A TRANSMISSION PROBLEM FOR FLUID-STRUCTURE INTERACTION IN
THE EXTERIOR OF A THIN DOMAIN

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1. Introduction. In this article, we study the interaction between an infinitely long cylinder coated with a sheath of an elastic material and a compressible, inviscid fluid, with lubricated contact on the interface. The problem is thus posed in \( \mathbb{R}^2 \). The thickness of this sheath is a small parameter in this problem, and we are particularly interested in the asymptotic behavior of the solutions to the fluid-solid interaction problem as this parameter approaches zero. We attempt to resolve not only the fluid pressure in the exterior, but also the elasto-dynamic oscillations in the thin region. This is in contrast to earlier work, where the elastic region is replaced by an effective boundary condition on the fluid-solid interface for the acoustic problem in the exterior. The incident waves are time-harmonic in nature, enabling us to study a time-independent scattering problem.

It was shown in (26) that for certain frequencies, the inviscid fluid-structure interaction problem does not have unique solutions. Even in the absence of a pressure field in the exterior fluid region, the elastic domain may support so-called traction-free oscillations, whose normal components and tractions across the fluid-structure interface are continuous. This imposes constraints on our existence theory; we are only able to demonstrate solvability for the fluid-structure problem away from certain natural eigenfrequencies. Fortunately, such oscillations occur in highly specific situations and heavily depend upon the symmetry properties of the region. Therefore, our analysis holds for all but a highly specific class of fluid-structure problems. Moreover, these Jones frequencies form a discrete spectrum. If the structure under consideration supports Jones oscillations, the analysis is valid except for at most a discrete set of frequencies.

The existence of solutions is usually demonstrated by casting the problem into variational form, and then using standard techniques from Fredholm theory. We follow this procedure, but encounter difficulty in showing the invertibility of a certain operator. The use of Korn's inequality enables us to get around this problem.

We begin the article by describing the fluid-structure problem in Section 2, with a brief discussion on issues of uniqueness for solutions. The governing equations will be the Navier-Lamé system for the elastic displacements in the thin region, and the Helmholtz equation in the fluid, coupled via transmission conditions across the fluid-structure interface.

In order to present our key ideas, without getting lost in the technical details, in Section 3 we develop the analysis in the context of a very simple model scalar transmission problem. After precisely formulating this illustrative example in Section 3, we reduce the exterior problem by means of two integral equations. Using a variational formulation for the reduced problem leads to key estimates.

In Section 4, we return to the fluid-structure problem of interest, and formulate it carefully in a Hilbert space setting. We study the uniqueness and existence of solutions via variational techniques. Finally, a formal asymptotic scheme is described. The asymptotic procedure is justified by using a result (19) concerning the Korn's inequality in a thin region. The proof follows a technique introduced earlier in the context of simpler problems, (18), (38).

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In this paper, we are able to justify the formal asymptotics by making a fairly strong technical assumption. This assumption can be avoided by introducing an artificial layer, as described in (2) for a scalar Helmholtz transmission problem. We shall present this technique in the context of the fluid-structure interaction in future work, (20).