Phase transformation which would never occur in the deterministic setting becomes possible in the presence of stochastic perturbations. For instance, the stable state $u \equiv -1$ of the deterministic Allen-Cahn equation can be stochastically driven to the opposing state, $u \equiv +1$. We consider the Allen-Cahn phase transformation problem and the related large deviation action functional,

$$\int_0^T \int_{[0,L]^d} |\dot{u} + (-\Delta u + V'(u))|^2 \, dx \, dt.$$ 

When $L$ is fixed and $T \to \infty$, the action minimizer is easy to identify. When time is short or joint limits are considered, however, the story is more complicated. In the particularly interesting case of the sharp interface limit, the competing costs of interface nucleation and interface propagation lead to a reduced functional with a special form.

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