partial^2\mathbf{v} - \beta\mathbf{g}\,(T - T_o) \\
\boldsymbol{\partial}\cdot\mathbf{v} &= 0 \\
\partial_t T + \mathbf{v}\cdot\boldsymbol{\partial} T &= \varkappa\,\partial^2 T
\end{aligned}
$$

initial conditions: $\mathbf{v}(\mathbf{x}, t=0) = 0$ & $T(\mathbf{x}, t=0) = -\,\mathrm{sign}\,(z)\,\Theta/2$

- width of mixing layer: the temperature field rules the dynamics

$$L(t) \sim \beta g \Theta t^2$$

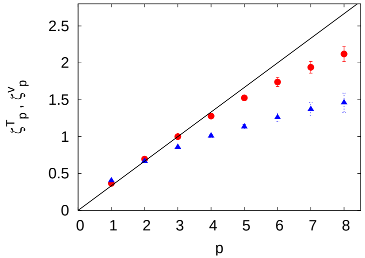
- at scales $r \ll L$: the dynamics is only ruled by the non-linear term
  - moments of field increments:

$$
\begin{aligned}
S_p^v(\mathbf{r}, t) &= \langle|\delta_r v|^p\rangle \sim (\beta g \Theta)^{2/3}\, r^{p/3} t^{p/3} \\
S_p^T(\mathbf{r}, t) &= \langle|\delta_r T|^p\rangle \sim \Theta^{2/3}(\beta g)^{-1/3} r^{p/3} t^{p/3}
\end{aligned}
$$

Numerical results: spatial exponents

dimensional predictions: $S_p^v(\mathbf{x},t) \sim r^{\zeta_p^v}$ & $S_p^T(\mathbf{x},t) \sim r^{\zeta_p^T}$

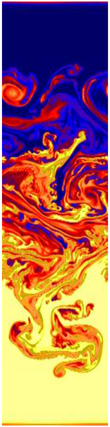
with $\zeta_p^v = \zeta_p^T = p/3$



- exponents values are the same as those obtained in a classical stationary Navier–Stokes turbulence + passive scalar [T. WATANABE & T. GOTOH, *New J. Phys.*, **6** (2004)]

⇒ RT and NS turbulence belong to the same class of universality

A glance at the 2D miscible case



$$\begin{aligned} \partial_t \mathbf{v} + \mathbf{v} \cdot \boldsymbol{\partial} \mathbf{v} &= -\boldsymbol{\partial} p + \nu \boldsymbol{\partial}^2 \mathbf{v} - \beta \mathbf{g} (T - T_o) \\ \boldsymbol{\partial} \cdot \mathbf{v} &= 0 \\ \partial_t T + \mathbf{v} \cdot \boldsymbol{\partial} T &= \varkappa \partial^2 T \end{aligned}$$

$\Rightarrow$ temperature field rules the dynamics at larger and smaller scales

M. CHERTKOV, *Phys. Rev. Lett.*, **91** (2003)

numerical results

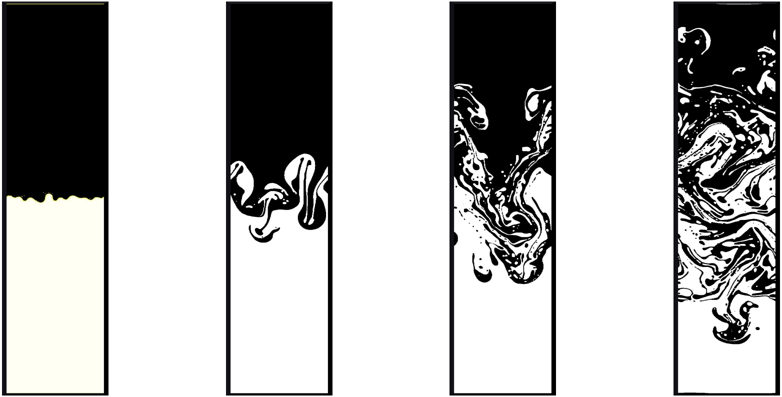
- velocity statistics follows dimensional predictions
- temperature statistics displays strong non-dimensional corrections

A. CELANI, A. MAZZINO & L. VOZELLA, *Phys. Rev. Lett.* **96** (2006)

2D immiscible configuration

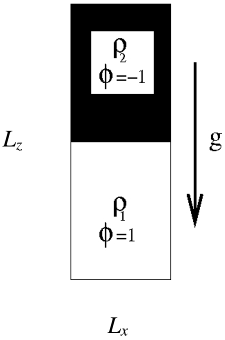
- the presence of surface tension introduces serious problems in numerical description ⇒ How do you numerically describe the instability evolution?

Time evolution:



⇒ full details: A. Celani, A. Mazzino, P. Muratore-Ginanneschi & L. Vozella *J. Fluid Mech.* **622** (2009)

Numerical description: phase-field method



- sharp interface $\Rightarrow$ diffuse interface
- phase field $\phi(\boldsymbol{x})$ and surface tension σ
- Ginzburg–Landau free energy
  $\mathcal{F}[\phi] = \int_\Omega \left(\frac{\Lambda}{2} |\boldsymbol{\partial}\phi(\boldsymbol{x})|^2 + \frac{\Lambda}{4\varepsilon^2}(\phi^2 - 1)^2 \right) d\boldsymbol{x}$
- equilibrium: $\phi(z) = \pm \tanh\left(\frac{z}{\sqrt{2}\varepsilon}\right)$ and $\sigma = \frac{2\sqrt{2}}{3}\frac{\Lambda}{\varepsilon}$
- out of equilibrium:

$$\partial_t \phi + \boldsymbol{v} \cdot \boldsymbol{\partial}\phi = \gamma\, \Lambda \partial^2 \left[-\partial^2 \phi + \frac{(\phi^3 - \phi)}{\varepsilon^2} \right]$$

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\partial} \boldsymbol{v} = -\frac{\boldsymbol{\partial} p}{\rho_o} + \nu \partial^2 \boldsymbol{v} - \frac{\Lambda}{\rho_o} \partial^2 \phi \boldsymbol{\partial} \phi - \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \phi \boldsymbol{g}$$

$$\boldsymbol{\partial} \cdot \boldsymbol{v} = 0$$

- sharp-interface limit

$$\varepsilon, \Lambda \quad \rightarrow \quad 0 \quad \textit{with } \sigma \sim \frac{\Lambda}{\varepsilon} \textit{ fixed}$$

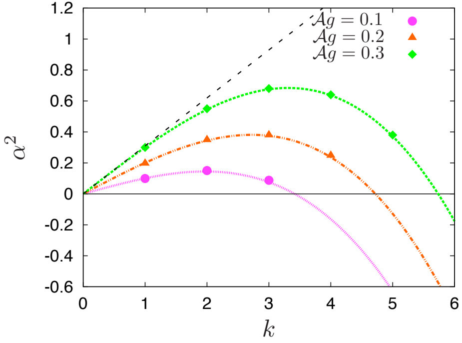
$$\gamma \quad \rightarrow \quad 0$$

J.W. Cahn & J.E. Hilliard *J. Chem. Phys.* **28** (1958)
A.J. Bray *Advances in Physics* **51**(2) (2002)

Numerical results: linear stage and growth rate

$t = 0$: $\eta(x, 0) = \eta_0 \sin(kx)$ with

$\frac{\eta_o}{\lambda} \ll 1$ (small disturbance) and $\frac{\eta_o}{\varepsilon} \gg 1$ (sharp-interface limit)



phase-field formulation ⇒

perturbation amplitude:

$$\eta(t) = \eta_0 \cosh(\alpha t)$$

with:

squared growth rate:

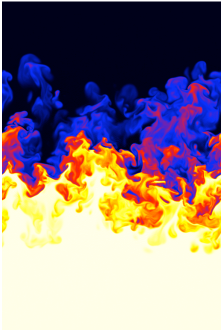
$$\alpha^2 = \mathcal{A}gk - \frac{\sigma}{\rho_2 + \rho_1} k^3$$

$$\left(\mathcal{A} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right)$$

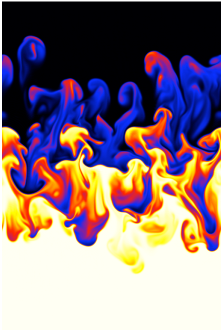
- Good agreement with the theory

⇒ The phase-field method works fine

A glance at the 3D viscoelastic configuration



NEWTONIAN SIMULATION



VISCOELASTIC SIMULATION

⇒ For details, see poster by Guido Boffetta

→ G. BOFFETTA, A. MAZZINO, S. MUSACCHIO AND L. VOZELLA *J. Fluid Mech.* **643** (2010)
G. BOFFETTA, A. MAZZINO, S. MUSACCHIO AND L. VOZELLA *submitted to PRL*.