ON GRADIENT YOUNG MEASURES SUPPORTED ON A POINT AND A WELL

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ABSTRACT. There are essentially two facets to the two well problem in the study of phase transitions in crystals: classifying microstructures when there is a rank-1 connection between the wells and trying to establish that only trivial microstructures should be observed when there is no rank-1 connection. Dealing with the latter has been remarkably hard in space dimension 3. We prove that spatially homogeneous gradient Young measures supported in $SO(3)H \cup 1$ must reduce to a Dirac mass provided there are no rank-1 connections in this set and $H$ is sufficiently near the identity. This follows exclusively from the minor relations without any small strain hypothesis on the sequence of gradients. This provides some insight on why the general case ($SO(3)H \cup SO(3)$) is still open and the special role of $SO(3)$ symmetry in this problem.

1. Introduction

Let $H$ denote a $n \times n$ real positive definite matrix with positive determinant and $1$ the identity matrix. In what follows there will be no loss of generality in assuming that $H = \text{diag}(\mu_1, \ldots, \mu_n)$ with $\mu_i > 0$ for all $i$.

The following conjecture is open:

Conjecture 1.1 (Kinderlehrer). Let $\nu_x$ denote a gradient Young measure\(^1\) supported in $SO(n) \cup SO(n)H$, which in particular means it has the form

$$\nu_x = (1 - \lambda(x))\rho_1(x) \otimes 1 + \lambda(x)\rho_2(x) \otimes 1_H$$

where $\lambda(x) \in [0,1]$ a.e. in $\Omega$ and $\rho_1, \rho_2$ are probability densities.

Then either the wells $SO(n)$ and $SO(n)H$ have a rank-1 connection, i.e.,

$$\text{rank}(RH - 1) \leq 1$$

\(^1\)Specifically the reader can assume that $\nu_x$ is generated by a uniformly bounded sequence $(\nabla u_j)_{j \in \mathbb{N}} \subset W^{1,\infty}(\Omega, \mathbb{R}^n)$ where the $u_j$'s are maps defined in $\Omega$, a non-empty open subset of $\mathbb{R}^n$, with values in $\mathbb{R}^n$. Less stringent constraints can be considered.