The Sharp Rate of Numerical Methods for Degenerate ODEs

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Abstract. In this paper we propose a numerical method, in the spirit of finite difference approximations, for the following ODE:

$$\frac{1}{2}\sigma^2(x)u''(x) + b(x)u'(x) - u(x) = -f(x),$$

where σ is allowed to be *degenerate*. We note that in general the solution u is *not* smooth. By using some probabilistic approach, we obtain the rate of convergence of our approximation and show that it is sharp.