High SNR Analysis of MIMO Broadcast Channels

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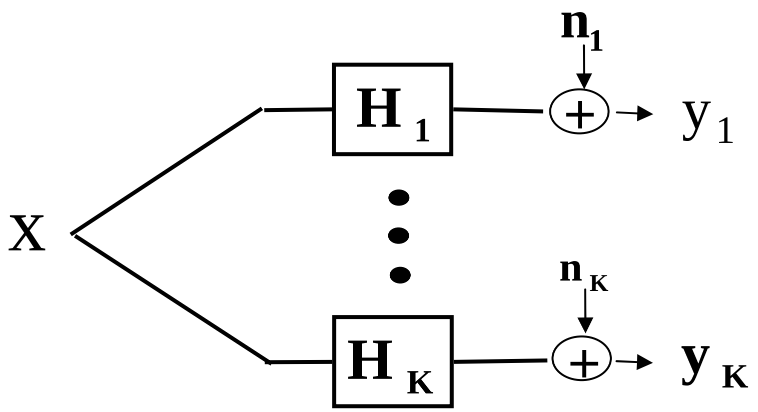
MIMO Downlink Channel

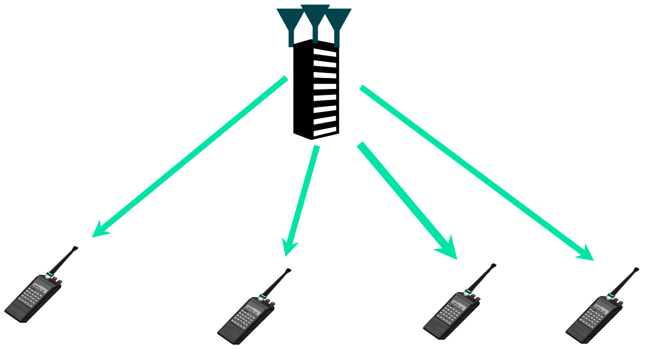


$$\mathbf{y}_i = \mathbf{H}_i \vec{\mathbf{x}} + \mathbf{n}_i$$

- M transmit antennas, 1 receive antenna (each mobile)
  - $\mathbf{H}_i$ is a 1 x M row vector
  - $\mathbf{H} = \left[\mathbf{H}_1^{\mathrm{T}} \cdots \mathbf{H}_K^{\mathrm{T}}\right]^T$

- Transmit power constraint P
- Circularly symmetric complex Gaussian noise
- Slowly fading channel, perfect CSI at RX & TX

Motivation



- Multiple-Antenna Downlink:
  - Common model for many systems: cellular, Wi-Fi, 802.16, …
  - TDMA currently used, but very sub-optimal
  - Optimal to simultaneously transmit to multiple users without time/freq/code separation

- Need better understanding of optimal/near-optimal strategies

MIMO BC Capacity

- Capacity achieved by dirty paper coding (DPC) [CS, VJG, VT, YC, WSS]
- Sum rate capacity (i.e. maximum throughput):

$$C_{BC}^{sumrate}(H,P) = \max_{Q_i : \sum_{i=1}^{K} Tr(Q_i) \le P} \log \left| I + \sum_{i=1}^{K} H_i^T Q_i H_i \right|$$

- No simplified expression, multiplexing gain gives:

$$C_{BC}^{sumrate}(H,P) \approx \min(M,K) \log_2 P$$

- Multiplexing gain <u>not sufficient</u> to capture effect of fading distribution, extra antennas, …

High SNR Approximation

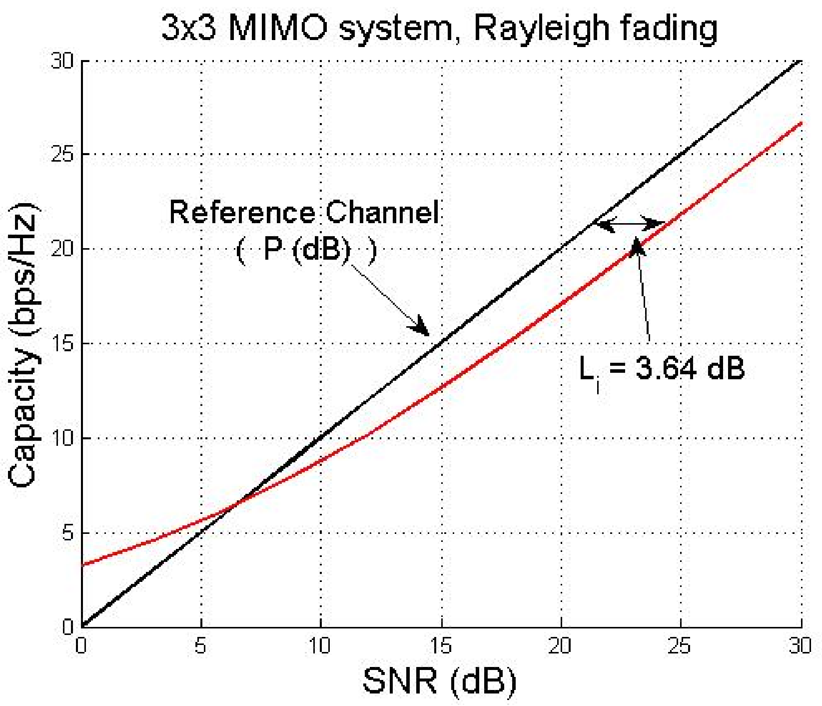
- First introduced for CDMA channels [Shamai & Verdu]

$$\begin{aligned} C(P) &\approx S_\infty(\log_2 P - L_\infty) + o(1) \\ &\approx S_\infty\left(\frac{P_{dB}}{3} - L_\infty\right) + o(1) \end{aligned}$$

$$S_\infty = \lim_{P\to\infty} \frac{C(P)}{\log_2 P} \qquad \text{(multiplexing gain)}$$

$$L_\infty = \lim_{P\to\infty}\left(\log_2 P - \frac{C(P)}{S_\infty}\right) \qquad \text{(additive offset in dB's)}$$

- Extended to point-to-point MIMO channels for Rayleigh, Ricean, correlated fading [Lozano,Tulino,Verdu]

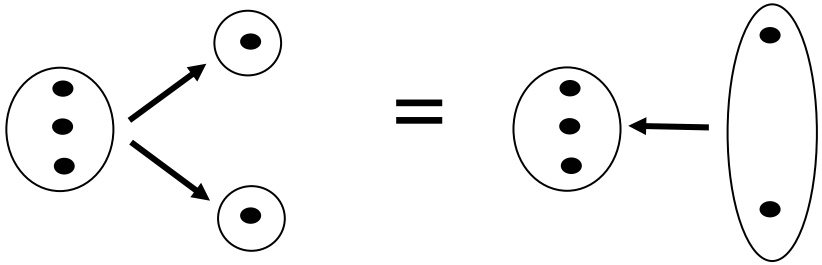


Power offset important at moderate SNR's

Point-to-Point/BC Equivalence

- Thm: $C_{sum}(H,P) \approx C_{MIMO}(P)$ at high SNR if $M \leq K$ (more TX ant's than receivers)
- Pf: Use Caire & Shamai's result that proves sum rate capacity converges to cooperative MIMO capacity (absolutely) at high SNR

- True for each instantiation H, as well as in expected value sense



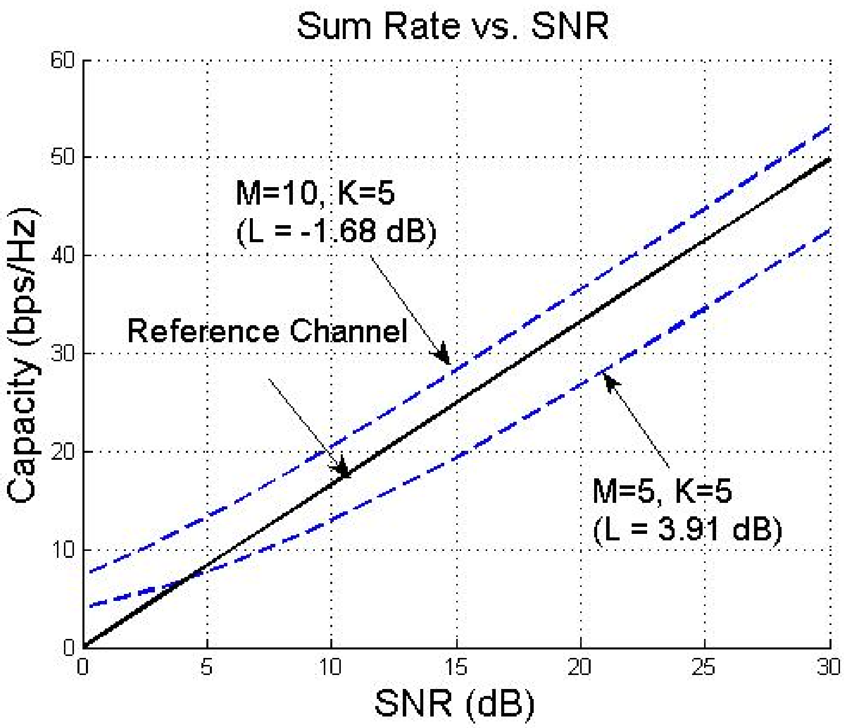
M TX antenna,
K receiver MIMO BC

K x M MIMO with CSI at RX
(opposite direction)

MIMO BC at High SNR

- Directly apply known MIMO results
- Example: Increasing TX antennas beyond K does not increase multiplexing gain, but provides SNR boost

- Power offset only depends on <u>aggregate</u> number of RX antennas
  - Ants/RX unimportant if aggregate # ≤ M



Downlink Beamforming at High SNR

- No interference pre-cancellation performed (less computationally complex), same multiplexing gain as DPC, MIMO (min(M,K))

- Thm: When M ≥ K, rate loss of BF:

$$C_{BC}^{sumrate}(H,P) - C_{BC}^{BF}(H,P) \approx \log_2 \frac{\left|HH^H\right|}{\prod_{i=1}^{K} \| g_i \|^2}$$

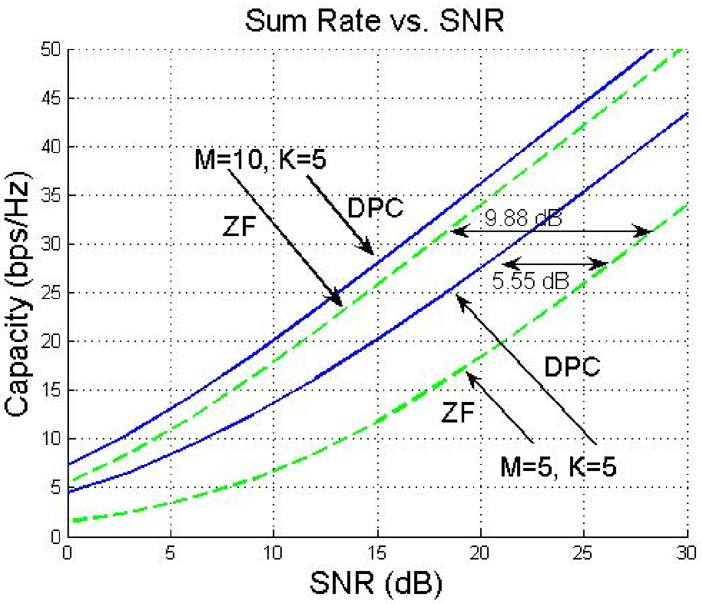
  where g_i is proj. of H_i onto nullspace of $\{H_j\}_{j \neq I}$

- In Rayleigh fading, average power offset: $\frac{3\log_2 e}{M}\sum_{j=1}^{K-1}\frac{j}{M-j}$ dB

- When M=K, power offset ≈ 3 $\log_2$ M dB
  - Huge penalty to using ZF when K = M because random square matrices poorly conditioned as size increases

Performance of Beamforming

- In M=5, K=5 system, BF incurs 5.55 dB penalty
- In M=10, K=5 system, BF only incurs 1.26 dB penalty
- BF viable alternative if # of TX ant's > # of receviers



- Asymptotic power penalty of BF converges as M,K grow large with M = β·K for β>1 (from CDMA results)

β	1.33	1.5	2
Power Penalty	3.7 dB	2.8 dB	1.7 dB

Extensions

- More receivers than antennas (K > M)
  - No direct MIMO equivalence
  - # of antennas per RX does affect power offset
- Quantify power penalty of BF for non-Rayleigh fading (e.g. Ricean, correlated)
  - Need distribution orthogonal projections of rows of H onto other rows of H
- Study entire capacity region at high SNR
  - Incorporate multi-user scheduling ??