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INTRODUCTION

The behavior of materials and waves in a dynamical setting offers many challenging nonlinear problems. The emphases of this workshop include phase transitions, change of type, asymptotic properties, and the interaction of waves with themselves and with matter. Attempts are made to reconcile theory with experiment. Methods of calculation will also be considered.

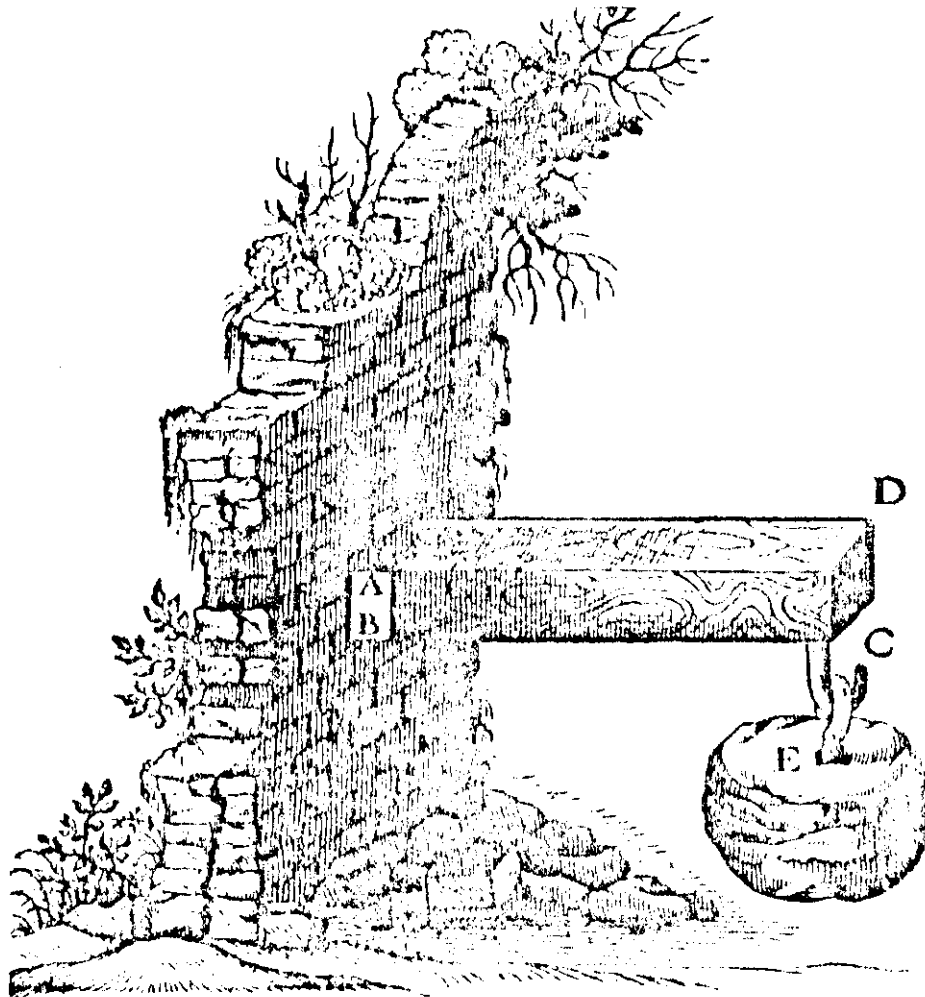
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Presence and absence of weak singularities in nonlinear waves

Michael Beals

Results and methods in the study of the propagation of smoothness and development of singularities for nonlinear strictly hyperbolic equations are surveyed. General semilinear and quasilinear equations are treated; a solution $u \in H^s(0)$ is assumed to exist on $0 \subset \mathbb{R}^n$ for s sufficiently large. As in the linear case, additional smoothness is propagated along null bicharacteristics, but only up to order roughly $2s - \frac{n}{2}$ in general. Nonlinear singularities develop, due to the interaction of linear singularities along characteristics which cross, and (if $n > 2$) due to self-interaction along a single characteristic. Natural conditions which curtail the singularities due to self-interaction in higher dimensions are examined.

**"Existence and asymptotic behaviour for strong
solutions of the Navier-Stokes equations in the
whole space",**

by H. Beirão da Veiga

Abstract. We consider the initial value problem for the non-stationary Navier-Stokes equation in the whole space \mathbb{R}^n . In the sequel, a denotes the initial velocity, f the exterior force field, p the pressure, v the velocity field, and $\mu > 0$ the viscosity. We assume that a and f are divergence free. $T \in]0, +\infty]$, is fixed. Our main concern will be the asymptotic behaviour of the solutions. We start by proving that if $a \in L^\alpha$, $f \in L^1(0, T; L^\alpha)$, where $L^\alpha \equiv [L^\alpha(\mathbb{R}^n)]^n$, then there exists a unique strong solution $v \in C([0, T_\alpha[; L^\alpha)$, which norm in L^α verifies the estimate $\|v(t)\|_\alpha \leq y(t)$, $\forall t \in [0, T_\alpha[$. T_α denotes the time existence of the maximal solution $y(t)$ of the o.d.e. $y' = k y^q + |f(t)|_\alpha$, with Cauchy data $y(0) = \|a\|_\alpha$. Here, $q = (3\alpha - n) / (\alpha - n)$, and $k = c\mu^{-\theta}$, $\theta = (n + \alpha) / (\alpha - n)$.

Concerning the global existence and the decay as $t \rightarrow +\infty$, we prove that if the norm of a in $L^2 \cap L^\alpha$, and the norm of f in $L^1(0, T; L^2) \cap L^\infty(0, T; L^\alpha)$ are sufficiently small (for brevity the bounds are not written here), then the solution exists in $[0, T]$ and verifies the estimate

$$\|v\|_{C(0, T; L^\alpha)} \leq c \left[\|a\|_2 + \|f\|_{L^1(0, T; L^2)} \right]^{-\beta},$$

where $\beta = 2(\alpha - n) / \alpha(n - 2)$. In particular, if $f \equiv 0$ and if $\|a\|_2^\beta \|a\|_\alpha \leq c_2$, then $\|v(t)\|_\alpha \leq c \|a\|_2 (\mu t)^{-\gamma}$, $\forall t \in]0, +\infty[$, where $\gamma = (\alpha - 2)n / 4\alpha$.

Similar results hold also for the limit case $\alpha = n$. Related results were proved in [1] and [2].

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A CONFLUENCE OF EXPERIMENT AND THEORY FOR WAVES OF
FINITE STRAIN IN THE SOLID CONTINUUM

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ABSTRACT:

With \mathbf{T}_R , $\mathbf{R}^T = \mathbf{R}^{-1}$, $\mathbf{V} = \mathbf{V}^T$, as the first Piola-Kirchhoff stress tensor, rigid body rotation, and pure homogeneous deformation respectively, experiment on finite strain plasticity discloses that great order and simplification prevail. One perceives this order not by attempting to relate a Cauchy stress tensor with Cauchy-Green strain tensors, which leads to great complication in the interpretation and unification of data, but by introducing and relating a stress tensor $\sigma = \mathbf{R}\mathbf{T}_R^T$ and a strain tensor $\mathbf{E} = \mathbf{V} - \mathbf{1}$, where $\sigma = \sigma^T$ and $\mathbf{E} = \mathbf{E}^T$. For isotropic solids in which the strain energy W depends only on the invariants of the strain tensor, \mathbf{E} , it follows that the energy statement, $dW = \text{trace } \mathbf{T}_R^T d\mathbf{F}$, and the polar decomposition theorem, $\mathbf{F} = \mathbf{V}\mathbf{R}$, lead directly to $dW = \text{trace } \sigma d\mathbf{E}$, thus correlating with the quantities of importance in experiment. The deformation gradient \mathbf{F} has components $\partial x^k / \partial X^m$, where X refers to the undeformed reference configuration.

Large plastic deformation in ordered solids is subject to an internal constraint, $\text{trace } \mathbf{E} = 0$, and its related stress which does no work. Together, these characterize every aspect of experiment. An admissible form for this stress which does no work, compatible with the internal constraint and with experiment in general, is $-(1/3)(\text{trace } \sigma)\mathbf{1}$. In these terms, this stress is not a hydrostatic component. The total stress \mathbf{S} becomes

$$\mathbf{S} = \sigma - (1/3)(\text{trace } \sigma)\mathbf{1}. \quad (1)$$

Introducing $T^2 = 2\text{II}^S$, and $(d\Gamma)^2 = \text{II}_{d\mathbf{E}}$, extensive experiment two decades ago revealed that the second invariants of the total stress and the increment of strain are related in a global statement

$$d\Gamma = 2TdT/\beta_a^2. \quad (2)$$

where β_a is a measured material constant.

The study of the response to non-proportional stress paths, from small strain to over 50%, has shown that the generalized strain Γ is path dependent. The incremental constitutive equations that follow are

$$dE = 2SdT/\beta_a^2. \quad (3)$$

Noting from the above that $T_R = \sigma R$, equilibrium equations become

$$\partial \sigma R / \partial X + \rho_R b = \rho_R \ddot{x}, \quad (4)$$

where ρ_R is the density in the undeformed reference configuration and b represents the body forces.

Within the framework of this incremental theory of finite strain plasticity is a quantum structure whose stable and unstable states are characterized by a specific set of relative reference configurations; they are designated by a set of values of the second invariant of the strain tensor II_E . Related to the stable stress in this transition structure are the metallurgical recipes for high strength metal alloys.

In the terms described above, fourteen documented experimental situations are introduced and briefly described. For the finite strain of crystalline solids, they encompass the dynamical problems in continuum physics that have exhibited a genuine confluence between continuum experiment and internally consistent continuum theory. The solutions to these dynamical problems in continuum physics constitute what I believe can be stated with assurance in this context. The problems discussed are:

- I. One Dimensional Waves of Finite Strain: Unaxial Stress (Compression)
- II. One Dimensional Waves of Finite Strain: Unaxial Stress (Tension)

- III. On the Interaction of Waves of Loading and Unloading
- IV. Wave Propagation at Finite Strain in a Relative Reference Configuration
- V. Finite Strain Waves at a Linear Elastic-Plastic Boundary
- VI. Non-Symmetrical Impact: The Analysis of an Historical Problem
- VII. The Incremental Wave in Pre-Stressed Solids: Metals
- VIII. The Incremental Wave in Pre-Stressed Solids: Rubber
- IX. The Surface Angle Measurement of Large Radial Motion for Waves in Rods
- X. Wave Propagation in a Three Dimensional Axially-Symmetric Field
- XI. The Elastic Limit and Outer Yield Surface in Dynamic Plasticity
- XII. On the Role of Dimensionless Universal Parameters at Finite Strain
- XIII. McReynolds-Dillon Slow Waves during Finite Strain in Ordered Solids
- XIV. On the Generation of Radial Shear Waves at a Plastic-Elastic Boundary

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Shallow-Water and Sediment Transport

Jerry Bona

In shallow-water zones the evolution of waves is strongly influenced by the topography over which they propagate. If the bed topography is made up of a movable substance such as sand, then it may in turn be shifted by the waves' action. The entire system in view, comprising both the water and the bottom surfaces, admits the possibility of complex self interaction.

A particular dynamical model of wave-bottom interaction will be proposed that is especially appropriate for describing the formation of sand bars in near-shore zones and sand ridges on continental shelves. The model is analyzed and a numerical scheme given for the approximation of its solutions. A confrontation of model predictions and laboratory and field data will be presented.

The acousto-elastic effects

Joseph B. Keller

Abstract. A theoretical analysis of the acoustoelastic effect is presented. It is based upon the theory of sound wave propagation in a stressed heterogeneous weakly anisotropic elastic medium composed of grains. The effect of residual stress is included, and shown to be different from that of applied stress. The statistics of grain orientation and of grain correlation are taken into account. The acoustoelastic coefficients and the effects of dispersion, attenuation and symmetry of the medium are determined.

ONE DIMENSIONAL FINITE AMPLITUDE PULSE PROPAGATION
IN ELECTROELASTIC SEMICONDUCTORS

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ABSTRACT:

The propagation of high frequency finite amplitude pulses in electroelastic semiconductors is examined using the techniques of modulated simple wave theory. The differential equations which govern the propagation of high frequency pulses are derived and their consequences are examined. Small amplitude high frequency pulses are examined in detail and the influence of the biasing electric field on the evolutionary behaviour of such waves is discussed in depth. The behaviour of pulses of finite amplitude which propagate into a region which is initially in a homogeneous steady state is also examined and the behaviour of such pulses is studied in a number of particular situations. In particular the conditions under which a finite amplitude pulse may become saturated are determined.

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ON WEAK SOLUTIONS FOR TRANSONIC FLOW

Cathleen S. Morawetz

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The flow of a two-dimensional compressible inviscid irrotational fluid past an arbitrary airfoil is studied using weak solutions and compensated compactness. To simplify the problem the airfoil is symmetric and the flow is reduced to flow past a smooth bump as a wall and inside a box.

If the speed is below cavitation speed on the boundary, the potential on the two end walls $x = 0$, $x = a$ is prescribed; its normal derivative is zero on $y = b$, and on the bump $y = Y(x)$. If no solution exists with the speed less than or equal to cavitation speed on $x = a$ or $y = Y(x)$ a free boundary is introduced on which the speed is cavitation speed and ϕ is constant.

Viscosity is introduced via the momentum equations. This leads to $v\nabla\rho \cdot \nabla\phi = -\rho(|\nabla\phi|^2 - q_B^2(\rho))$ where ρ is density, v is viscosity, q_B is speed as given by Bernoulli's law. The additional boundary condition $\rho = \rho_0$ on $x = 0$ is introduced.

Of course, the remaining equation is conservation of mass: $\operatorname{div} \rho \nabla\phi = 0$. If the solution exists for $v > 0$ it has several estimates that are independent of v and for example

$$|\nabla\phi|^2 - q_B^2(\rho) \rightarrow 0 \text{ in } L^2.$$

One establishes limits for ϕ and ψ , the stream functions. However to prove that the weak solution limits for ϕ, ψ satisfy the conservation of mass with $\rho = \rho(q)$ where q is the weak limit of $|\nabla\phi|$ requires the use of compensated compactness.

We need the additional hypothesis that in the interior as $v \rightarrow 0$ the speed remains less than cavitation speed and greater than zero. Then by introducing solutions of the Tricomi-like equation $K(\mu)H_{\theta\theta} + H_{\mu\mu} = 0$, $\mu H(\mu) \leq 0$, one is able to prove that as $v \rightarrow 0$, $f(q, \theta) \rightarrow \int f(q, \theta) d\nu$ and $d\nu$, the Young measure, is

a Dirac measure. The convergence is in H_{loc}^{-1} . Here θ is flow angle which is assumed bounded.

Reference

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EXTENDED THERMODYNAMICS OF IDEAL GASES

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ABSTRACT:

The purpose of Extended Thermodynamics of ideal gases is to describe the viscous and heat conductivity properties of gases by equations that are matrices in some points to the usual Navier-Stokes and Fourier equations. There are thirteen fields to be determined in Extended Thermodynamics, viz. mass density, velocity, stress and heat flux and thirteen field equations are needed. The form of the field equations is strongly motivated by the Kinetic Theory of Gases and they are closed by constitutive relations. The form of the constitutive functions is restricted by three requirements

- i) material objectivity
- ii) entropy inequality
- iii) hyperbolicity

It turns out that all constitutive functions can be determined from the thermal and caloric equations of state in equilibrium and from measurements of viscosity and heat conductivity. The resulting field equations predict finite speeds for shear waves and thermal waves and they describe the effects of inertia on stress and heat flux. The entropy flux is calculated and used to show that temperature is not in general continuous at a "thermometric" surface.

PHASE TRANSITIONS, ADMISSIBILITY AND STABILITY IN 1-D
NONLINEAR VISCOELASTICITY

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ABSTRACT:

A simple model proposed by Ericksen for elastic bars admitting phase transitions is closely related to equations of isentropic motions of a van der Waals gas and equations for shearing motion in fluids admitting hysteretic phase changes. In order to cope with the ill-posedness of the Cauchy problem in this model and the question of elastic stability, we consider the associated viscoelastic model of rate type, where the elastic stress is a nonmonotone function of the strain. From the limit of vanishing viscosity we derive an admissibility criterion of Shearer [2] for elastic waves of discontinuous strain. Shearer showed that this criterion implies the Riemann problem in the elastic model can always be uniquely solved.

Nonmonotonicity implies that equilibria with discontinuous strain occur with or without viscosity. Traditional static energy criteria for the stability of such equilibria require the solution to be a strong minimizer of a stored-energy functional, implying that the stored-energy density is the same in each phase. From a study of asymptotic behavior of solutions in the viscoelastic model (a study initiated by Dafermos, and Andrews and Ball [1]), with boundary conditions corresponding to a "soft" loading device, we find that equilibria which are extrema for the energy functional with positive second variation are dynamically stable. Such equilibria admit the coexistence of "metastable" and "stable" states of strain (the stored energy density may be higher in one phase than another), and occur as stable asymptotic limits of smooth solutions. Also, a precise description of the viscous smoothing of solutions in time versus persisting limited regularity in space is obtained from a simple existence theory based on the abstract theory of semilinear parabolic equations.

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Some related works:

R. Hagan and M. Slemrod, The viscosity-capillarity admissibility criterion for shocks and phase transitions, Arch. Rat. Mech. Anal. 83 (1983)333.

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Recent Results in Nonlinear Wave Equations

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We will discuss long time existence and asymptotic behaviour of small solutions of nonlinear wave equations. In particular we will show that in 3+1 dimensions a large class of equations admit global solutions whose asymptotic behaviour is the same as the linear problem.

DYNAMICAL PROBLEMS IN CONTINUUM PHYSICS

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ABSTRACT:

This talk discusses recent work of V. Roytburd and M. Slemrod on the equations $u_t + p(w)_x = 0$, $w_t - u_x = 0$, where p is taken to be of the type: van der Waals equation of state with the Maxwell line construction. We give L^∞ estimates for a regularized problem and discuss passage to the limit via L. Tartar's method of compensated compactness.

SOME EXISTENCE, UNIQUENESS AND NON UNIQUENESS RESULTS FOR
WEAKLY HYPERBOLIC EQUATIONS IN THE GEVREY CLASSES

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ABSTRACT:

We consider the Cauchy problem

$$(1) \quad u_{tt} - \nabla_x(a(t,x)\nabla_x u) + b(t,x)\nabla_x u + c(t,x)u = 0$$

$$(2) \quad u(0,x) = \phi(x), \quad u_t(0,x) = \psi(x)$$

with coefficients a, b, c which are C^∞ on $R_t \times R_x^n$ and satisfy the weak hyperbolicity condition:

$$a(t,x) \text{ is a real symmetric matrix } > 0.$$

It is well known (e.g. consider the equation $u_{tt} + u_x = 0$) that such a problem may be non well-posed in the class $(C^\infty(R_x^n))$. However Pb. $\{(1), (2)\}$ is well-posed in the class $A(R_x^n)$ of the real analytic functions, or more generally in any Gevrey class $E^s(R_x^n)$ with $1 < s < 2$, provided that the coefficients are analytic, or Gevrey, in the x -variables. The situation gets better when $b(t,x) \equiv 0$. For instance, the Cauchy problem for the equation

$$(3) \quad u_t - \nabla_x(a(t))\nabla_x u + c(t,x)u = 0$$

is "nearly well-posed" in C^∞ , in the sense that it is well-posed in any Gevrey class E^s with $s > \bar{s}$, if $c(t, \cdot)$ belongs to $E^{\bar{s}}$ for some $\bar{s} > 1$ (see [1]). On the other hand the well-posedness in C^∞ can fail, even for the simple equation

$$(4) \quad u_{tt} - a(t)u_{xx} = 0;$$

indeed we can find a C^∞ function $a(t) > 0$ and two C^∞ functions $\phi(x), \psi(x)$ for which Pb. $\{(4), (2)\}$ has no local solution in the class of distributions (see [2]).

A considerably stronger example of non well-posedness can be given, if we permit the coefficients to depend also on the x -variables: there exist two C^∞ functions, $a(t)$ and $c(t,x)$, with $a(t) > 0$ for $t > 0$ and $a(t) = c(t,x) = 0$ for $t \leq 0$, such that Pb. {(3),(2)} has non-unique C^∞ solutions (see [3]). Similar examples of non uniqueness can be constructed for strictly hyperbolic equations of the form (3), with $a(t)$ Hölder continuous and strictly positive. A peculiarity of such examples with respect to others (cf. [4]) is in that here the "frozen" equations

$$u_{tt} - a(\bar{t})u_{xx} + c(\bar{t},\bar{x})u = 0$$

are hyperbolic for each (\bar{t},\bar{x}) , uniformly with respect to (\bar{t},\bar{x}) (in the sense that the characteristic roots have imaginary parts which are uniformly bounded).

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ON THE DYNAMICS OF A COLLISIONLESS PLASMA

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ABSTRACT:

The dynamics of a hot dilute plasma are described by the Vlasov-Maxwell system, where the only forces are electromagnetic. Starting with smooth initial conditions, singularities could only be created by particles traveling at speeds arbitrarily close to the speed of light.

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