In this paper we present a generalization of the classical bitangential Nevanlinna-Pick theory in which one bounds not the norm of the interpolating functions but their structured singular value. This work was motivated by some problems arising robust control of systems with structured uncertainty. This approach is based on the commutant lifting theory of Sz.-Nagy and Foias (1968) and extends previous work of Bercovici, Foias and Tannenbaum (1990) on structured matrix Nevanlinna-Pick interpolation.

1. Introduction

The problem of interpolation with bounded analytic functions has been studied since the turn of the century (Carathéodory 1907, Carathéodory and Féjer 1911, Schur 1912). A classical problem is to find necessary and sufficient conditions for the existence of an analytic function \( f : \mathbb{D} \to \overline{\mathbb{D}} \) such that \( f(z_k) = w_k, \ 1 \leq k \leq n \) where \( z_1, \ldots, z_n, w_1, \ldots, w_n \in \mathbb{D} \) are given points in the open unit disk. This problem was first solved by Pick (1916) for the case of finitely many points. Later, Nevanlinna (1919) extended this result for countable sets and also presented a recursive algorithm to parameterize all the solutions. The initial approaches to these problems were purely complex analytic.

Several decades later, in a landmark paper, Sarason (1967) developed an operator theoretic framework in which one could deduce many results on interpolation with bounded analytic functions, even in the case of points with infinite multiplicity (which he called generalized interpolation in \( H^\infty \)). Shortly after, these results were significantly broadened by Sz.-Nagy and Foias (1968) with their commutant lifting theorem which allowed interpolation with bounded analytic operator valued functions. Since these seminal papers, there has been significant advances on interpolation theory from an operator theoretic point of view (e.g. Ball and Helton 1983, Dym 1989, Foias and Frazho 1990, Ball, Gohberg and Rodman 1990a, Ball, Gohberg and Kaashoek 1992, Nudelman 1993, Frazho and Kherat 1993, Ivanchenko and Sakhnovich 1994).

Besides its intrinsic mathematical value, interpolation by bounded analytic functions has had a major impact in a number of areas of applied sciences such as circuits and systems theory (Delsarte, Genin and Kamp 1984, Helton 1987), control theory (Francis and Tannenbaum 1988), and geophysics (Foias 1978) among others. Indeed, this work on structured interpolation has been motivated by a problem in robust control, e.g., the synthesis of linear feedback systems in the presence of structured uncertainty (Doyle 1982).

In order to put this result in perspective, consider the standard Nevanlinna-Pick theory in the matrix case. Given a set of distinct points \( \alpha_i \in \mathbb{D}, \ 1 \leq i \leq n \), together with a set of \( N \times N \) complex matrices \( F_1, \ldots, F_n \) we want to find necessary and sufficient conditions for the existence of an analytic (in the disc) matrix valued function \( F(z) \) with \( F(\alpha_i) = F_i \ (1 \leq i \leq n) \) such that \( \| F \| \leq 1 \). The existence of such \( F \) can be reduced to the determination of the positivity of certain Nevanlinna-Pick matrix. This fact can be deduced,