On the Number of Positive Solutions to a Class of Integral Equations *

Long Wang
Center for Systems and Control, Department of Mechanics and Engineering Science
Peking University, Beijing 100871, P. R. CHINA

Wensheng Yu
Laboratory for Complex Systems and Intelligent Control, Institute of Automation
Chinese Academy of Sciences, Beijing 100080, P. R. CHINA

Lin Zhang
Department of Automation, Tsinghua University
Beijing 100084, P. R. CHINA

Abstract: By using the complete discrimination system for polynomials, we study the number of positive solutions in $C[0,1]$ to the integral equation

$$
\phi(x) = \int_0^1 k(x,y)\phi^n(y)dy,
$$

where $k(x,y) = \phi_1(x)\phi_1(y) + \phi_2(x)\phi_2(y)$, $\phi_i(x) > 0$, $\phi_i(y) > 0$, $0 < x, y < 1$, $i = 1, 2$, are continuous functions on $[0,1]$, $n$ is a positive integer. We prove the following results: when $n = 1$, either there does not exist, or there exist infinitely many positive solutions in $C[0,1]$; when $n \geq 2$, there exist at least $1$, at most $n + 1$ positive solutions in $C[0,1]$. Necessary and sufficient conditions are derived for the cases: 1) $n = 1$, there exist positive solutions; 2) $n \geq 2$, there exist exactly $m(m \in \{1, 2, \ldots, n + 1\})$ positive solutions. Our results generalize the existing results in the literature, and their usefulness is shown by examples presented in this paper.

Keywords: Integral Equations, Positive Solutions, the Complete Discrimination System for Polynomials, the Number of Solutions

*Supported by the National Natural Science Foundation of China (No. 69925307), National Key Project of China, National Key Basic Research Special Funds of China (No. G1998020302) and National Laboratory of Intelligent Technology and Systems of Tsinghua University. Corresponding author: Professor Long Wang, Email: longwang@mech.pku.edu.cn