Scaling properties of vortex ring formation at a circular tube opening

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ABSTRACT. A vortex sheet model is used to study vortex ring formation at the edge of a circular tube. We determine properties of the vortex ring as a function of the generating piston motion and investigate the extent to which similarity theory for planar vortex sheet separation applies. We find that the ring diameter, core size and circulation are well predicted by the planar theory, even at large times when the ring has travelled significantly downstream. The axial ring translation is a superposition of an upstream component predicted by the theory and a downstream component which is linear in the piston stroke. The front of the fluid volume exiting the tube is also linear in the piston stroke and travels with 75% of the piston velocity.

I. INTRODUCTION

A typical experiment of vortex ring formation at a tube opening is shown in Fig. 1a. It consists of a circular tube immersed in fluid and a piston inside the tube which moves, ejecting fluid from the opening. This causes the boundary layer on the inner tube wall to separate at the edge as an axisymmetric shear layer. The separated layer rolls up and forms a vortex ring. One objective of vortex ring experiments has been to describe the ring properties as a function of the generating conditions (Shariff & Leonard\textsuperscript{1}). For the tube geometry, the relevant condition is the piston velocity. In this paper we use a numerical vortex sheet model to investigate the dependence of the ring trajectory, circulation, size and shape on piston velocities of the form $U_p(t) \sim t^m$.

Theoretical predictions based on similarity theory exist for the planar flow shown in Fig 1b. Here, flow past a semi-infinite flat plate causes the separation and roll-up of a planar shear layer. After an initial time-interval the viscous shear layer thickness is small relative to the length scales of the flow and the separated layer is well approximated by a vortex sheet. The vortex sheet separation and roll-up is known to be self-similar: for starting flows that satisfy a power law in time, the position of the spiral center, the spiral size and the shed circulation have known power law behaviour. Pullin\textsuperscript{2} discusses the similarity theory and computes the self-similar planar separation using a numerical method.

The similarity results are based on the form of the potential flow past the plate. Near the edge, the axisymmetric potential flow out of a tube is similar to the planar flow past the plate. It is therefore plausible to assume that at small times the axisymmetric separation in Fig. 1a is approximated by the planar separation in Fig. 1b. The planar similarity theory can then be used to predict the vortex ring trajectory, shape and circulation, for the case of power law piston velocity (Saffman\textsuperscript{3}, Pullin\textsuperscript{4}). However, there is a significant difference between the two flows. In the planar case, the self-similar spiral center travels along a straight line with negative horizontal velocity (dotted line in Fig. 1b). The vortex ring on the other hand has a downstream velocity component and travels along a curved trajectory with positive axial velocity (dotted curve in Fig. 1a). The position of the vortex significantly affects for example the circulation shedding rate at the edge. It is thus not clear a priori to what extent the planar similarity theory describes the axisymmetric ring formation process.

Nitsche & Krasny\textsuperscript{5} developed a vortex sheet model for vortex ring formation at the edge of a circular tube and simulated an experiment by Didden\textsuperscript{6}. Comparisons with the experimental measurements showed that the model accurately recovers the formation process. In this paper we apply the model to simulate the vortex ring formation for piston velocities $U_p \sim t^m$. We consider $m = 0, 1/2, 1, 2$. We investigate the flow and determine the extent to which it is described by the planar similarity theory.