Probabilistic interpretation and stochastic particle method for vortex equations

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We are interested in proving Monte-Carlo approximations for Navier-Stokes equations in bounded domains of R^2 . The difficulty comes from the boundary conditions. If we only assume the nullity of the normal component of the velocity on the boundary, a similar approach to the one in the whole space can be used. We interpret in a probabilistic point of view the associated vortex equation and study some stochastic particle systems, which justify the famous vortex algorithm firstly introduced by Chorin.

We are then interested to the case of a no-slip condition on the boundary. Cottet proved that the equivalent vortex formulation gives a vortex equation with Neumann's condition on the boundary, which badly depends on the solution.

We simplify the model by studying in details the case of a fixed Neumann condition. We associate with this equation a nonlinear reflected process with space-time random births on the boundary of the domain. Next, we approximate the solution of this vortex equation by interacting diffusive particles with normal reflecting boundary conditions and space-time random births on the boundary. That allows us to prove the pathwise convergence of an easily simulable particle algorithm.

In a last part, we explain how the results have to be adapted in the Navier-Stokes case.

Key words: Vortex equation on a bounded domain; Monte-Carlo approximation; Interacting particle systems with reflection; random births on the boudary

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