DECAY AND ANALYTICITY OF SOLITARY WAVES

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ABSTRACT. Considered here are detailed aspects of solitary-wave solutions of nonlinear evolution equations including the Euler equations for the propagation of gravity waves on the surface of an ideal, incompressible, inviscid fluid. Two properties will occupy our attention. The first, described already in an earlier paper, concerns the regularity of these travelling waves. In the context of certain classes of model equations for long waves in nonlinear dispersive media, we showed that solitary waves are obtained as the restriction to the real axis of functions analytic in a strip of the form \( \{ z : -a < \Re(z) < a \} \) in the complex plane. In this direction, the scope of our previous discussion of model equations is broadened considerably. Moreover, it is also shown that solitary-wave solutions of the full Euler equations have the properties that the free surface is given by the restriction to the real axis of a function analytic in a strip in the complex plane and the velocity potential is the restriction to the flow domain of a function that is analytic in an open set in complex 2-space \( \mathbb{C}^2 \). The second issue considered is the asymptotic decay of solitary waves to a quiescent state away from their principal elevation. A theorem pertaining to the evanescence of solutions of certain types of one-dimensional convolution equations is formulated and proved, showing that decay is related to the smoothness of the Fourier transform of the convolution kernel \( k \), as well as the nonlinearity present in the equation. It is demonstrated that if the Fourier transform \( \hat{k} \in H^s \) for some \( s > 1/2 \), the rate of decay of a solution is at least as fast as that of the kernel \( k \) itself. This result is used to establish asymptotic properties of solitary-wave solutions of a broad class of model equations, and of solitary-wave solutions of the full Euler equations.

1. Introduction. This paper is concerned with two aspects of solitary waves that are a reflection of their natural appearance as smooth, steadily propagating disturbances of elevation or depression, asymptotically approaching a constant level on either side of their crests. These attributes of the real phenomenon find mathematical expression in regularity theory and decay results for solitary-wave solutions of nonlinear wave equations. It is our purpose here to investigate these mathematical properties in the context of model equations of Korteweg-de Vries-type, regularized long-wave-type and nonlinear Schrödinger-type, as well as for the time-dependent Euler equations for the propagation of gravity waves on the surface of an inviscid, incompressible Euler fluid.

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