The Exponential Stability of a Coupled Hyperbolic/Parabolic System Arising in Structural Acoustics

George Avalos*

Abstract

We show here the uniform stabilization of a coupled system of hyperbolic and parabolic PDE's which describes a particular fluid/structure interaction system. This system has the wave equation, which is satisfied on the interior of a bounded domain \( \Omega \), coupled to a "parabolic-like" beam equation holding on \( \partial \Omega \), and wherein the coupling is accomplished through velocity terms on the boundary. Our result is an analog of that in [12] which shows the exponential stability of the wave equation via Neumann feedback control, and like that work, depends upon a trace regularity estimate for solutions of hyperbolic equations.

1 Introduction

1.1 Statement of the Problem and Motivation

Let \( \Omega \) be a bounded domain of \( \mathbb{R}^n, n \geq 2 \), with Lipshitz boundary \( \Gamma = \Gamma_0 \cup \Gamma_1, \Gamma_i \) open, \( \Gamma_0 \) and \( \Gamma_1 \) both nonempty, with \( \overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset \). This paper is a continuation of our study, initiated in ([1]), of the following system consisting of a coupling between a wave and plate-like equation:

\[
\begin{align*}
  z_{tt} &= \Delta z \quad \text{on} \ (0, \infty) \times \Omega \\
  z(0, x) &= z_0, \ z_t(0, x) = z_1 \quad \text{on} \ \Omega \\
  z(t, x) &= 0 \quad \text{on} \ (0, \infty) \times \Gamma_0 \\
  \frac{\partial z(t, x)}{\partial \nu} + \alpha z_t(t, x) &= v_t \quad \text{on} \ (0, \infty) \times \Gamma_1 \quad \text{with} \ \alpha \geq 0; \\
  v_{tt} &= -\Delta^2 v - \Delta^2 v_t - z_t \quad \text{on} \ (0, \infty) \times \Gamma_1 \\
  v(0, x) &= v_0, \ v_t(0, x) = v_1 \quad \text{on} \ \Gamma_1 \\
  v(t, x) &= \frac{\partial v(t, x)}{\partial \nu} = 0 \quad \text{on} \ (0, \infty) \times \partial \Gamma_1.
\end{align*}
\]

Note how this coupling above of two qualitatively different equations is accomplished by the velocity terms \( z_t \) and \( v_t \) on the active portion of the boundary \( \Gamma_1 \).

In [2], issues of well-posedness for (1)–(2) were settled, with initial data \([\overline{z_0}, \overline{v_0}] \equiv [z_0, z_1, v_0, v_1]\) determining the solution \([\overline{z}, \overline{v}] \equiv [z, z_t, v, v_t]\) to be in \( H^{1}_{\Gamma_0}(\Omega) \times L^2(\Omega) \times H^2_0(\Gamma_1) \times L^2(\Gamma_1) \) (where \( H^{1}_{\Gamma_0}(\Omega) = \{ z \in H^1(\Omega) \ \exists \ z = 0 \text{ on } \Gamma_0 \} \)). Here, we are concerned with the exponential decay of the

*Institute for Mathematics and its Applications, University of Minnesota, 514 Vincent Hall, 206 Church Street S. E., Minneapolis, MN 55455–0436