Exponential Stability of a Thermoelastic System Without Mechanical Dissipation

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Abstract

We show herein the uniform stability of a thermoelastic plate model with no added dissipative mechanism on the boundary (uniform stability of a thermoelastic plate with added boundary dissipation was shown in [3]). The proof is constructive in the sense that we make use of a multiplier with respect to the coupled system involved so as to generate a fortiori the desired estimates; this multiplier is of an operator theoretic nature, as opposed to the more standard differential quantities used for such work. Moreover, the particular choice of multiplier becomes clear only after recasting the pde model into an associated abstract evolution equation. With this direct technique, we also obtain an exponential stability estimate pertaining to the special case in which rotational inertia is neglected, and which leads to an associated analytic semigroup. This result was originally derived through a contradiction argument (see [9]).

1 Introduction

1.1 Statement of the Problem

Let Ω be a bounded open subset of $\mathbb{R}^2$ with Lipshitz boundary $\Gamma$. We consider here the following thermoelastic system taken from J. Lagnese’s monograph [3]:

$$\begin{cases}
\omega_{tt} - \gamma \Delta \omega_{tt} + \Delta^2 \omega + \alpha \Delta \theta = 0 & \text{in } (0, \infty) \times \Omega; \\
\beta \theta_t - \eta \Delta \theta + \sigma \theta - \alpha \Delta \omega_t = 0 & \\
\frac{\partial \theta}{\partial \nu} + \lambda \theta = 0 & \text{on } (0, \infty) \times \Gamma, \lambda \geq 0; \\
\omega(t = 0) = \omega^0, \omega_t(t = 0) = \omega^1, \theta(t = 0) = \theta^0 \text{ on } \Omega; \\
\omega = (1 - \kappa) \frac{\partial \omega}{\partial \nu} = 0 & \text{on } (0, \infty) \times \Gamma. \\
\kappa (\Delta \omega + (1 - \mu) B_1 \omega + \alpha \theta) = 0
\end{cases}$$

(1)

Here, the parameter $\kappa$ is either 0 or 1; $\alpha$, $\beta$, $\gamma$ and $\eta$ are strictly positive constants with $\gamma$ proportional to the thickness of the plate and assumed to be small; the constant $\sigma \geq 0$ and the boundary operator $B_1$ is given by

$$B_1 \omega \equiv 2 \nu_1 \nu_2 \frac{\partial^2 \omega}{\partial x \partial y} - \nu_1^2 \frac{\partial^2 \omega}{\partial y^2} - \nu_2^2 \frac{\partial^2 \omega}{\partial x^2};$$

(3)

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