Stability of matter through an electrostatic inequality

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Abstract. Stability of matter is proved using an electrostatic inequality which is a manifestation of screening.

1. Introduction

Nonrelativistic matter is described by the Hamiltonian

$$ H = - \sum_{i=1}^{N} \Delta^{(i)} + \sum_{\substack{i,j=1 \atop i \neq j}}^{N+M} \frac{e_i e_j}{|x_i - x_j|}, $$

accounting for $N$ fermionic electrons $i = 1, \ldots, N$ and $M$ nuclei $i = N + 1, \ldots N + M$ with positions $x_i \in \mathbb{R}^3$ and charges $e_i = -1$, resp. $1 \leq e_i \leq \text{const}$. Stability of matter is the statement:

Theorem 1. There is a constant $C$ such that

$$ H \geq -C(N + M). $$

This result has first been proved by Dyson and Lenard [3] and subsequently by Lenard [10], Federbush [5], Eckmann [4], Lieb and Thirring [14] and Fefferman [6]. We refer to [11] for the implications of this result. More recently, stability of matter in magnetic field has been proved [7] (but see [13] for another proof), thereby extending previous results [8, 12]. Here we propose a fairly direct proof of (1) based on an electrostatic inequality: Essentially, Coulomb energies are lowered as $\mathbb{R}^3$ is decomposed into simplices and the interaction is restricted to pairs belonging to the same simplex. This procedure is then repeated until only a few nuclei are left in each simplex.