EXTINCTION AND POSITIVITY FOR A SYSTEM
OF SEMILINEAR PARABOLIC VARIATIONAL INEQUALITIES

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EXTINCTION AND POSITIVITY FOR A SYSTEM OF SEMILINEAR PARABOLIC VARIATIONAL INEQUALITIES

AVNER FRIEDMAN† AND MIGUEL A. HERRERO‡

Abstract. A simple model of chemical kinetics with two concentrations \( u \) and \( v \) can be formulated as a system of two parabolic variational inequalities with reaction rates \( v^p \) and \( u^q \) for the diffusion processes of \( u \) and \( v \) respectively. It is shown that if \( p q < 1 \) and the initial values of \( u \) and \( v \) are “comparable” then at least one of the concentrations becomes extinct in finite time. On the other hand for any \( p = q > 0 \) there are initial values for which both concentrations do not become extinct in any finite time.

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§1. Introduction. It is well known that nonnegative solutions of various initial and boundary value problems associated to the semilinear heat equation

\[
(1.1) \quad u_t - u_{xx} + u^p = 0 \quad \text{with} \quad 0 < p < 1
\]

vanish identically in finite time; see [7] [2] [3] [1]. This phenomenon is termed extinction, and is clearly illustrated by the explicit solution

\[
(1.2) \quad u = ((1 - p)(T - t)_+)^{1/(p - 1)} , \quad T > 0
\]

where \( s_+ = \max\{s, 0\} \). This particular solution plays an important role in describing the asymptotic behavior of the extinction process; cf. [5] [6].

In this paper we consider a semilinear parabolic system which may be thought of as a toy model in chemical kinetics. Let \( u(x, t) \) and \( v(x, t) \) denote the concentrations of two species which diffuse and react in a one-dimensional domain \( L = \{-1 < x < 1\} \) according to

\[
(1.3) \quad u_t - u_{xx} + v^p = 0 \quad , \quad v_t - v_{xx} + u^q = 0 \quad (p > 0, q > 0).
\]

Since the concentrations must be nonnegative, we are led to the following variational inequality formulation:

\[
(1.4) \quad u \geq 0 , \quad v \geq 0 \quad \text{in} \quad Q = (-1,1) \times (0,\infty),
\]

\[
(1.5) \quad u(u_t - u_{xx} + v^p) = 0 , \quad v(v_t - v_{xx} + u^q) = 0 \quad \text{a.e. in} \ Q.
\]

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We also impose initial conditions

\[(1.6) \quad u(x, 0) = u_0(x), \ v(x, 0) = v_0(x) \quad \text{in} \quad [-1, 1]\]

where \(u_0 \geq 0, \ v_0 \geq 0,\) and boundary conditions which are either Dirichlet

\[(1.7) \quad u(\pm 1, t) = v(\pm 1, t) = 0 \quad \text{for} \quad t > 0,\]

or Neumann

\[(1.8) \quad \frac{\partial u}{\partial x}(\pm 1, t) = \frac{\partial v}{\partial x}(\pm 1, t) = 0 \quad \text{for} \quad t > 0.\]

We shall briefly refer to the problem (1.3)--(1.7) as \((DP)\) and to (1.3)--(1.6),(1.8) as \((NP)\).

For simplicity we assume that

\[(1.9) \quad u''_0(x), \ v''_0(x) \quad \text{belong to} \quad L^\infty[-1, 1],\]

where \(u_0, v_0\) are just continuous in \([-1, 1]\).

We define a solution of \((DP)\) to a pair of functions \((u, v)\) be such that (1.3)--(1.7) hold and

\[(1.10) \quad \iint_{Q_T} (|w_x|^r + |w_{xx}|^r + |w_t|^r) < \infty \quad \text{for} \ w = u, v \ \text{and for all} \ r > 1, T > 0.\]

For \((NP)\) we replace (1.7) by (1.8) and require, in addition to (1.10), that

\[(1.11) \quad u_x, v_x \quad \text{are continuous in} \ \bar{Q}.\]

We observe that, if \(pq < 1,\) then for any \(T > 0\) \((NP)\) has a solution

\[(1.12) \quad u = c_1(T - t)^{\frac{p+1}{1-pq}}, \ v = c_2(T - t)^{\frac{q+1}{1-pq}}\]

with

\[(1.13) \quad c_1 = \left(\frac{(1 - pq)^{1+p}}{(p+1)(q+1)^p}\right)^{\frac{1}{1-pq}},\ c_2 = \left(\frac{(1 - pq)^{1+q}}{(p+1)^q(q+1)}\right)^{\frac{1}{1-pq}}.\]

This example and the extinction results for the single equation (1.1) suggest that one may expect the extinction phenomenon to hold for the system (1.3)--(1.5) whenever \(pq < 1.\) As
it will turn out, however, this is not always the case. But before going any further we need to define the concept of extinction more carefully. If
\[ u(x, 0) \equiv 0 , \quad v(x, 0) \not\equiv 0 \]
then there exists a solution \((u, v)\) such that \(u(x, t) \equiv 0\) if \(t > 0\), whereas \(v(x, t) > 0\) for all \(-1 < x < 1, \ t > 0\). Also, if
\[ u(x, 0) = \text{const} = c_1 , \quad v(x, 0) = \text{const} = c_2 , \quad pq < 1 , \]
then, for \((NP)\), there exists a solution \((u, v)\) which is a function of \(t\) only and, for general \(c_1, c_2\), only one component becomes zero in finite time (the case (1.13) in exceptional).

These examples show that in order to capture the phenomenon of extinction one should define:

A solution \((u, n)\) has finite extinction time \(T\) if \(T\) is the smallest positive number such that either \(u(x, t) \equiv 0\) for all \(t > T\), or \(v(x, t) \equiv 0\) for all \(t > T\).

In §3 we give a sufficient condition on the data \(u_0, v_0\) which ensures extinction, and in §4 we give an example (with \(p = q\)) where there is no extinction. (The same initial data as in that example also leads to a positivity result of one component in case \(p \geq q\).) In §2 we briefly establish existence of solutions to \((DP)\) and \((NP)\).

§2. Existence.

**Theorem 2.1.** Let \(u_0, v_0\) be nonnegative functions satisfying (1.9). Then there exists a solution \((u, v)\) to \((DP)\) (respectively \((NP)\)).

**Proof.** We consider only \((DP)\); the case \((NP)\) is similar. For any \(0 < \varepsilon < 1\) let \(\beta_\varepsilon(s)\) be a \(C^\infty\) function such that
\[ \beta_\varepsilon'(s) \geq 0 , \quad \beta_\varepsilon(s) = 0 \quad \text{if} \ s \geq 0 , \quad \lim_{\varepsilon \to 0} \beta_\varepsilon(s) = -\infty \quad \text{if} \ s < 0 . \]

Let \(f_{\varepsilon, p}, f_{\varepsilon, q}\) be smooth, nonnegative, monotone nondecreasing and bounded functions satisfying:
\[ \lim_{\varepsilon \to 0} f_{\varepsilon, p}(s) = s_+^p , \lim_{\varepsilon \to 0} f_{\varepsilon, q}(s) = s_+^q . \]

Consider the system of penalized equations:

\[ u_t - u_{xx} + \beta_\varepsilon(u) + f_{\varepsilon, p}(v) = 0 \quad \text{in} \quad Q , \]

(2.1)
\[ v_t - v_{xx} + \beta_\varepsilon(v) + f_{\varepsilon, q}(v) = 0 \quad \text{in} \quad Q \]
with the same data as for \((DP)\). One can easily prove (as in \([4: \text{Chap. 1}]\)) that this problem has a solution \((u_\varepsilon, v_\varepsilon)\) and

\[
 u_\varepsilon \leq \|u_0\|_{L^\infty}, \quad v_\varepsilon \leq \|v_0\|_{L^\infty};
\]

a standard energy inequality can be used to establish uniqueness. It follows that \(f_{\varepsilon,p}(v_\varepsilon)\) and \(f_{\varepsilon,q}(u_\varepsilon)\) are bounded uniformly in \(\varepsilon\) and then (as in \([4; \text{p. 25}]\))

\[
 \beta_\varepsilon(u_\varepsilon) \quad \text{and} \quad \beta_\varepsilon(v_\varepsilon)
\]

are bounded uniformly in \(\varepsilon\). We can then deduce that for any sequence \(\varepsilon \to 0\) there is a subsequence which converges to a solution of \((DP)\).

We note that the question of uniqueness of the solutions is open.

\section{Extinction result.}

\textbf{Theorem 3.1.} Suppose that \(pq < 1\) and

\[
 (3.1) \quad u_0(x) \geq \left( \frac{q + 1}{p + 1} \right)^{\frac{q\beta}{p + 1}} v_0(x)^{\frac{p\beta}{q + 1}}, \quad p \geq q.
\]

Then there exist a solution \((u, v)\) of \((DP)\) (respectively \((NP)\)) such that \(v(x, t) \equiv 0\) for \(t \geq T,\) for some \(T > 0.\)

\textbf{Proof.} Consider the auxiliary functions

\[
 \theta = cv^\beta, \quad \theta_\lambda = c(v + \lambda)^\beta \quad \text{where} \quad c = \left( \frac{q + 1}{p + 1} \right)^{\frac{q\beta}{p + 1}}, \quad \beta = \frac{p + 1}{q + 1}, \quad \lambda > 0.
\]

Notice that \(\theta_\lambda\) satisfies

\[
 \theta_{\lambda,t} - \theta_{\lambda,xx} = -c\beta(v + \lambda)^{\beta-1}(v_t - v_{xx}) - c\beta(\beta - 1)(v + \lambda)^{\beta-2}v_x^2 \quad (\beta \geq 1).
\]

Dropping the last term and letting \(\lambda \to 0\) we get

\[
 (3.2) \quad \theta_t - \theta_{xx} \leq -c\beta v^{\beta-1}u^q
\]

in some weak sense. Recalling \((1.4)\) we deduce that

\[
 (3.3) \quad (u - \theta)_t - (u - \theta)_{xx} \geq c\beta v^{\beta-1}u^q - v^p \quad \text{in} \quad Q.
\]
We now replace \( u_0(x) \) by \( u_0(x) + \tau \) \((\tau > 0)\) and in the case of \((DP)\) replace the conditions \( u(\pm 1, t) = 0 \) by \( u(\pm 1, t) = \tau \). We continue to denote by \((u, v)\) the corresponding solution of (1.3)--(1.5). Observe that

\[
\text{if } u \geq cv^\beta \text{ then } c\beta v^{\beta-1} u^q \geq v^p,
\]

so that the right-hand side of (3.3) is \( \geq 0 \). Using this fact, and the strong maximum principle (which holds for our solution \( u \), in view of the regularity (1.10)) we can deduce that if \( u(x, t) \geq cv(x, t)^\beta \) for \(-1 \leq x \leq 1\), \(0 \leq t \leq s\) then also \( u(x, s) > cv(x, s)^\beta \) for \(-1 \leq x \leq 1\); here we needed the modification of the Dirichlet conditions at \( x = \pm 1 \). Since \( u(x, 0) = u_0(x) + \tau > cv_0(x)^\beta \) for \(-1 \leq x \leq 1\) (by (3.1)), it follows that

\[
 u(x, t) > cv(x, t)^\beta, \quad -1 \leq x \leq 1
\]

for small \( t \) and then also for all \( t > 0 \).

Letting \( \tau \to 0 \) we obtain \( u \geq cv^\beta \) for the solution \((u, v)\) of \((DP)\) or \((NP)\). Substituting this into the differential equation for \( v \) (on the set \( \{v = 0\}, v_t = 0 \) and \( v_{xx} = 0 \) a.e.), we get

\[
v_t - v_{xx} + kv^\alpha \leq 0 \quad \text{with} \quad \alpha = \frac{q(p+1)}{q+1}, \quad k = c^q.
\]

The assumption \( pq < 1 \) implies that \( \alpha < 1 \), and therefore there exists a \( T > 0 \) such that \( v(x, t) \equiv 0 \) for \( t \geq T \).

§4. Non-extinction and positivity.

We begin with a result on non-extinction.

**Theorem 4.1.** Assume that \( p = q \) and

\[
(4.1) \quad u_0(x) = v_0(-x) \quad \text{for} \quad -1 \leq x \leq 1,
\]

\[
(4.2) \quad u_0(x) \geq u_0(-x), \quad u_0(x) \neq u_0(-x) \quad \text{for} \quad -1 \leq x \leq 0.
\]

Then there exists a solution of \((DP)\) (respectively \((NP)\)) such that

\[
(4.3) \quad u(x, t) > 0 \quad \text{in} \quad Q^- \equiv (-1, 0) \times (0, \infty),
\]

\[
(4.4) \quad v(x, t) > 0 \quad \text{in} \quad Q^+ \equiv (0, 1) \times (0, \infty).
\]

Thus the solution does not have finite extinction time.

**Proof.** We consider the system

\[
(4.5) \quad u_t(x, t) - u_{xx}(x, t) + \beta_e(u(x, t)) + f_{e,p}(u(-x, t)) = 0,
\]

\[
(4.6) \quad v_t(x, t) - v_{xx}(x, t) + \beta_e(v(x, t)) + f_{e,p}(u(x, t)) = 0.
\]
with the same initial and boundary data as before. It is easy to show that \((4.5)\) has a solution with the required data. If we set \(v(x,t) = u(-x,t)\) then \(v\) satisfies \((4.6)\) and the required data (here we used \((4.1)\)). Finally, since

\[
 f_{\varepsilon,p}(u(-x,t)) = f_{\varepsilon,p}(v(x,t)) ,
\]

the pair \((u,v)\) is a solution of the penalized problem \((2.1)\).

Denoting this solution by \((u_\varepsilon,v_\varepsilon)\) we thus have

\[
(4.7) \quad v_\varepsilon(x,t) = u_\varepsilon(-x,t)
\]

and

\[
 u_\varepsilon \to u , \quad v_\varepsilon \to v \quad \text{as} \quad \varepsilon \to 0 ,
\]

where \((u,v)\) is a solution of \((DP)\) (respectively \((NP)\)).

The function

\[
 z_\varepsilon(x,t) = u_\varepsilon(x,t) - v_\varepsilon(x,t)
\]

satisfies:

\[
 z_t - z_{xx} + c_1 z + c_2 z = 0
\]

where

\[
 c_1 = \frac{\beta_\varepsilon(u_\varepsilon(x,t)) - \beta_\varepsilon(v_\varepsilon(x,t))}{u_\varepsilon(x,t) - v_\varepsilon(x,t)} , \quad c_2 = \frac{f_{\varepsilon,p}(v_\varepsilon(x,t)) - f_{\varepsilon,p}(u_\varepsilon(x,t))}{u_\varepsilon(x,t) - v_\varepsilon(x,t)}
\]

if \(u_\varepsilon(x,t) \neq v_\varepsilon(x,t)\) and \(c_1 = c_2 = 0\) otherwise. Notice that \(c_1, c_2\) are bounded functions for any fixed \(\varepsilon\). Since

\[
 z_\varepsilon(x,0) = u(x) - u_0(-x) \geq 0 , \quad \neq 0 \quad \text{in} \quad (-1,0) ,
\]

and \(z_\varepsilon\) satisfies homogeneous boundary conditions at \(x = -1,0\), the strong maximum principle yields \(z_\varepsilon > 0\) in \(Q^-\). Letting \(\varepsilon \to 0\) we obtain a solution \((u,v)\) satisfying:

\[
(4.8) \quad u(x,t) = v(-x,t) \quad \text{in} \quad Q ,
\]

\[
(4.9) \quad u(x,t) \geq u(-x,t) \quad \text{in} \quad Q^- .
\]

For any \(T > 0\) we consider the function

\[
 z(x,t) = u(x,t) - u(-x,t) \quad \text{in} \quad Q_T^- = (-1,0) \times (0,T) .
\]
In the subset where \( u(x, t) \geq u(-x, t) > 0 \), we have
\[
    z_t - z_{xx} = u(x, t)^p - u(-x, t)^p \geq 0 \quad \text{by (4.9)}.
\]
In the subset where \( u(x, t) > u(-x, t) = 0 \), \( u_t(-x, t) = 0, u_{xx}(-x, t) = 0 \) a.e. so that
\[
    z_t - z_{xx} = u_t - u_{xx} = -u(-x, t)^p = 0 \quad \text{a.e.}
\]
Finally in the subset where \( u(x, t) = u(-x, t) = 0 \) we have a.e.
\[
    z_t = z_{xx} = u_t - u_{xx} = 0, \quad \text{since} \quad u_t = 0, \ u_{xx} = 0 \quad \text{a.e.}
\]
We conclude that
\[
    z_t - z_{xx} \geq 0 \quad \text{in} \quad Q_T^{-}.
\]
Since also
\[
    z(x, 0) \geq 0, \neq 0 \quad \text{in} \quad (-1, 0)
\]
and \( z \) satisfies homogeneous boundary conditions,
\[
    z(x, T) > 0 \quad \text{if} \quad -1 < x < 0.
\]
This implies that \( u(x, T) > 0 \) if \(-1 < x < 0\), and since \( T \) is arbitrary, (4.3) is satisfied. Recalling (4.8), the assertion (4.4) also follows.

Theorem 4.1 can be used to derive a positivity result in case \( p \neq q \):

**Theorem 4.2.** Assume that \( p \geq q \), (4.1), (4.2) are satisfied, and

\[
(4.10) \quad \max\{\|u_0\|_{L^\infty}, \|v_0\|_{L^\infty}\} \leq 1.
\]

Then there exists a solution of (DP) (respectively (NP)) such that (4.3) holds.

**Proof.** We choose \( f_{\varepsilon, p}, f_{\varepsilon, q} \) such that
\[
(4.11) \quad f_{\varepsilon, p}(s) \leq f_{\varepsilon, q}(s) \quad \text{if} \quad 0 \leq s \leq 1.
\]

Denote by \( u_\varepsilon, v_\varepsilon \) the solution of the penalized problem as defined in §2 and by \( v_{\varepsilon, 1} \) the solution of
\[
v_t - v_{xx} + \beta_\varepsilon(v) + f_{\varepsilon, p}(u_\varepsilon) = 0 \quad \text{in} \quad Q
\]
with the same data as \( v_\varepsilon \). Since \( \|u_\varepsilon\|_{L^\infty} \leq 1, \|v_\varepsilon\|_{L^\infty} \leq 1 \) (here we use (4.10)), it follows from (4.11) that
\[
(4.12) \quad v_{\varepsilon, 1} \geq v_\varepsilon \quad \text{in} \quad Q.
\]
Next we define $u_{\varepsilon,1}$ to be the solution of

$$u_t - u_{xx} + \beta_\varepsilon(u) + f_{\varepsilon,p}(u_{\varepsilon,1}) = 0 \quad \text{in} \quad Q$$

with the same data as $u$. Recalling (4.12) and the fact that the $f_{\varepsilon,p}$ are monotone nondecreasing, we deduce that

$$u_{\varepsilon,1} \leq u_{\varepsilon} \quad \text{in} \quad Q.$$

Iterating this procedure we obtain sequences of nonnegative functions, bounded above by 1, $\{u_{\varepsilon,j}\}$ and $\{v_{\varepsilon,j}\}$ such that

$$v_{\varepsilon} \leq v_{\varepsilon,1} \leq v_{\varepsilon,2} \leq \cdots,$$

$$u_{\varepsilon} \geq u_{\varepsilon,1} \geq u_{\varepsilon,2} \geq \cdots,$$

and $u_{\varepsilon,j}$ are uniformly bounded from below by a negative constant independent of $\varepsilon, j$. The limits $U_{\varepsilon} = \lim_{j \to \infty} u_{\varepsilon,j}$, $V_{\varepsilon} = \lim_{j \to \infty} v_{\varepsilon,j}$ satisfy

$$v_{\varepsilon} \leq V_{\varepsilon}, \quad u_{\varepsilon} \geq U_{\varepsilon};$$

(4.13)

further, $(U_{\varepsilon}, V_{\varepsilon})$ form the solution of the penalized problem with $p = q$. By uniqueness for the penalized problem,

$$U_{\varepsilon}(x, t) = V_{\varepsilon}(-x, t).$$

By the proof of Theorem 4.1, for any limits

$$U = \lim_{\varepsilon \to 0} U_{\varepsilon}, \quad V = \lim_{\varepsilon \to 0} V_{\varepsilon}$$

there holds

$$U(x, t) > 0 \quad \text{if} \quad -1 < x < 0, \quad t > 0.$$

Since, by (4.13), the limit $u = \lim u_{\varepsilon}$ satisfies $u \geq U$, the assertion of the theorem follows.

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