

# Optimization Using Surrogates for Engineering Design

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Thanks to: **AFOSR, Boeing, LANL, SANDIA, ExxonMobil, NSF**

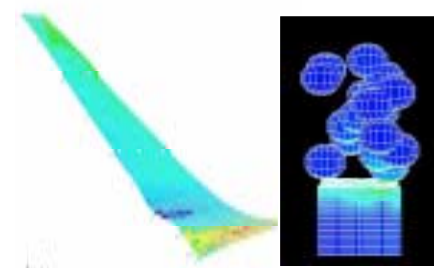
# Design Explorer Applications



Helicopter Rotor Design



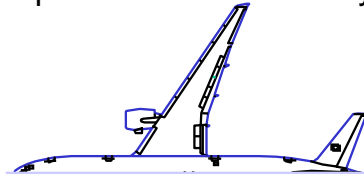
Space Station Power System



Shot peen forming of wing skins



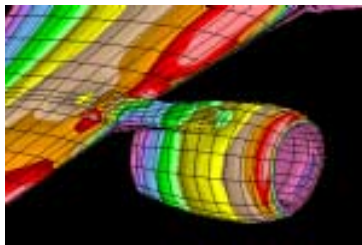
Aerospike Nozzle



Multidisciplinary wing planform design



3-D Fighter Aerodynamics



Engine Nozzle Performance



777 Engine Duct Seals



Machining, riveting, and drilling database

# A strawman surrogate approach

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*But, what if no improvement was found?*

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- ◆ May be able to interpolate to gradients as well - Alexandrov

## Form of the DACE interpolants

Polynomials introduce extraneous extremes that trap strawman

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Correlation parameters for each site estimated by MLE.

# Surrogate Management Framework

Given initial surrogates  $s_f, s_C$  and  $p_0 \in M_0$ , a grid on  $\mathbb{R}^n$ , let  $P_0 \subset M_0$  be  $p_0$  and the points of  $M_0$  adjacent to  $x_0$

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1. (Strawman) Search on  $s_f, s_C$  to find an unfiltered  $x_{k+1} \in M_k$  and then set  $M_{k+1} = M_k$  and update the surrogates;

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2. Else if  $p_k$  is the only unfiltered point in  $P_k$ ; Then set  $x_{k+1} = x_k$  and  $M_{k+1} = M_k/2$  and update the surrogates; Else return to 1

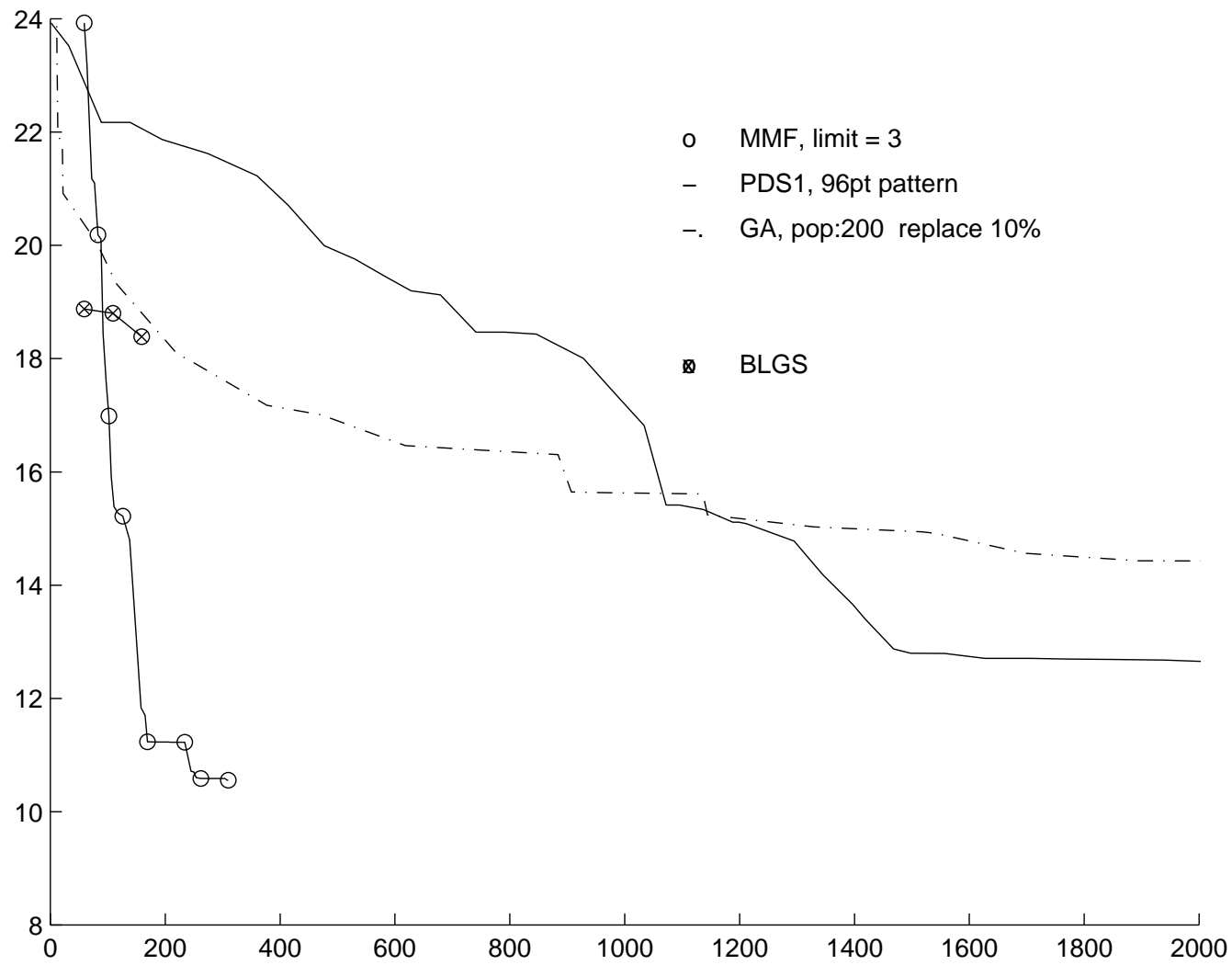
## Just like FPS, but...

**Poll step:** Polling is done on the actual (barrier) functions, which guarantees convergence of the algorithm (through the sequence of unsuccessful poll steps).

**Search step:** Use surrogates  $s_f, S_C$  to find some promising candidates where  $f, C$  will be evaluated.

Update  $s_f, S_C$  with the new evaluations of  $f, C$ .

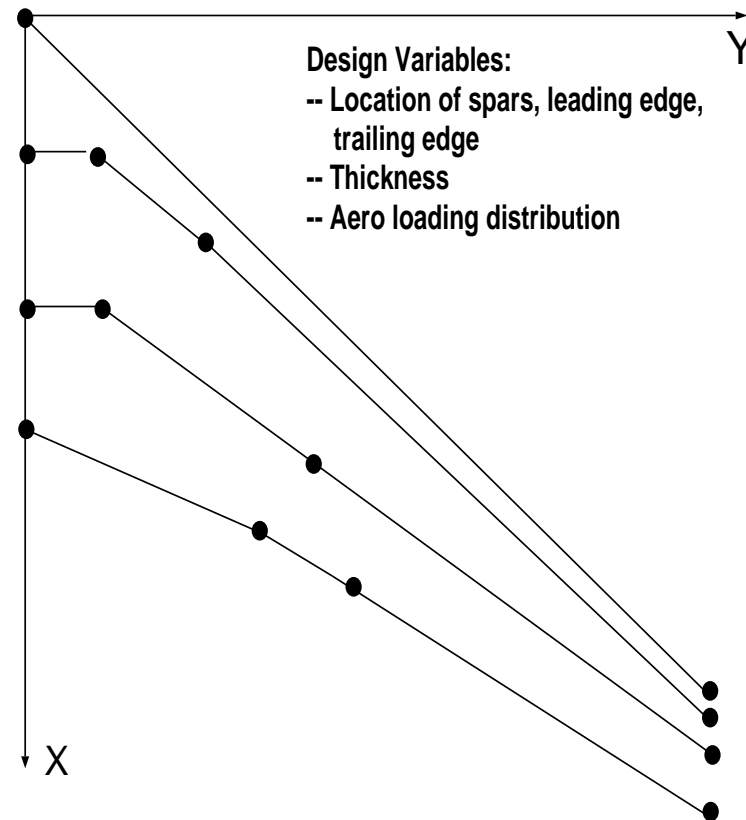
# Comparisons on 31d helicopter example



# Boeing wing planform design

Infeasible baseline design.

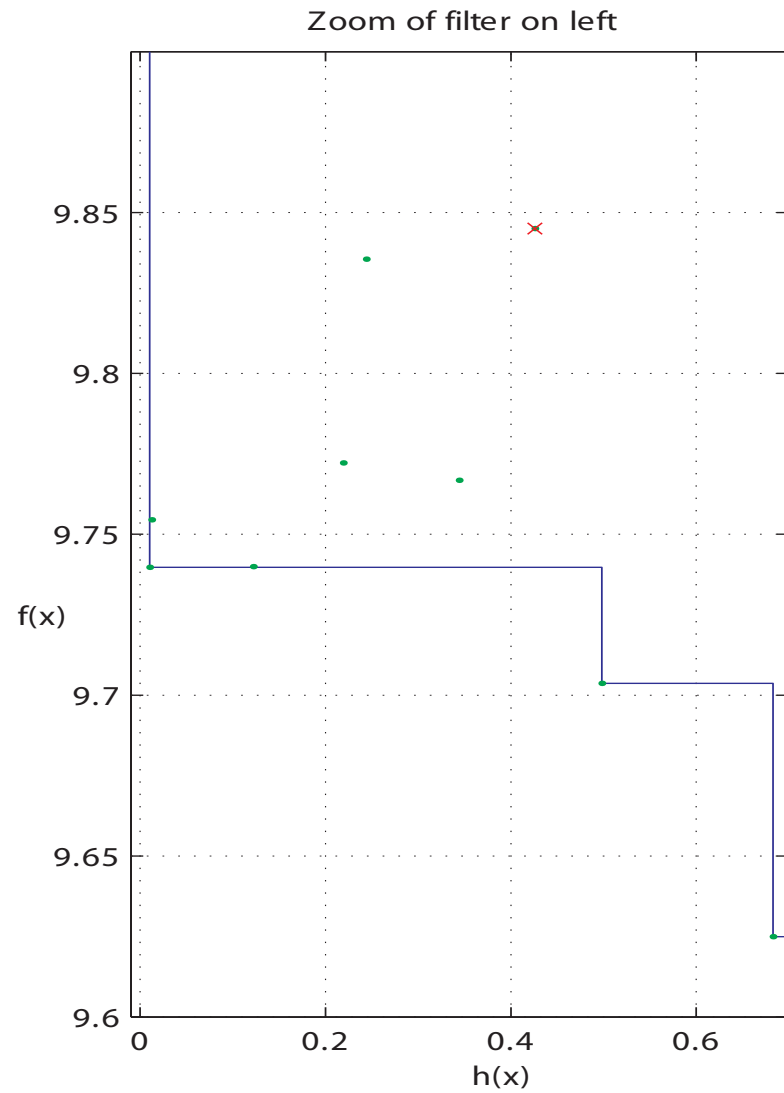
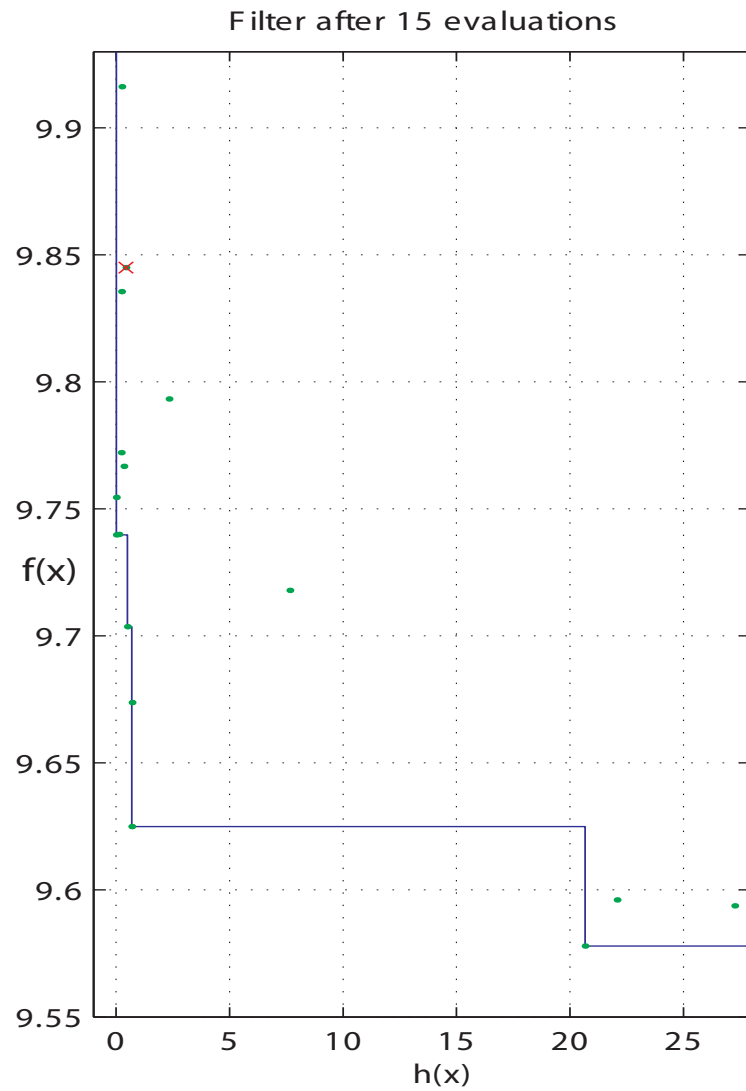
	$n$	# of ctrs	# of fevals
A	15	11	304
B	15	11	292



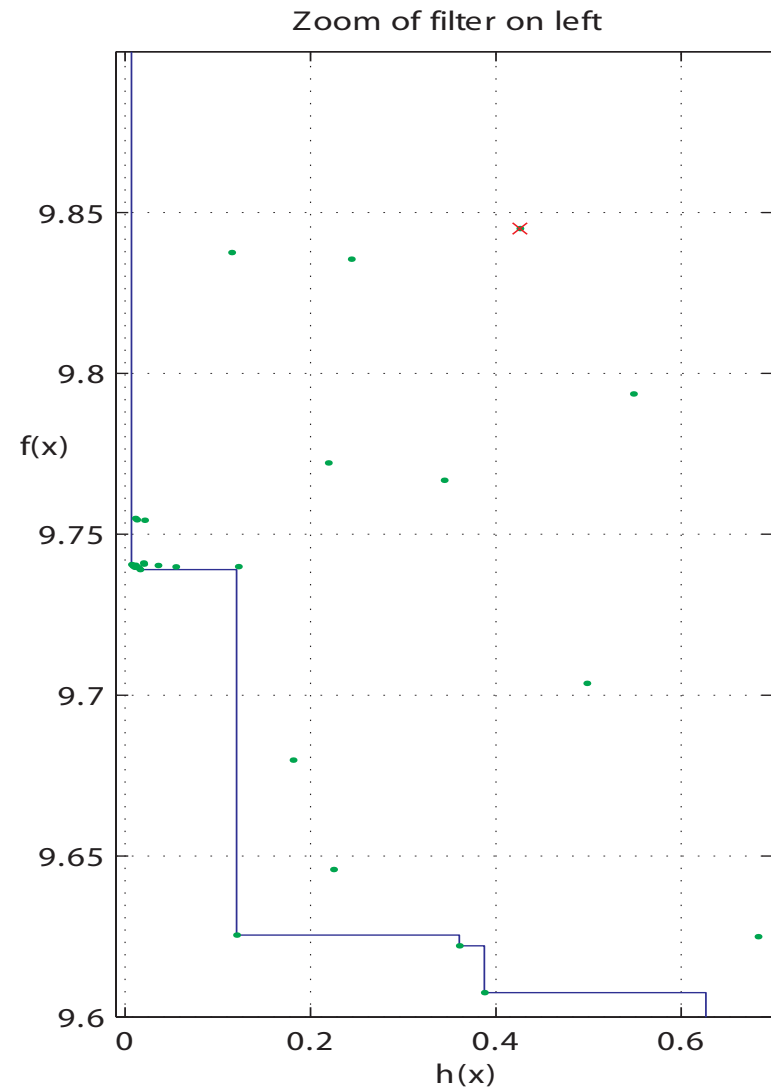
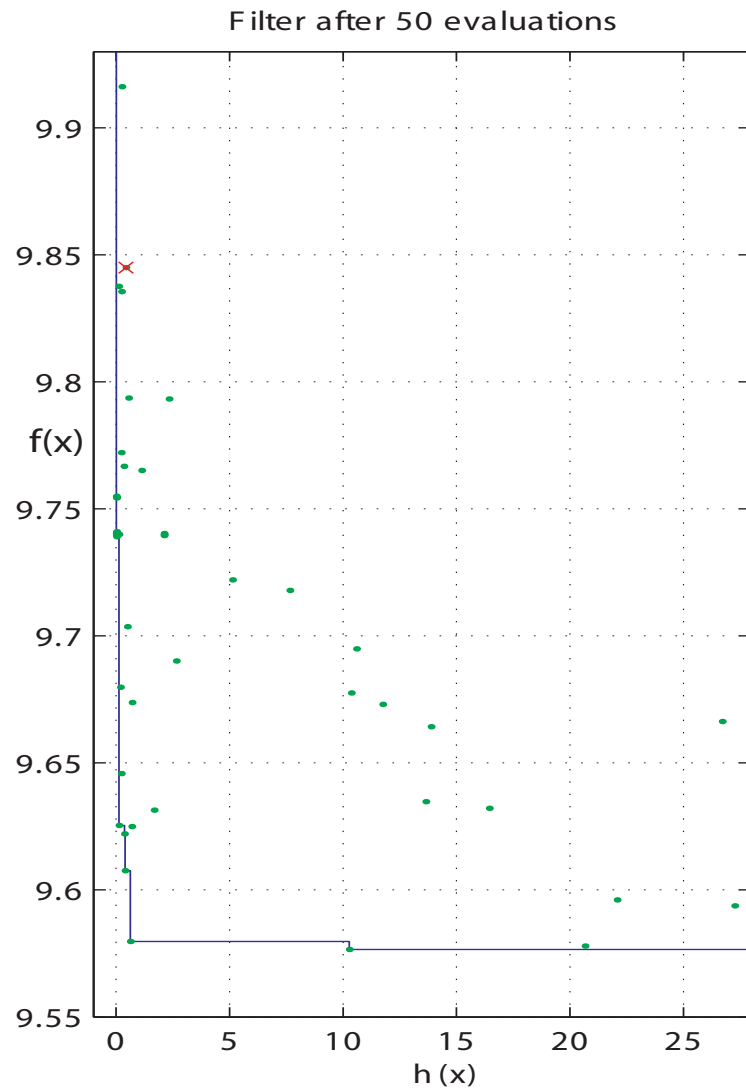
# New Boeing SonicCruiser platform



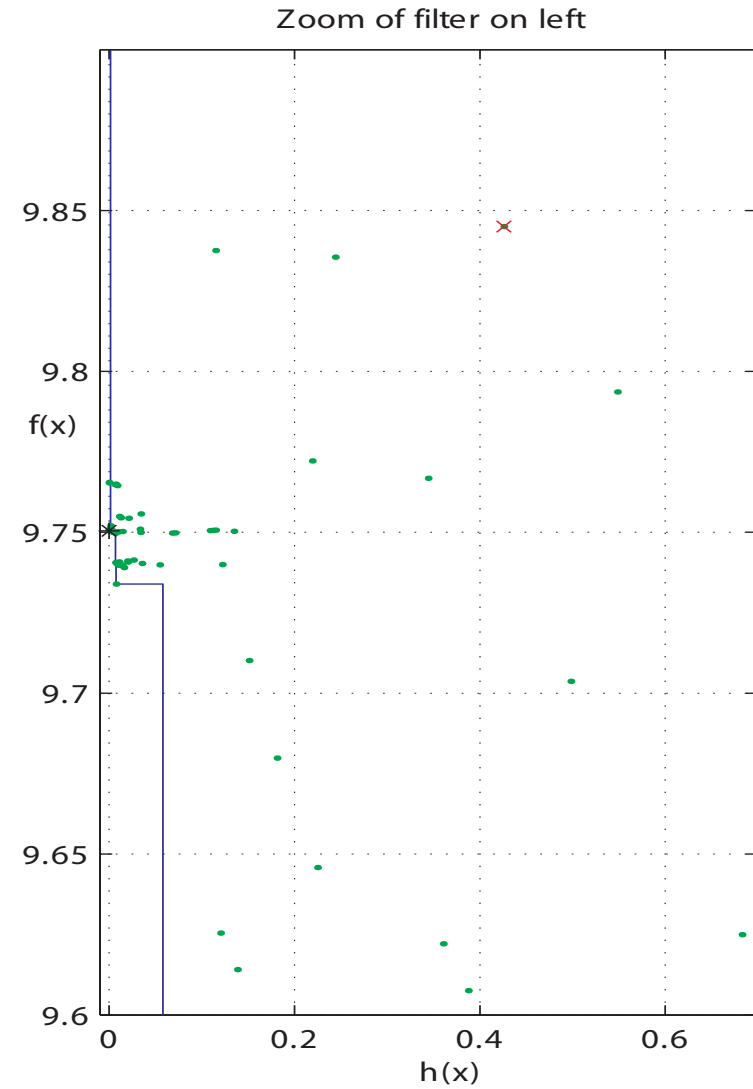
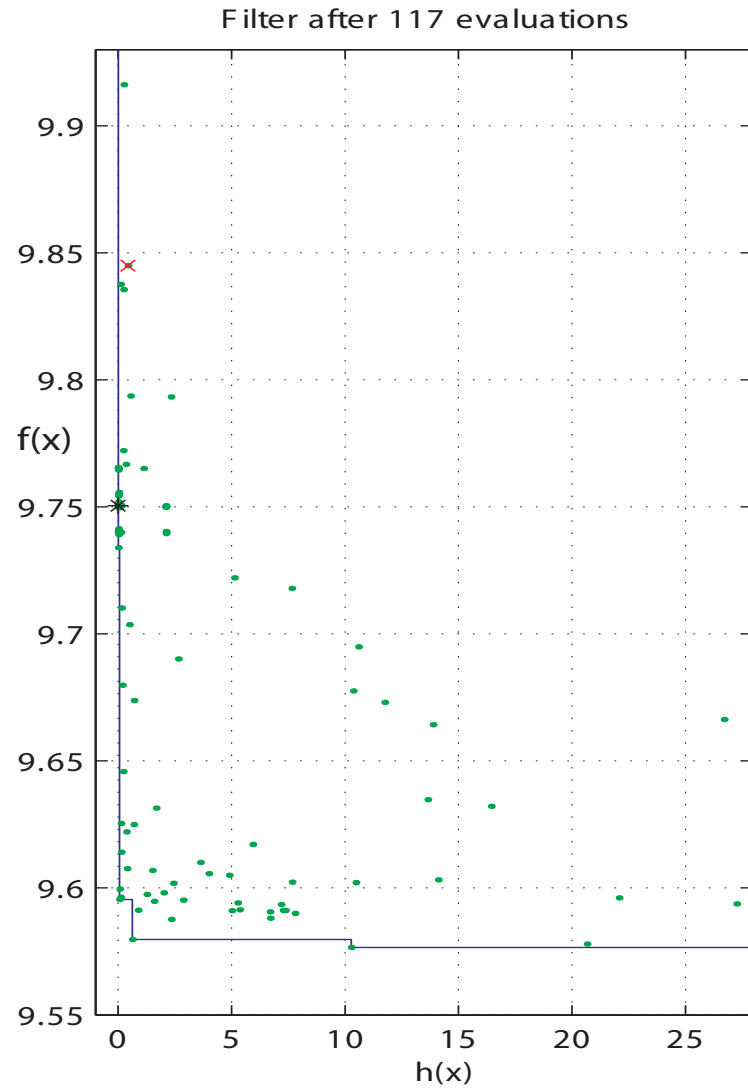
# A planform filter



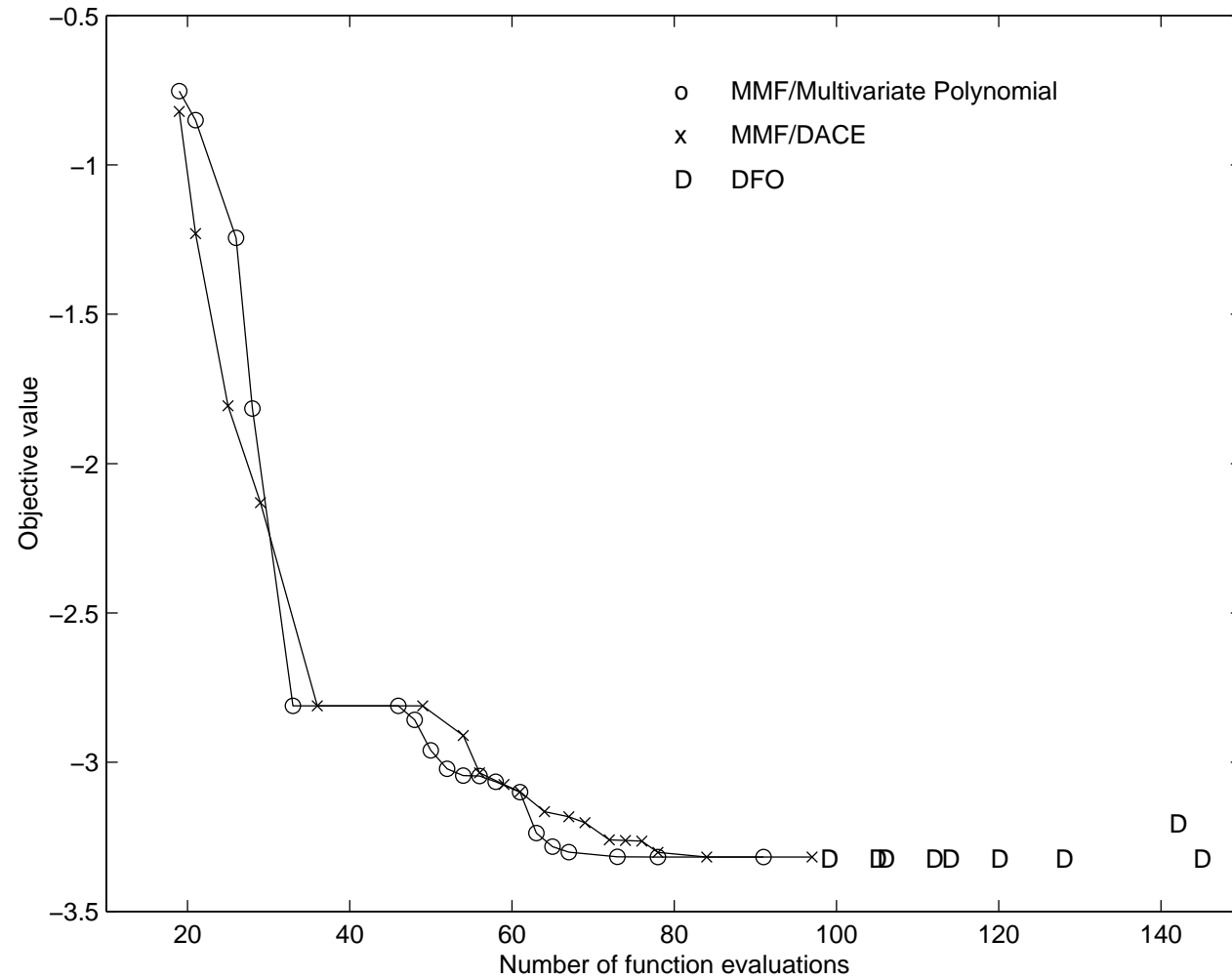
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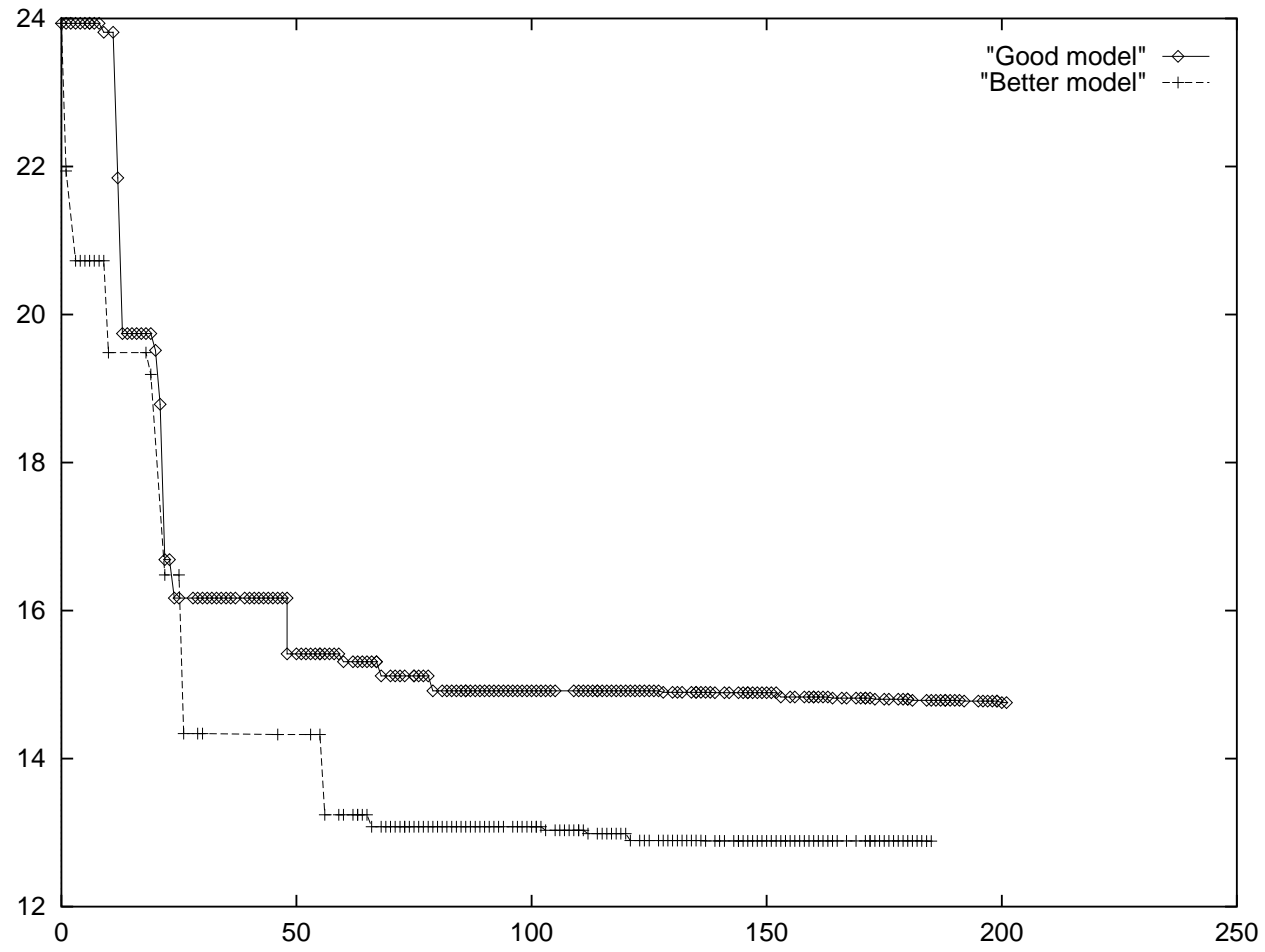
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# Polynomial vs DACE surrogates



# Recovering from a bad surrogate



# Software

- ◆ A simplified algorithm with local steps only is used 100k times each day in Boeing's commercialized 2NA parts nesting software
- ◆ Design Explorer is a Boeing proprietary implementation of our SMF with proprietary surrogates and a specialized search. Commercialization is underway

# NOMAD

- ◆ Current NOMAD C++ implements the filter approach. Presently in production use at ExxonMobil. Available to beta users
- ◆ NOMADm is a downloadable MatLab implementation in production use at Seimens VDO for designing engine control laws
- ◆ Should handle uncertainty optimization models

# Conclusions

- ◆ Clarke's nonsmooth analysis tools simplify, shorten, and strengthen the pattern search analysis
- ◆ Filters handle nonlinear constraints without using derivatives, Lagrange multipliers or penalty parameters
- ◆ SMF seamlessly incorporates user heuristic SEARCHES, and POLLING provides robustness and rigor
- ◆ SMF needs less accurate surrogates than designers want anyway

# Plans

- ◆ Boeing and ExxonMobil want more design variables than present interpolatory surrogates can handle. Requires rethinking the surrogate/optimization interface. High payoff, high risk, high entertainment value

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- ◆ Put filter approach to categorical variables from NOMADm into NOMAD and test on pipe bending problems. **Open ended, high entertainment value**

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- ◆ Put NOMAD into Boeing Design Explorer. High payoff, low risk
- ◆ Parallelize NOMAD. Open ended , high payoff, low risk
- ◆ Use NOMAD or Design Explorer to investigate various uncertainty formulations and explore tighter integration

# Recommendations

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- ◆ Learn all you can about optimization, statistics, linear algebra, software engineering and numerical methods
- ◆ Don't change the problem to fit the mathematics you know, develop the mathematics the problem needs

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Don't expect support up front. Remember, even meeting with you to formulate a joint project is support.