A NOTE ON BOUNDARY BLOW-UP PROBLEM OF $\Delta u = u^p$

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Abstract. Assume that $\Omega$ is a bounded domain in $\mathbb{R}^n$ with $n \geq 2$. We study positive solutions to the problem, $\Delta u = u^p$ in $\Omega$, $u(x) \to \infty$ as $x \to \partial \Omega$, where $p > 1$. Such solutions are called boundary blow-up solutions of $\Delta u = u^p$. We show that a boundary blow-up solution exists in any bounded domain if $1 < p < \frac{n}{n-2}$. In particular, when $n = 2$, there exists a boundary blow-up solution to $\Delta u = u^p$ for all $p \in (1, \infty)$. We also prove the uniqueness under the additional assumption that the domain satisfies the condition $\partial \Omega = \partial \Omega$.

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