An Inverse Obstacle Problem: A Uniqueness Theorem for Spheres

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Abstract

In the scattering of time-harmonic acoustic or electromagnetic waves, whether an impenetrable sound-soft obstacle \( \Omega \) can be completely determined by the scattering amplitude (or the far field pattern) \( A_\Omega(\xi, k) \) given for \( |\xi|^2 = |k|^2 \) at fixed wave number \( |k| \) and fixed incident plane wave direction \( k \) is still a question. In this paper, we show that any sphere in \( \mathbb{R}^n (n \geq 3) \) can be uniquely determined by its scattering amplitude \( A_\Omega(\cdot, k) \) given at two linearly independent incident directions \( \hat{k}_1 \) and \( \hat{k}_2 \) with one fixed wave number \( |k| \). We also show that two spheres in \( \mathbb{R}^n (n \geq 2) \) with same scattering amplitude \( A_\Omega(\cdot, k) \) at only one fixed \( k \in \mathbb{R}^n \) must coincide.

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