Transpose-free Multiple Lanczos and Its Application in Padé Approximation

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Abstract

We consider the task of model reduction via Padé approximation on a multi-input multi-output (MIMO) linear dynamical system

$$\mathbf{C}\frac{d\mathbf{x}}{dt} = -\mathbf{G}\mathbf{x}(t) + \mathbf{R}\mathbf{w}(t), \qquad \mathbf{y}(t) = \mathbf{L}^H\mathbf{x}(t)\,,$$

where $\mathbf{C}, \mathbf{G} \in \mathcal{C}^{N\times N}$, $\mathbf{R} \in \mathcal{C}^{N\times m}$, $\mathbf{L} \in \mathcal{C}^{N\times n}$, and $\mathbf{w}(t), \mathbf{y}(t)$ and $\mathbf{x}(t)$ are vector-valued functions of length m, n and N, respectively.

Corresponding to this system is the matrix-valued transfer function $\mathbf{F}(z)$ mapping the input $\mathbf{W}(z)$ to the output $\mathbf{Y}(z) = \mathbf{F}(z)\mathbf{W}(z)$ in frequency domain, where

$$\mathbf{F}(z) = \mathbf{L}^H(z\mathbf{C} + \mathbf{G})^{-1}\mathbf{R} = \sum_{k=0}^{\infty} \mathbf{M}_k (z - z_0)^k,$$

where we show the expansion of the transfer function in a power series about $z = z_0$. We seek a new lower order Padé approximant

$$\tilde{\mathbf{C}}\frac{d\tilde{\mathbf{x}}}{dt} = -\tilde{\mathbf{G}}\tilde{\mathbf{x}} + \tilde{\mathbf{R}}\mathbf{w}(t), \qquad \tilde{\mathbf{y}}(t) = \tilde{\mathbf{L}}^H\tilde{\mathbf{x}}(t) \tag{1}$$

with transfer function

$$\tilde{\mathbf{F}}(z) = \tilde{\mathbf{L}}^H(z\tilde{\mathbf{C}} + \tilde{\mathbf{G}})^{-1}\tilde{\mathbf{R}} = \sum_{k=0}^{\infty} \tilde{\mathbf{M}}_k (z - z_0)^k,$$

for which the first l moments agree with the original: $\tilde{\mathbf{M}}_i = \mathbf{M}_k$ for $k = 1, \ldots, l$, for a given l. It is well known that the Lanczos procedure is intimately related to the computation of (1). A famous method called Matrix Padé via Lanczos (MPVL) was proposed by Feldmann and Freund using the novel two-sided Lanczos procedure with multiple starting vectors which requires matrix vector products with both $\mathbf{A}$ and $\mathbf{A}^H$ where $\mathbf{A}$ is the system matrix.

In this paper, a transpose-free two-sided Lanczos procedure is developed for multiple starting vectors. The procedure is mathematically equivalent to the original two-sided one, but avoids the use of the transpose of the system matrix $\mathbf{A}$. The new procedure is then applied to the computation of the matrix Padé approximation (1). The result is a method which can be labeled Transpose-Free Matrix Padé Via Lanczos (TFMPVL). Under certain circumstances, TFMPVL will actually reduce the total number of matrix-vector products needed by MPVL. It is illustrated with some numerical examples.

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