Optimal Blowup Rates for the Minimal Energy Null Control for the Structurally Damped Abstract Wave Equation

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Abstract
The null controllability problem for a structurally damped abstract wave equation—a so-called elastic model—is considered with a view towards obtain optimal rates of blowup for the associated minimal energy function $E_{\min}(T)$, as terminal time $T \downarrow 0$. Key use is made of the underlying analyticity of the elastic generator $\mathcal{A}$, as well as of the explicit characterization of its domain of definition. We ultimately find that the blowup rate for $E_{\min}(T)$, as $T$ goes to zero, depends on the extent of structural damping.

1 Introduction

Let $\mathcal{A} : D(\mathcal{A}) \subset H \to H$ be a strictly positive, self-adjoint operator; of course, $H$ is Hilbert. Therewith, we consider the structurally damped and controlled abstract model

\[
\begin{aligned}
& v_{tt} + \mathcal{A} v + \rho \mathcal{A}^\alpha v_t = u \quad \text{on } (0, T) \\
& [v(0), v_t(0)] = [v_0, v_1] \in D(\mathcal{A}^\frac{1}{2}) \times H
\end{aligned}
\]  

(1)

where here, $0 \leq \alpha < 1$, and $\rho > 0$. Also, the “control” $u(t)$ is a function in $L^2(0, T; H)$. So as it appears, this model constitutes an abstract wave equation, under the influence of the structural damping term $\rho \mathcal{A}^\alpha v_t$. (This form of interior damping is referred to as being of Kelvin-Voight type.) It is now wellknown that for damping parameter $\alpha$ in the range $\frac{1}{2} \leq \alpha \leq 1$, the system’s underlying generator $\mathcal{A} : D(\mathcal{A}) \subset D(\mathcal{A}^\frac{1}{2}) \times H \to D(\mathcal{A}^\frac{1}{2}) \times H$ is of analytic character (see [3]). Consequently, those controlled partial differential equations which can be described by the abstract system (1) manifest parabolic-like dynamics.

For this model, we wish to consider the null controllability problem. This problem can be broadly stated as that of finding a control function $u$, such that the corresponding solution of (1) is brought from the initial state to rest at terminal time $T$. Because the abstract system (1) models parabolic-like behaviour, including an infinite speed of propagation, one should expect that if this system is indeed null controllable within the given class of control inputs $u$, the property should hold true in arbitrary short time $T > 0$. This expectation is fully in line with what is known about the canonical parabolic controllability problem; namely the problem of controlling the heat equation, be it via boundary or interior control (see e.g., [2], [12], [15]). Denoting

\[
\mathcal{X} = D(\mathcal{A}^\frac{1}{2}) \times H, 
\]  

(2)

we are accordingly led to our working definition of null controllability: