SOLITON’S REBUILDING IN
ONE-DIMENSIONAL SCHRÖDINGER MODEL
WITH POLYNOMIAL NONLINEARITY*

VALERY E. GRIKUROV †

Abstract. The solitons of the one-dimensional NLS equation are considered. We suppose that the nonlinearity 1) provides the existence of both stable and unstable solitons and 2) limits the possible solitonic parameters by some critical relation (the simplest representative is a two-terms polynomial). The first part of the paper is devoted to the perturbation of an unstable soliton, and the solution describing its rebuilding is proposed. In the second part the collision of solitons is numerically simulated, and the rebuilding of solitons is established: if solitonic parameters are close to the critical ones, then in a short time after the collision the new stuff of solitons is formed.

Key words. nonlinear Schrödinger (NLS) equation, nonintegrable, polynomial nonlinearity, soliton, stability, perturbation series, collisions, long-time behaviour

AMS(MOS) subject classifications. 35Q40, 35Q51, 35Q55, 35Q60, 65C20, 65Mxx

1. Introduction. The nonlinear Schrödinger (NLS) equation

\[ \psi_t = -\Delta \psi + F \left( |\psi|^2 \right) \psi \tag{1.1} \]

arises in the mathematical description of the propagation of lasers through nonlinear optic materials and Langmuir waves in a plasma. Under certain conditions on the nonlinearity function \( F(\xi) \) the equation (1.1) admits a one-parametric family of localized, finite energy travelling wave solutions — solitary waves or, simply, solitons. These solutions were extensively studied in recent years since its are of special importance due to the distinguished role they sometimes play in the long-time asymptotics of the initial value problem for nonlinear evolution equations.

The brief review of the known results should be started from the famous “cubic” one-dimensional NLS \( F(\xi) = \pm \xi \) which is a completely integrable system. This implies, in particular, that, for the focusing case (the minus sign), the long-time asymptotics of the Cauchy problem with rapidly vanishing initial data consists of a finite number of solitons and a dispersive radiation which is governed by the free linear equation. For example, if the initial function equals to the sum of well separated solitons, when the radiation is (almost) zero and the long-time solitons (possibly) differ from the initial ones only in phase shifts (in this context one can talk about superstability of solitons). Among a lot of monographs and papers on this subject we refer to [2], [1].

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† Dept. Comp. & Math. Phys., Inst. on Phys., St.Petersburg Univ. 1 Uljanovskaia Ave., Petrodvoretz 198904 Russia

grikurov@phim.niif.spb.su