Modeling Portfolio Credit Risk

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Outline

• Credit Risk Overview
• Brief History of Quantitative Credit Modeling
• Risk Modeling for Credit Portfolios
• Monte Carlo Simulation of Credit Portfolios
Credit Risk

- The risk that a counterparty in a financial contract may not pay obligations – i.e. the risk of default.
- Recent examples: Enron, K-Mart, United Airlines
- Affects corporate bonds and loans and associated derivatives, plus some governmental bonds.
- Time frame for risk usually in years – what is the probability of default over the next year?
- Other credit risky instruments: Asset Backed Securities (ABS) based on credit cards, mortgages, student loans, aircraft leasing, etc.
Modeling Tools Used in Credit Risk

- Empirical Data Handling Methods: Model Specification, Model Calibration
- Statistics: Regression, Hypothesis Testing, Model Validation
- Stochastic Processes
- Numerical Methods: Monte Carlo
- Applied Math: Multivariate Probability, Linear Algebra, Fourier Transforms, etc.
People Interested In Credit Risk

• Bankers – Portfolio and Risk Managers, Lenders
• Asset Managers, Insurance Companies, Hedge Funds, Corporations with Trade Credit
• Debt Structures
• Regulators and Rating Agencies
• Finance Researchers – Corporate Finance
• Economists
• Technology Developers
• Quants
Simple Corporate Finance Model

Corporations finance projects by through
- Equity (stock): ownership of part of the firm
- Debt: borrowing with obligation to repay

Firm Value = Market Value + Market Value
            of Equity             of Debt

Default occurs when
Firm Value < Liabilities
Structural Model for Default Probability

- Default Probability can be inferred from:
  - \( A = \) Firm Value
  - \( D = \) Liabilities
  - \( \sigma = \) Firm Value Volatility

- \( DD = \) Distance to Default
  \[
  DD = \frac{\log(A/D)}{\sigma}
  \]
  Number of standard deviations away from default

- \( DD \) is a good measure for ranking firms. Need empirical default data to get default probabilities.
Modeling Questions for Structural Models

• How can firm value be determined? – not observable in the markets
• How do firm value, liabilities, volatility evolve over time?
• How are they correlated with other companies?
• How does default probability determine market price of a bond, loan, etc.? Other factors include:
  - Recovery
  - Market Price of Risk
  - Liquidity
  - Size Premium/ Rating effect
  - Option features of instrument
Selective Highlights of Quantitative Credit Modeling History

• Merton Model (1973-74) – Black and Scholes and Merton developed framework based on observation that Equity has a call option on the firm value. Much academic work follows, but difficulties in getting default probabilities from models.
• Banks use scoring or internal ratings models; Altman’s Z score adds econometric insight.
• Rating Agencies provide qualitative assessment.
• Vasicek-Kealhofer model (1989) – Structural model in Merton framework calibrated to default data; first widely successful structural model.
Selective Highlights of Quantitative Credit Modeling History

- Portfolio credit models combining default probabilities and correlations become more widely used by mid 1990s.
- Jarrow-Turnbull (1995) – Introduction of “Reduced Form” models, describing stochastic process for evolution of credit risk in terms of default intensities as a component of yield spread. Duffie-Singleton (1999) further develop the idea to give the most commonly used reduced form model.
- Private firm default models develop in late 1990s.
- Today: Many models for default probabilities, correlations, portfolio modeling, and credit derivatives.
Risk Modeling for Credit Portfolios

\[ \omega = \$ \text{ exposure size} \]

\[ V(x,t) = \text{Value of loan in credit state } x \text{ at time } t \text{ relative to par} \]

\[ \Pi(t) = \text{Value of portfolio} \]

\[ \Pi(t) = \sum_{i=1}^{N} \omega_i V_i(x_i,t) \]

\((x_1, \ldots, x_N)\) correlated random variables
Risk Modeling for Credit Portfolios

\[ L(t) = \text{Loss} \]

\[ L(t) = E(\Pi(t) \mid \text{no defaults}) - \Pi(t) \]

\[ EL = \text{Expected Loss} \]

\[ UL = \text{Unexpected Loss} = \sigma(\Pi) \]

\[ C = \text{Capital} \sim \text{Value at Risk} \]

\[ RC_i = \frac{\partial UL}{\partial \omega_i} = \text{Risk Contribution} \]

\[ UL = \sum_{i=1}^{N} \omega_i RC_i \]
Simple Homogenous Default/No-Default Model

• Assume all loans have same exposure, default probability, correlation, and recovery

\[ L(t) = \omega \cdot LGD \cdot \sum_{i=1}^{N} \chi(x_i) \]

\[ x_i \text{ iid, uniform on } [0,1] \]

\[ \chi(x) = 1 \text{ if } x \leq EDF \]

\[ 0 \text{ if } x > EDF \]
Simple Homogenous Default/No-Default Model

• Easy to compute EL and UL as functions of EDF, LGD and correlation (joint default frequency JDF)
• Question: How much diversity can be achieved for a given EDF and correlation? How many loans do you need before the portfolio is well diversified?
• Probability distribution of loss is not obvious because of correlations. For single factor models, Semi-Analytic methods allow the distribution to be expressed as a 1 dimensional integral.
Monte Carlo Simulation for Credit Portfolios

- **Single Horizon Analysis:**
  - Draw random, correlated credit states at horizon for all obligors
  - Value each facility at the horizon as a function of its credit state
  - Compute portfolio value/loss at horizon
  - Repeat many times to build probability distribution of losses and estimate associated statistics such as risk contribution.
Monte Carlo Simulation for Credit Portfolios

• Computational Problem: Capital is often required at the 10 bp level (i.e., 99.9% VaR). This requires 100,000 simulation runs for reasonable accuracy; very slow for large portfolios. Accurate Tail Risk Contribution requires even more runs.

• Solution: Importance Sampling. Put more sample points in loss tail and weight samples non-uniformly.

• Difficulties: Identifying ways to generate large (but not too large) loss sample points; making sure the rest of the distribution is still adequately sampled.
Monte Carlo Simulation for Credit Portfolios

- Importance Sampling usually depends on tailoring sampling to function (here the portfolio).
- Credit portfolio property: Tail of loss distribution created by multiple correlated defaults. Increasing correlations will lead to more large loss events.
- If firm value returns are assumed Gaussian, default correlations can be increased by increasing value return correlations.