The Strong Stability of a Semigroup Arising from a coupled Hyperbolic/Parabolic System

George Avalos* Irena Lasiecka†

Abstract

We consider here a coupled system of hyperbolic and parabolic PDE's which arises in a given fluid/structure interaction. The system is transformed into an abstract differential equation, and from this operator theoretic model, questions of strong stability for the equation are addressed. A distinctive feature of the problem is that the resolvent of the operator is not compact, and hence a treatment with the standard Nagy–Foias theory or Lasalle Invariance Principle is not available. Instead, we show that a powerful stability result of Arendt–Batty applies, and which consequently proves strong decay of the energy functional.

1 Introduction

1.1 Statement of Problem and Motivation

Let $\Omega$ be an bounded open subset of $\mathbb{R}^n$, $n \geq 2$, with Lipshitz boundary $\Gamma$, and $\Gamma_0$ a segment of $\Gamma$. We consider here the problem of finding functions $z(t, x)$ and $v(t, x)$ which solve the following system comprised of a “coupling” between a wave and elastic plate–like equation:

\[
\begin{align*}
    z_{tt} &= \Delta z & \text{on } (0, \infty) \times \Omega \\
    z(0, x) &= z^0, z_t(0, x) = z^1 & \text{on } \Omega \\
    z(t, x) &= 0 & \text{on } (0, \infty) \times \Gamma \setminus \Gamma_0 \\
    \frac{\partial z(t, x)}{\partial \nu} + \alpha z_t(t, x) &= v_t & \text{on } (0, \infty) \times \Gamma_0 \text{ with } \alpha \geq 0; \\
    v_{tt} &= -\Delta^2 v - \Delta^2 v_t - z_t & \text{on } (0, \infty) \times \Gamma_0 \\
    v(0, x) &= v^0, v_t(0, x) = v^1 & \text{on } \Gamma_0 \\
    v(t, x) &= \frac{\partial v(t, x)}{\partial \nu} = 0 & \text{on } (0, \infty) \times \partial \Gamma_0.
\end{align*}
\]

We are interested here in the stability of the solutions $[\mathbf{z}, \mathbf{v}]^T \equiv [z, z_t, v, v_t]^T$ to (1)–(2); viz. we are concerned with the decay of the “energy” $E(\mathbf{z}, \mathbf{v}, t)$ of the system (1)–(2) as $t \to \infty$, where

\[
E(\mathbf{z}, \mathbf{v}, t) = \int_{\Omega} \left[ |\nabla z(t)|^2 + |z_t(t)|^2 \right] d\Omega + \int_{\Gamma_0} \left[ |\Delta v(t)|^2 + |v_t(t)|^2 \right] d\Gamma_0.
\]