INVERSE CONDUCTIVITY PROBLEM WITH ONE MEASUREMENT: UNIQUENESS OF BALLS IN $\mathbb{R}^3$

By

Hyeonbae Kang

and

Jin Keun Seo

IMA Preprint Series # 1421
July 1996

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
UNIVERSITY OF MINNESOTA
514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455
INVERSE CONDUCTIVITY PROBLEM WITH ONE MEASUREMENT: UNIQUENESS OF BALLS IN $\mathbb{R}^3$

Hyeonbae Kang and Jin Keun Seo

Abstract. We prove that the location and size of an unknown ball $D$ entering the conductivity equation $\text{div}((1 + (k - 1)\chi_D)\nabla u) = 0$ in a bounded domain $\Omega \subset \mathbb{R}^3$ is uniquely determined by any single nonzero Cauchy data $(u, \frac{\partial u}{\partial \nu})$ on $\partial \Omega$.

1. Introduction

Let $\Omega$ and $D$ be both bounded Lipschitz domains in $\mathbb{R}^3$, with connected boundaries and $\overline{D} \subset \Omega$. Assume that the conductivity coefficient of $D$ is $k \neq 1, k > 0$ and the conductivity coefficient of $\Omega \setminus D$ is 1. We consider the inverse problem to recover an unknown domain $D$ from the relationship between a given current flux $g$ on $\partial \Omega$ and the measurement of the resulting potential $u$ on $\partial \Omega$. For a given current density $g \in L^2(\partial \Omega), \int_{\partial \Omega} g = 0$, the resulting potential $u$ satisfies the Neumann problem:

\begin{align}
\text{div}((1 + (k - 1)\chi_D)\nabla u) &= 0 \quad \text{in } \Omega, \\
\frac{\partial u}{\partial \nu} &= g \quad \text{on } \partial \Omega
\end{align}

where $\nu$ is the unit outer normal vector to the boundary and $u$ is normalized by $\int_{\partial \Omega} u = 0$. Here, $\chi_D$ is the characteristic function of $D$.

We are interested in the uniqueness question of the inverse problem in this paper: whether a single Cauchy data $(u|_{\partial \Omega}, g)$ is sufficient to determine $D$ uniquely. The global uniqueness results are obtained when $D$ is restricted to be convex polyhedron in three dimensional space and polygons and disks in the plane. (See [BFI], [BFS], [FI], [IP], [KS], [S].) But the uniqueness has not been known even for balls in three dimensions.

Authors are partially supported by GARC-KOSEF, KOSEF, and BSRI, 1996.
dimensional space and it has been open for a while (see a review paper of Isakov [I]). In this paper, we prove this uniqueness for balls in three dimensional space.

**Main Theorem.** Let $D_1$ and $D_2$ be balls whose closure are contained in $\Omega \subset \mathbb{R}^3$. Let $g \in L^2(\partial\Omega)$, $\int_{\partial\Omega} g = 0$ be any nonzero function. If the solutions $u_j$ of the Neumann problem (1.1)-(1.2) with $D = D_j$ satisfy $u_1 = u_2$ on $\partial\Omega$, then $D_1 = D_2$.

### 2. Properties of the Solution

Let $\Omega$ be a simply connected bounded Lipschitz domain in $\mathbb{R}^3$. Let $D = B_d(a)$ the ball centered at $a$ with radius $d$. In this section we derive some special properties of the solution to the equation (1.1). We first review the representation formula for the solution to (1.1)-(1.2).

**Theorem 2.1 [KS].** If $u$ is a weak solution to the Neumann problem (1.1)-(1.2), then there are a unique harmonic function $H \in W^{1,2}(\Omega)$ and a density function $\varphi_D \in L^2_0(\partial D)$ so that $u$ can be expressed as

\begin{equation}
(2.1) \quad u(X) = H(X) + S_D \varphi_D(X) \quad \text{for } X \in \Omega
\end{equation}

where

\begin{equation}
(2.2) \quad H(X) = S_\Omega(-g) + D_\Omega(f), \quad f = u|_{\partial\Omega},
\end{equation}

\begin{equation}
(2.3) \quad (\lambda I - K_D^*) \varphi_D = \frac{\partial H}{\partial n} |_{\partial D} \quad \text{on } \partial D
\end{equation}

where $\lambda = \frac{k+1}{2(k-1)}$. Here $S_\Omega$ and $D_\Omega$ denote the single and double layer potential operators, respectively and

$$K_D^* \varphi(P) = \int_{\partial D} \langle \nu_P, \nabla \Gamma(P - Q) \rangle \varphi(Q) d\sigma_Q, \quad P \in \partial D$$

where $\Gamma(X)$ is the fundamental solution of $\Delta$.

This representation formula holds for a Lipschitz domain $D$. Necessary terminology and a proof can be found in [KS] and [KSS].
Let \( u = H + S_D \varphi_D \) be the weak solution of (1.1)-(1.2). Let

\[(2.4) \quad u^i = u \chi_D \quad \text{and} \quad u^e = u \chi_{\Omega \setminus \overline{D}}.\]

Then \( u \) satisfies the transmission conditions:

\[(2.5) \quad u^e = u^i \quad \text{and} \quad \frac{\partial u^e}{\partial \nu} = k \frac{\partial u^i}{\partial \nu} \quad \text{on} \quad \partial D.\]

Denote \( \hat{x} = \frac{x}{|x|} \). Let \( Y_n^m(\hat{x}) \) \((m = -n, -n+1, \ldots, 0, 1, \ldots, n)\) be the orthonormal spherical harmonics of degree \( n \). (See [CK] for the spherical harmonics.) Then \(|x|^n Y_n^m(\hat{x})\) is harmonic in \( \mathbb{R}^3 \).

**Lemma 2.2.** Suppose that, for \(|x - a| < d\),

\[
u^i(x) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_n c_n^m |x - a|^n Y_n^m(\hat{x} - a).\]

Then \( u = H + S_D \varphi D \) in \( \Omega \) can be expanded as follows

\[(2.6) \quad H(x) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_n c_n^m |x - a|^n Y_n^m(\hat{x} - a), \quad |x - a| < \inf \{|y - a| : y \in \partial \Omega\},\]

\[(2.7) \quad S_D \varphi D(x) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \beta_n c_n^m \frac{d^{2n+1}}{|x - a|^{n+1}} Y_n^m(\hat{x} - a), \quad |x - a| > d\]

\[(2.8) \quad S_D \varphi D(x) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \beta_n c_n^m |x - a|^n Y_n^m(\hat{x} - a) \quad |x - a| < d.\]

where

\[
\alpha_n = \frac{(k+1)n+1}{2n+1} \quad \text{and} \quad \beta_n = \frac{(1-k)n}{2n+1}.
\]

**Proof.** Using the transmission conditions (2.5), we can see that

\[u^e(x) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left( \alpha_n |x - a|^n + \beta_n \frac{d^{2n+1}}{|x - a|^{n+1}} \right) c_n^m Y_n^m(\hat{x} - a), \quad d < |x - a| < \inf \{|y - a| : y \in \partial \Omega\}.\]

The result follows from the uniqueness of \( H \) and \( S_D \varphi D \). \( \square \)

When \( D = B_d(a) \), we denote

\[x^*(D) := a + \frac{d^2(x - a)}{|x - a|^2}, \quad x \in \mathbb{R}^3.\]

For a set \( A \), we set

\[A^*(D) := \{x^*(D) : x \in A\}.
\]
Lemma 2.3.

\begin{equation}
K^*_D \varphi(P) = -\frac{1}{2d} S_D \varphi(P), \quad P \in \partial D
\end{equation}

for every \( \varphi \in C(\partial D) \).

\begin{equation}
\langle x - a, \nabla S_D \varphi_D(x) \rangle = -\frac{k-1}{k+1} \langle x - a, \nabla H(x) \rangle - \frac{1}{k+1} S_D \varphi_D(x), \quad x \in D,
\end{equation}

\begin{equation}
S_D \varphi_D(x) = \frac{d}{|x-a|} S_D \varphi_D(x^*(D)), \quad x \in \mathbb{R}^3.
\end{equation}

\textbf{Proof.} Let us assume the center \( a \) of \( D \) is 0 for convenience. It follows from a direct computation that, for \( P \in \partial D \),

\[ \frac{\langle P - Q, \nu(P) \rangle}{|P - Q|^3} = \frac{1/d(d^2 - \langle P, Q \rangle)}{(2d^2 - 2\langle P, Q \rangle)^{3/2}} = \frac{1}{2d|P - Q|} \]

and hence (2.9) follows.

From (2.3), (2.9), and the jump formula for the single layer potential, we obtain for \( P \in \partial D \)

\[ \langle \hat{P}, \nabla S_D \varphi_D(P) \rangle = -\frac{1}{2} \varphi_D(P) + K^*_D \varphi_D(P) \]

\[ = -\frac{1}{2\lambda} (\lambda I - K^*_D) \varphi_D(P) + (1 - \frac{1}{2\lambda}) K^*_D \varphi_D(P) \]

\[ = -\frac{1}{2\lambda} \langle \hat{P}, \nabla H(P) \rangle - \frac{2\lambda - 1}{4\lambda d} S_D \varphi_D(P) \]

\[ = -\frac{k-1}{k+1} \langle \hat{P}, \nabla H(P) \rangle - \frac{1}{(k+1)d} S_D \varphi_D(P). \]

Since \( \langle x, \nabla S_D \varphi_D(x) \rangle \) and \( \langle x, \nabla H(x) \rangle \) are harmonic in \( D \),

\[ \langle x, \nabla S_D \varphi_D(x) \rangle = -\frac{k-1}{k+1} \langle x, \nabla H(x) \rangle - \frac{1}{k+1} S_D \varphi_D(x) \quad \text{in} \ D. \]

We now prove (2.11): for \( |x| < d \)

\[ \frac{d}{|x|^2} S_D \varphi_D \left( \frac{d^2 x}{|x|^2} \right) = \sum_{n=0}^{\infty} \sum_{n=-m}^{m} \beta_n c_n^m |x|^n Y_n^m(\hat{z}) \]

\[ = u_1(x) - H(x) \]

\[ = S_D \varphi_D(x). \]

This completes the proof. \( \square \)

From Lemma 2.3, we have the following corollary.
Corollary 2.4. Let $\rho$ be the smallest positive number $s$ so that the harmonic function $S_D\varphi_D$ in $\mathbb{R}^3 \setminus \overline{D}$ can be harmonically extended to the set $\mathbb{R}^3 \setminus \overline{B_s(a)}$. Let $\gamma$ be the biggest positive number $s$ so that the harmonic function $S_D\varphi_D$ in $D$ can be harmonically extended to the set $B_s(a)$. Then

\begin{align}
\rho < d < \gamma, \\
\gamma = \frac{d^2}{\rho}.
\end{align}

Proof. Since the series of $H$ in (2.6) converges for $|x - a| < \inf\{|y - a| : y \in \partial\Omega\}$, (2.12) follows from (2.7) and (2.8). (2.13) follows from (2.11). This completes the proof. □

3. The proof of the Main Theorem

Let $D_1 = B_{d_1}(a_1)$ and $D_2 = B_{d_2}(a_2)$. We may assume, by translation and rotation if necessary, $a_1 = 0$ and $a_2 = (a, 0, 0)$. For a given nonzero $g \in L^2_0(\partial\Omega)$, let $u_j, j = 1, 2$, be weak solutions to the Neumann problem (1.1)-(1.2) for $D = D_j$. Suppose that $u_1 = u_2 = f$ on $\partial\Omega$. It suffices to prove $a = 0$. Indeed, if $a = 0$, then either $D_1 \subset D_2$ or $D_2 \subset D_1$ in which cases (monotone cases) it is well known that $D_1 = D_2$ (see [A] or [BFI]).

To obtain a contradiction, suppose that $a > 0$ without loss of generality. By the representation formula, there exist unique $H$ and $\varphi_j \in L^2(\partial D_j)$ so that

\begin{equation}
 u_j = H + S_{D_j}\varphi_j \quad \text{in } \Omega.
\end{equation}

Put

\begin{align}
S_j^i(x) &= S_{D_j}\varphi_j(x) \quad \text{for } |x - a_j| \leq d_j \\
S_j^e(x) &= S_{D_j}\varphi_j(x) \quad \text{for } |x - a_j| \geq d_j.
\end{align}

Let $\rho_j$ and $\gamma_j$ as in Corollary 2.4 for $D = D_j$ $(j = 1, 2)$. Since

\begin{equation}
S_1^e = S_{D_1}\varphi_1 = S_{D_2}\varphi_2 = S_2^e \quad \text{in } \mathbb{R}^3 \setminus \overline{D_1 \cup D_2},
\end{equation}
it follows from the unique continuation of the harmonic functions that the harmonic function $S_j^\varepsilon$ can be extended to the set $\mathbb{R}^3 \setminus B_{\rho_1}(a_1) \cap B_{\rho_2}(a_2)$. Put

(3.2) \[ T_j = (\mathbb{R}^3 \setminus B_{\rho_1}(a_1) \cap B_{\rho_2}(a_2))^*(D_j) \quad j = 1, 2. \]

Then by (2.11), $S_j^i$ can be harmonically extended to $T_j$.

**Lemma 3.1.** The harmonic function $H$ can be extended to any convex subset of $T_1 \cup T_2$ containing $a_j$.

**Proof.** Since $S_j^1$ is harmonic in $T_j$, one sees from (2.10) that $\langle x - a_j, \nabla H(x) \rangle$ can be extended to $T_j$. Since

\[ H(x) - H(a_j) = \int_0^1 (t(x - a_j), \nabla H(t(x - a_j) + a_j)) \frac{dt}{t}, \]

$H$ can be extended to any convex subset of $T_j$ containing $a_j$. This completes the proof. \(\square\)

Since $S_1^\varepsilon = S_2^\varepsilon$ in $\mathbb{R}^3 \setminus (B_{\rho_1}(a_1) \cap B_{\rho_2}(a_2))$, it must be

\[ \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2) \neq \emptyset \]

since, otherwise, $S_j^\varepsilon$ are harmonically extended to the entire space $\mathbb{R}^3$ in which case $S_j^\varepsilon = 0$. By the relation (2.13)

\[ \partial B_{\gamma_1}(a_1) \cap \partial B_{\gamma_2}(a_2) \neq \emptyset. \]

Let us denote

(3.3) \[ \eta := \langle Q, e_1 \rangle \quad \text{where} \quad Q \in \partial B_{\gamma_1}(a_1) \cap \partial B_{\gamma_2}(a_2). \]

(this is well defined because $a_1$ and $a_2$ lie on $e_1$-axis.) Because $a_1 = 0$ and $a_2 = (a, 0, 0), a > 0$, it is easy to see that

(3.4) \[ -\gamma_1 \leq a \leq \eta \leq \gamma_1 \leq \gamma_2 + a, \]

(3.5) \[ \{ x \in \partial B_{\gamma_1}(a_1) : \eta < \langle x, e_1 \rangle \} \subset B_{\gamma_1}(a_2), \]

(3.6) \[ \{ x \in \partial B_{\gamma_2}(a_2) : \langle x, e_1 \rangle < \eta \} \subset B_{\gamma_1}(a_1). \]
Let us fix $P_0 \in \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)$.

Case 1: $(P_0)^*(D_1) \neq (P_0)^*(D_2)$.

Suppose that $(P_0)^*(D_1) \neq (P_0)^*(D_2)$.

**Lemma 3.2.** If $P \in \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)$, then $P^*(D_1) \neq P^*(D_2)$ and

\[
\eta \leq \langle P^*(D_2), e_1 \rangle \quad \text{implies} \quad \eta < \langle P^*(D_1), e_1 \rangle,
\]

\[
\langle P^*(D_1), e_1 \rangle \leq \eta \quad \text{implies} \quad \langle P^*(D_2), e_1 \rangle < \eta.
\]

**Proof.** Let us denote by $L[Q_1, Q_2]$ the line joining the two points $Q_1$ and $Q_2$. By definition, the point $P$ lies on the intersection of the two line $L[P^*(D_1), a_1]$ and $L[P^*(D_2), a_2]$. So, if $P^*(D_1) = P^*(D_2)$, then the two points $P$ and $P^*(D_1) = P^*(D_2)$ must lie on the $e_1$-axis in which case the set $\partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)$ is a single point $P = P_0$. This contradicts to the assumption.

Now, we prove (3.7). If $\langle P^*(D_1), e_1 \rangle \leq \eta \leq \langle P^*(D_2), e_1 \rangle$, then two lines $L[P^*(D_2), a_2]$ and $L[P^*(D_1), a_1]$ do not have any intersecting point and this is not possible. Similarly, (3.8) can be obtained. $\Box$

**Lemma 3.3.** For at least one of $j = 1, 2$, the following holds:

\[
(\partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2))^*(D_j) \subset B_{\gamma_i}(a_i), \quad (i = 1, 2, \ i \neq j).
\]

**Proof.** By the symmetry it suffices to show (3.9) for one point $P \in \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)$, i.e., $P^*(D_j) \not\in B_{\gamma_i}(a_i) \quad (i \neq j)$.

Suppose that $P \in \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)$ and $P^*(D_2) \not\in B_{\gamma_1}(a_1)$. Then by (3.6)

\[
\eta \leq \langle P^*(D_2), e_1 \rangle
\]

and hence by (3.7)

\[
\eta < \langle P^*(D_1), e_1 \rangle.
\]

Then Lemma 3.3 follows from (3.5). $\Box$
Let us suppose that

\[(\partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2))^*(D_1) \subset B_{\gamma_2}(a_2).\]

Then

\[(3.10) \quad (\partial B_{\rho_1}(a_1) \cap \overline{B_{\rho_2}(a_2)})^*(D_1) \subset B_{\gamma_2}(a_2).\]

In fact, if \(x \in \partial B_{\rho_1}(a_1) \cap \overline{B_{\rho_2}(a_2)}\), then

\[\langle x^*(D_1), e_1 \rangle \geq \langle P^*(D_1), e_1 \rangle > \eta \quad \text{for} \ P \in \partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2).\]

Thus by (3.5), we obtain (3.10).

Since \((\partial B_{\rho_1}(a_1) \setminus \overline{B_{\rho_2}(a_2)})^*(D_1) \subset T_1\) by the definition of \(T_1\), we have

\[(\partial B_{\rho_1}(a_1))^*(D_1) \subset T_1 \cup B_{\gamma_2}(a_2)\]

and therefore

\[\overline{B_{\gamma_1}(a_1)} \subset T_1 \cup B_{\gamma_2}(a_2).\]

Since \(T_1 \cup B_{\gamma_2}(a_2)\) is an open set, there exists a positive constant \(\epsilon\) so that

\[B_{\gamma_1+\epsilon}(a_1) \subset T_1 \cup B_{\gamma_2}(a_2).\]

From Lemma 3.1, the harmonic function \(H\) can be extended to the set \(B_{\gamma_1+\epsilon}(a_1)\) and hence the series of \(H\) in (2.6) converges for \(|x-a_1| < \gamma_1 + \epsilon\). From Lemma 2.2, the series (2.8) for \(S_1^i\) also converges for \(|x-a_1| < \gamma_1 + \epsilon\). Hence, by Lemma 2.3, \(S_1^e\) can be extended to the set \(\mathbb{R}^3 \setminus \overline{B_{\rho_1}-\delta}\) where \(\delta = \frac{d^2}{\gamma_i} - \frac{d^2}{\gamma_i+\epsilon}\), which contradicts to the definition of \(\rho_1\). This completes the proof of the Main Theorem for the Case 1.

Case 2: \((P_0)^*(D_1) = (P_0)^*(D_2)\).

As we saw in the proof of Lemma 3.2, the two points \(P_0\) and \((P_0)^*(D_1) = (P_0)^*(D_2)\) lie on \(e_1-\text{axis}\) and the set \(\partial B_{\rho_1}(a_1) \cap \partial B_{\rho_2}(a_2)\) is a single point \(P = P_0\). In this case, there are only two possibilities:

1. \(\rho_1 + a = \rho_2\) and \(\gamma_1 + a = \gamma_2\),
2. \(\rho_1 = \rho_2 + a\) and \(\gamma_1 = \gamma_2 + a\).
We only deal with the first possibility (1). We have

\[ d_2^2 = \rho_2 \gamma_2 = (\rho_1 + a)(\gamma_1 + a) = (\rho_1 + a)(\frac{d_1^2}{\rho_1} + a). \]

It follows from a direct computation that

\[ d_2^2 = (\rho_1 + a)(\frac{d_1^2}{\rho_1} + a) \geq (d_1 + a)^2 \]

and therefore \( D_1 \subset D_2 \) which is the monotone case. Hence, by a result of [A] (or [BFI]) \( D_1 = D_2 \) and \( a = 0 \). This completes the proof. \( \square \).

REFERENCES


Hyonbae Kang, Department of Mathematics, Korea University, Seoul 136-701, Korea
E-mail address: kang@semi.korea.ac.kr

Jin Keun Seo, Department of Mathematics, Yonsei University, Seoul 120-749, Korea
E-mail address: seoj@bubble.yonsei.ac.kr
<table>
<thead>
<tr>
<th>#</th>
<th>Author/s</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1325</td>
<td>M. Luskin</td>
<td>Approximation of a laminated microstructure for a rotationally invariant, double well energy density</td>
</tr>
<tr>
<td>1326</td>
<td>Rakesh &amp; P. Sacks</td>
<td>Impedance inversion from transmission data for the wave equation</td>
</tr>
<tr>
<td>1327</td>
<td>O. Lafitte</td>
<td>Diffraction for a Neumann boundary condition</td>
</tr>
<tr>
<td>1328</td>
<td>E. Sobel, K. Lange, J.R. O’Connell &amp; D.E. Weeks</td>
<td>Haplotyping algorithms</td>
</tr>
<tr>
<td>1329</td>
<td>B. Cockburn, D.A. Jones &amp; E.S. Titiri</td>
<td>Estimating the number of asymptotic degrees of freedom for nonlinear dissipative systems</td>
</tr>
<tr>
<td>1330</td>
<td>T. Aktosun</td>
<td>Inverse Schrödinger scattering on the line with partial knowledge of the potential</td>
</tr>
<tr>
<td>1331</td>
<td>T. Aktosun &amp; C. Van der Mee</td>
<td>Partition of the potential of the one-dimensional Schrödinger equation</td>
</tr>
<tr>
<td>1332</td>
<td>B. Engquist &amp; E. Luo</td>
<td>Convergence of the multigrid method with a wavelet coarse grid operator</td>
</tr>
<tr>
<td>1333</td>
<td>V. Jakšič &amp; C.-A. Pillet</td>
<td>Ergodic properties of the Spin-Boson system</td>
</tr>
<tr>
<td>1334</td>
<td>S.K. Patch</td>
<td>Recursive solution for diffuse tomographic systems of arbitrary size</td>
</tr>
<tr>
<td>1335</td>
<td>J.C. Bronski</td>
<td>Semiclassical eigenvalue distribution of the non self-adjoint Zakharov-Shabat eigenvalue problem</td>
</tr>
<tr>
<td>1336</td>
<td>J.C. Cockburn</td>
<td>Bitangential structured interpolation theory</td>
</tr>
<tr>
<td>1337</td>
<td>S. Kichenassamy</td>
<td>The blow-up problem for exponential nonlinearities</td>
</tr>
<tr>
<td>1338</td>
<td>F.A. Grünbaum &amp; S.K. Patch</td>
<td>How many parameters can one solve for in diffuse tomography?</td>
</tr>
<tr>
<td>1339</td>
<td>R. Lipton</td>
<td>Reciprocal relations, bounds and size effects for composites with highly conducting interface</td>
</tr>
<tr>
<td>1340</td>
<td>H.A. Levine &amp; J. Serrin</td>
<td>A global nonexistence theorem for quasilinear evolution equations with dissipation</td>
</tr>
<tr>
<td>1341</td>
<td>A. Boutet de Monvel &amp; R. Purice</td>
<td>The conjugate operator method: Application to DIRAC operators and to stratified media</td>
</tr>
<tr>
<td>1342</td>
<td>G. Michele Graf</td>
<td>Stability of matter through an electrostatic inequality</td>
</tr>
<tr>
<td>1343</td>
<td>G. Avalos</td>
<td>Sharp regularity estimates for solutions of the wave equation and their traces with prescribed Neumann data</td>
</tr>
<tr>
<td>1344</td>
<td>G. Avalos</td>
<td>The exponential stability of a coupled hyperbolic/parabolic system arising in structural acoustics</td>
</tr>
<tr>
<td>1345</td>
<td>G. Avalos &amp; I. Lasiecka</td>
<td>A differential Riccati equation for the active control of a problem in structural acoustics</td>
</tr>
<tr>
<td>1346</td>
<td>G. Avalos</td>
<td>Well-posedness for a coupled hyperbolic/parabolic system seen in structural acoustics</td>
</tr>
<tr>
<td>1347</td>
<td>G. Avalos &amp; I. Lasiecka</td>
<td>The strong stability of a semigroup arising from a coupled hyperbolic/parabolic system</td>
</tr>
<tr>
<td>1348</td>
<td>A.V. Fursikov</td>
<td>Certain optimal control problems for Navier-Stokes system with distributed control function</td>
</tr>
<tr>
<td>1349</td>
<td>F. Gesztesy, R. Nowell &amp; W. Pötz</td>
<td>One-dimensional scattering theory for quantum systems with nontrivial spatial asymptotics</td>
</tr>
<tr>
<td>1350</td>
<td>F. Gesztesy &amp; H. Holden</td>
<td>On trace formulas for Schrödinger-type operators</td>
</tr>
<tr>
<td>1351</td>
<td>X. Chen</td>
<td>Global asymptotic limit of solutions of the Cahn-Hilliard equation</td>
</tr>
<tr>
<td>1352</td>
<td>X. Chen</td>
<td>Lorenz equations, Part I: Existence and nonexistence of homoclinic orbits</td>
</tr>
<tr>
<td>1353</td>
<td>X. Chen</td>
<td>Lorenz equations Part II: “Randomly” rotated homoclinic orbits and chaotic trajectories</td>
</tr>
<tr>
<td>1354</td>
<td>X. Chen</td>
<td>Lorenz equations, Part III: Existence of hyperbolic sets</td>
</tr>
<tr>
<td>1356</td>
<td>C. Liu</td>
<td>The Helmholtz equation on Lipschitz domains</td>
</tr>
<tr>
<td>1357</td>
<td>G. Avalos &amp; I. Lasiecka</td>
<td>Exponential stability of a thermoelastic system without mechanical dissipation</td>
</tr>
<tr>
<td>1358</td>
<td>R. Lipton</td>
<td>Heat conduction in fine scale mixtures with interfacial contact resistance</td>
</tr>
<tr>
<td>1359</td>
<td>V. Odisharia &amp; J. Peradze</td>
<td>Solvability of a nonlinear problem of Kirchhoff shell</td>
</tr>
<tr>
<td>1360</td>
<td>P.J. Olver, G. Sapiro &amp; A. Tannenbaum</td>
<td>Affine invariant edge maps and active contours</td>
</tr>
<tr>
<td>1361</td>
<td>R.D. James</td>
<td>Hysteresis in phase transformations</td>
</tr>
<tr>
<td>1362</td>
<td>A. Sei &amp; W. Symes</td>
<td>A note on consistency and adjointness for numerical schemes</td>
</tr>
<tr>
<td>1363</td>
<td>A. Friedman &amp; B. Hu</td>
<td>Head-media interaction in magnetic recording</td>
</tr>
<tr>
<td>1364</td>
<td>A. Friedman &amp; J.J.L. Velázquez</td>
<td>Time-dependent coating flows in a strip, part I: The linearized problem</td>
</tr>
<tr>
<td>1365</td>
<td>X. Ren &amp; M. Winter</td>
<td>Young measures in a nonlocal phase transition problem</td>
</tr>
<tr>
<td>1366</td>
<td>K. Bhattacharya &amp; R.V. Kohn</td>
<td>Elastic energy minimization and the recoverable strains of polycrystalline shape-memory materials</td>
</tr>
<tr>
<td>1367</td>
<td>G.A. Chechkin</td>
<td>Operator pencil and homogenization in the problem of vibration of fluid in a vessel with a fine net on the surface</td>
</tr>
<tr>
<td>1368</td>
<td>M. Carme Calderer &amp; B. Mukherjee</td>
<td>On Poiseuille flow of liquid crystals</td>
</tr>
<tr>
<td>1369</td>
<td>M.A. Pinsky &amp; M.E. Taylor</td>
<td>Pointwise Fourier inversion: A wave equation approach</td>
</tr>
<tr>
<td>1370</td>
<td>D. Brandon &amp; R.C. Rogers</td>
<td>Order parameter models of elastic bars and precursor oscillations</td>
</tr>
<tr>
<td>1371</td>
<td>H.A. Levine &amp; B.D. Sleeman</td>
<td>A system of reaction diffusion equations arising in the theory of reinforced random walks</td>
</tr>
<tr>
<td>1372</td>
<td>B. Cockburn &amp; P.-A. Gremaud</td>
<td>A priori error estimates for numerical methods for scalar conservation laws. Part II: Flux-splitting monotone schemes on irregular Cartesian grids</td>
</tr>
</tbody>
</table>
1373 B. Li & M. Luskin, Finite element analysis of microstructure for the cubic to tetragonal transformation
1374 M. Luskin, On the computation of crystalline microstructure
1375 J.P. Matos, On gradient young measures supported on a point and a well
1376 M. Nitsche, Scaling properties of vortex ring formation at a circular tube opening
1377 J.L. Bona & Y.A. Li, Decay and analyticity of solitary waves
1378 V. Isakov, On uniqueness in a lateral cauchy problem with multiple characteristics
1379 M.A. Kouritzin, Averaging for fundamental solutions of parabolic equations
1380 T. Aktosun, M. Klaus & C. van der Mee, Integral equation methods for the inverse problem with discontinuous wavespeed
1381 P. Morin & R.D. Spies, Convergent spectral approximations for the thermomechanical processes in shape memory allows
1382 D.N. Arnold & X. Liu, Interior estimates for a low order finite element method for the Reissner-Mindlin plate model
1383 D.N. Arnold & R.S. Falk, Analysis of a linear-linear finite element for the Reissner-Mindlin plate model
1384 D.N. Arnold, R.S. Falk & R. Winther, Preconditioning in $H(\text{div})$ and applications
1385 M. Lavrentiev, Nonlinear parabolic problems possessing solutions with unbounded gradients
1386 O.P. Bruno & P. Laurence, Existence of three-dimensional toroidal MHD equilibria with nonconstant pressure
1387 O.P. Bruno, F. Reitich, & P.H. Leo, The overall elastic energy of polycrystalline martensitic solids
1388 M. Fila & H.A. Levine, On critical exponents for a semilinear parabolic system coupled in an equation and a boundary condition
1390 J.M. Berg & H.G. Kwatny, Unfolding the zero structure of a linear control system
1391 A. Sei, High order finite-difference approximations of the wave equation with absorbing boundary conditions: A stability analysis
1392 A.V. Coward & Y.Y. Renardy, Small amplitude oscillatory forcing on two-layer plane channel flow
1393 V.A. Pliss & G.R. Sell, Approximation dynamics and the stability of invariant sets
1394 J.G. Cao & P. Roblin, A new computational model for heterojunction resonant tunneling diode
1395 C. Liu, Inverse obstacle problem: Local uniqueness for rougher obstacles and the identification of a ball
1396 K.A. Pericak-Spector & S.J. Spector, Dynamic cavitation with shocks in nonlinear elasticity
1397 G. Avalos & I. Lasiecka, Exponential stability of a thermoelastic system without mechanical dissipation II: The case of simply supported boundary conditions
1398 B. Brighi & M. Chipot, Approximation of infima in the calculus of variations
1399 G. Avalos, Concerning the well-posedness of a nonlinearly coupled semilinear wave and beam-like equation
1400 R. Lipton, Variational methods, bounds and size effects for composites with highly conducting interface
1401 B.T. Hayes & P.G. LeFloch, Non-classical shock waves in scalar conservation laws
1402 K.T. Joseph & P.G. LeFloch, Boundary layers in weak solutions to hyperbolic conservation laws
1403 Y. Diao, C. Ernst, & E.J.J. Van Rensburg, Energies of knots
1404 Xiaofeng Ren, Multi-layer local minimum solutions of the bistable equation in an infinite tube
1405 Vlastimil Pták, Krylov sequences and orthogonal polynomials
1406 T. Aktosun, M. Klaus, & C. van der Mee, Factorization of scattering matrices due to aptionition of potentials in one-dimensional Schrödinger-type equations
1408 D.N. Arnold, R.S. Falk, & R. Winther, Preconditioning discrete approximations of the Reissner-Mindlin plate model
1409 M.A. Kouritzin, On exact filters for continuous signals with discrete observations
1410 R. Lipton, The second Stekloff eigenvalue and energy dissipation inequalities for functionals with surface energy
1411 R. Lipton, The second Stekloff eigenvalue of an inclusion and new size effects for composites with imperfect interface
1412 W. Littman & B. Liu, The regularity and singularity of solutions of certain elliptic problems on polygonal domains
1413 C.R. Collins, Spurious oscillations are not fatal in computing microstructures
1414 M.A. Horn, Sharp trace regularity for the solutions of the equations of dynamic elasticity
1415 A. Friedman, B. Hu & Y. Liu, A boundary value problem for the Poisson equation with multi-scale oscillating boundary
1416 P. Baumann, D. Phillips & Q. Tang, Stable nucleation for the Ginzburg-Landau system with an applied magnetic field
1417 J.M. Berg, A strain profile for robust control of microstructure using dynamic recrystallization
1418 P. Klouček, Toward the computational modeling of nonequilibrium thermodynamics of the Martensitic transformations
1419 S. Chawla & S.M. Lenhart, Application of optimal control theory to in Situ bioremediation
1420 B. Li & M. Luskin, Nonconforming finite element approximation of crystalline microstructure
1421 H. Kang & J.K. Seo, Inverse conductivity problem with one measurement: Uniqueness of balls in $\mathbb{R}^3$