TCP Stability and Resource Allocation: Part II

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Issues with TCP

- Round-trip bias
- Instability under large bandwidth-delay product
- Transient performance under large bandwidth-delay product

Scalability Problem - I

- Consider a file transfer over a 10 Gbps fiber link with an RTT of 100 msecs.

- One packet is approx. 1000 bytes or 8,000 bits

- Link capacity is 1,250,000 packets/sec.

Scalability Problem - II

- Window size in equilibrium is 125,000 packets
- Suppose a packet loss occurs, window size becomes 62,500 packets
- Recall that the window size roughly increases by one for every RTT

Scalability Problem - III

- It will take at least 62,500 RTTs or 6,250 secs. to increase back to its original size

- Conclusion: severe loss in throughput

- Spurious congestion events such as packet corruption in optical fiber can trigger window reduction

Proposed Solutions

- Modify TCP’s AIMD behavior (HighSpeed TCP, Scalable TCP)
  - ➢Reduce less drastically than halving the window size
  - ➢Increase more rapidly when there is no congestion
- Use a completely different protocol based on the dual algorithm (FAST Protocol)
  - ➢Use delay as the congestion measure

High Throughput TCP (Vinnicombe)

$$\dot{x} = \kappa x(t-T)\left(w - x(t)q(t)\right)$$

- Compare with TCP-Reno (assume q << 1)

$$\dot{x} = x(t-T)\left(\underbrace{\boxed{\frac{1}{T^2}}}_{\text{RTT bias}}\underbrace{\boxed{\frac{1}{x(t)}}}_{\text{Lack of scalability}} - \frac{1}{2}q(t)x(t)\right)$$

High-Throughput TCP - II

- Window Interpretation

$$\dot{W} = \kappa T x(t-T)\left(1 - x(t)q(t)\right)$$

  - Increase window by a constant amount κTw for each ack
  - Decrease window by a factor κT for each lost packet
  - Scalability: Choose κT to be a constant

Linear System

$$\dot{x} = -\kappa \widehat{x} \left(\widehat{x} q(t) + \widehat{q} x(t - T) \right)$$

$$q(t) = f(\widehat{x}) + f'(\widehat{x}) x(t - T)$$

- Taking the Laplace transform

$$X(s) = H(s) x(0)$$

Stability

- System is stable if the roots of H(s)=0 lie in the complex left-half plane where

$$H(s) = s + \alpha_1 + \kappa\alpha_2 e^{-sT}$$

- Equivalently study the roots of 1+G(s)=0 where

$$G(s) = \kappa T \alpha_2 \frac{e^{-sT}}{sT + \alpha_1 T}$$

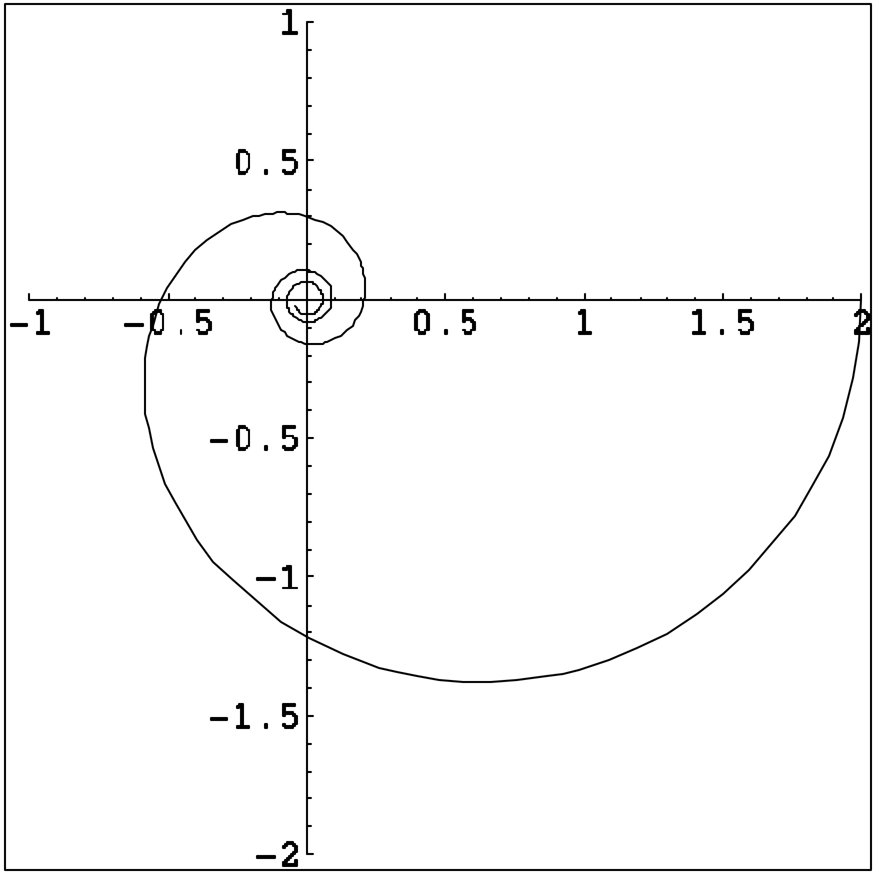
Nyquist Test

- If the plot of G(jω) does not encircle the point -1 as ω is varied from -1 to +1, then the system is stable
- Thus, we need to study the behavior of

$$\frac{e^{-j\omega}}{j\omega+\alpha}$$

as ω is varied

Nyquist Plot (α=0.5)



Sufficient Condition for Stability

- Key fact: The plot of $\frac{e^{-j\omega}}{j\omega + \alpha}$ does not encircle -2/π for any α>0

- Thus, the plot of

$$G(j\omega) = \kappa T \alpha_2 \frac{e^{-j\omega T}}{j\omega T + \alpha_1 T}$$

does not encircle -1 if

$$\kappa T \alpha_2 \leq \frac{\pi}{2}$$

Source and Router Conditions

- Source condition (recall scalability condition):

$$\kappa T \le \frac{\pi}{2\beta}$$

- Router condition:

$$\alpha_2 = \frac{\hat{x} f(\hat{x})}{f'(\hat{x})} \le \beta$$

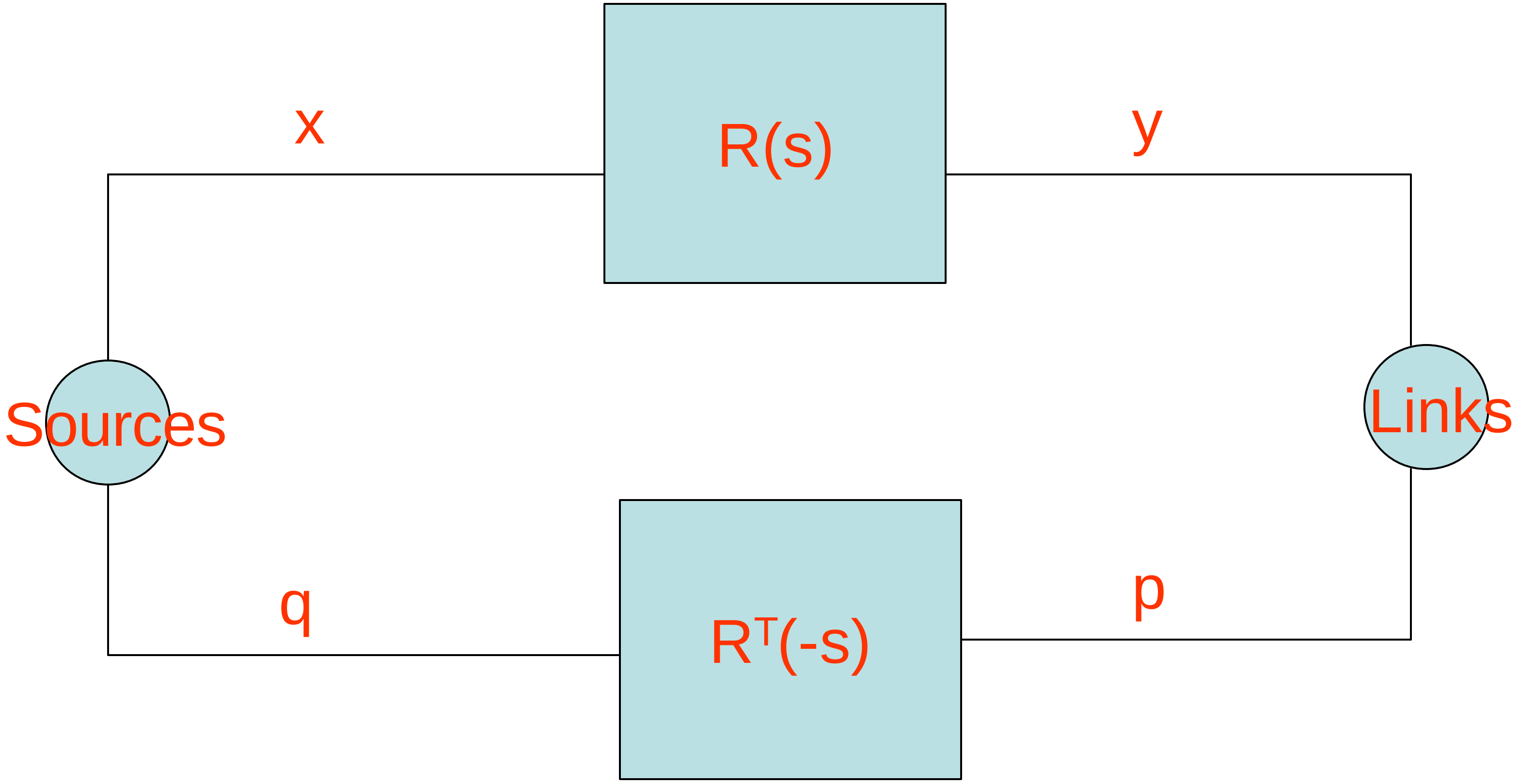
Decomposition for a Particular Price Function

- Consider the marking function

$$f(y) = \left(\frac{y}{c}\right)^{\beta}$$

- The router condition is automatically satisfied!

Network Analysis



Extension to Networks

- The same condition for local stability continues to hold for arbitrary topology networks with arbitrary delays!

- Similar conditions can be derived for dual and primal-dual algorithms

Global Stability - I

- Local stability in the presence of delays: Frequency-domain analysis, Nyquist criterion
- Global stability without delays: Lyapunov stability theorem
- Global stability with delays: ???

Global Stability - II

- Alternate proof of global stability without delays:

$$V(x) = \sum_r (x_r - \hat{x}_r)^2$$

- Show that $\dot{V} < 0$ and is equal to zero at the equilibrium point

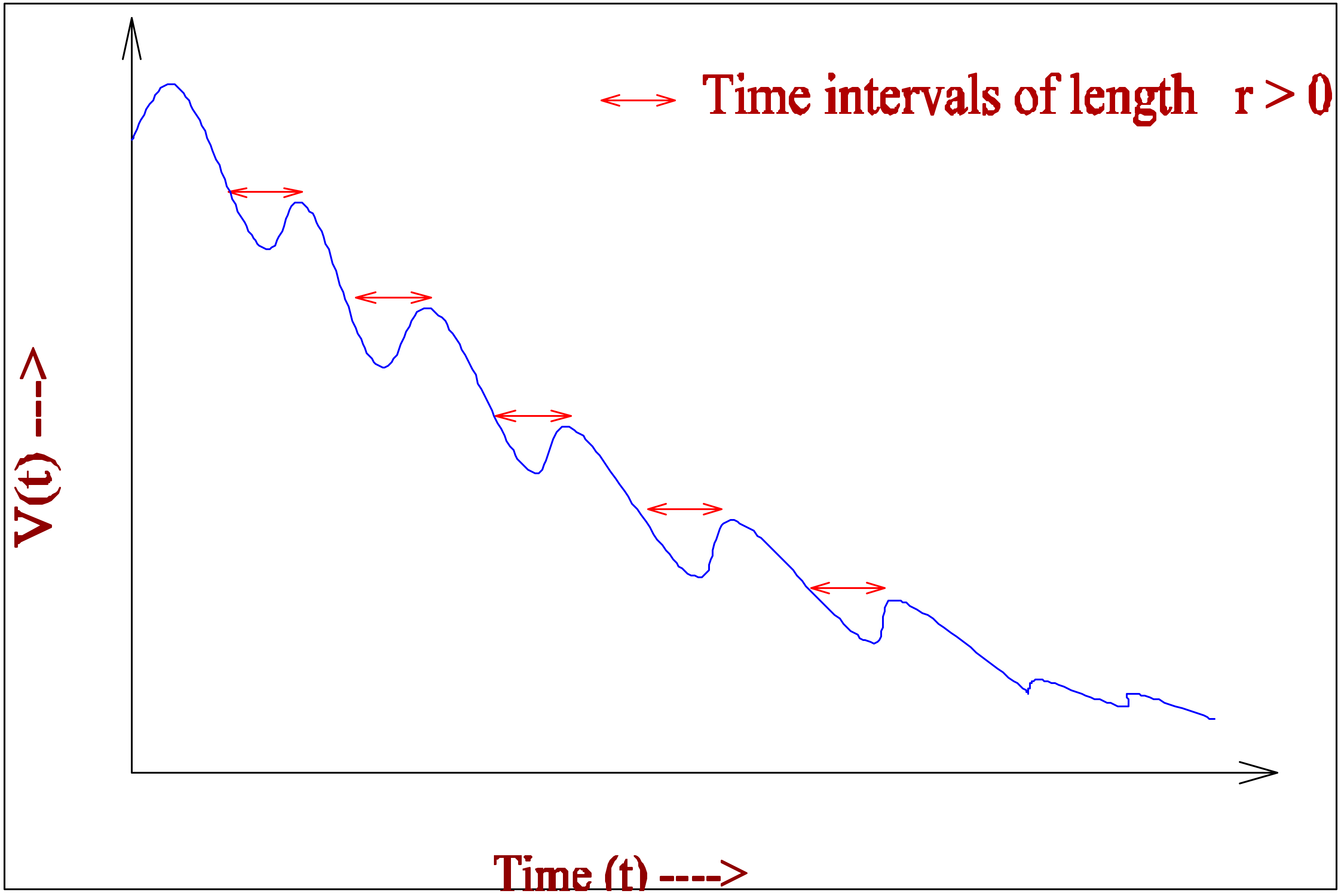
Razumikhin's theorem

- With delays, use the Lyapunov-Razumikhin function

$$V(t) = \max_{t-r \leq s \leq t} (x(t) - \hat{x})^2$$

- Show that V(t) decreases to zero

What does it mean?



Global Stability Results

- Can show results that are slightly weaker for some special cases: single link accessed by many sources, general topology networks with special price functions

- Open problem: results for general topology networks, general price functions

Stochastic Models

- Are deterministic models valid?
  - Not all sources respond to congestion signals (non-adaptive real-time sources)
  - Most file transfers are too short; stability analysis doesn’t make sense
  - The congestion feedback process is probabilistic
  - Inability to precisely model window flow control
- The real network has a lot of randomness

Discrete-Time Stochastic Models

- Consider N proportionally fair controllers over a single link.
- In the Nth system, the dynamics of the rth source is given by

$$x_r^{(N)}(k+1) = x_r^{(N)}(k) + \kappa(w - M_r(k))$$

- M_r is a random variable that is a function of $\sum_r x_r$

Model at the Link

- Each source generates packets according to Poisson(x_r)

- Each packet is marked with probability f(y(k)), where y(k) is the total number of packets that arrived at the link at time k

- $M_r(k)$=Poisson(x_r f(y))

Large Number of Sources Limit

- Under appropriate conditions

$$x^{(N)}(k) = \frac{1}{N}\sum_r x_r^{(N)}(k)$$

converges in probability to

$$x(k+1) = x(k) + \kappa(w - x(k)f(x(k)))$$

Arrivals and Departures

- Congestion time scale: Number of file transfers (connections) in the network is constant
- Connection-level time scale: File transfer requests (connections) arrive and depart. Resource allocation is performed instantaneously

Connection-Level Model

- Connections arrive according to a Poisson process of rate λ_r for route r

- Each connection for route r is a file whose size is drawn independently from an exponential distribution with mean $1/\mu_r$

- The first assumption is reasonable, the second assumption is not reasonable for the Internet

Necessary Condition for Stability

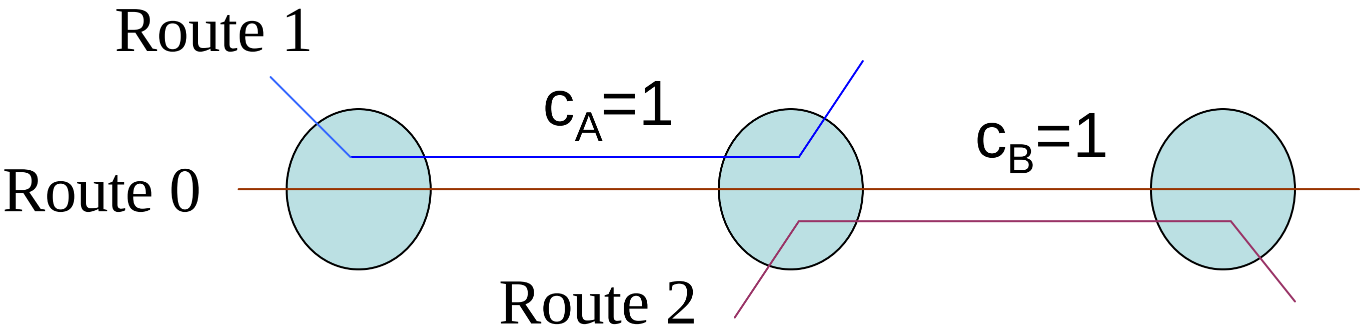
- For each link l, the total load on the link should be less than its capacity:

$$\sum_{r:\, l \in r} \rho_r < c_l$$

where $\rho_r = \lambda_r / \mu_r$

- Is this also sufficient? Answer: Depends on the resource allocation policy (Bonald and Massoulie)

Priority



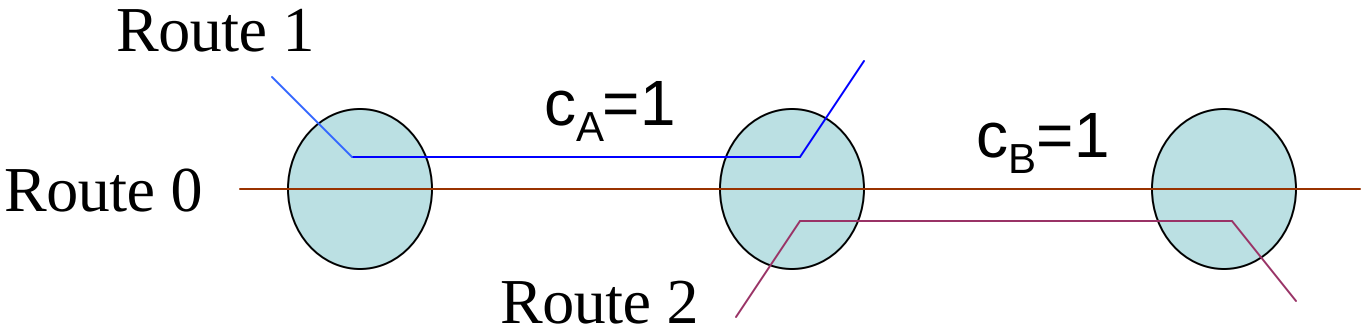
- Routes 1 and 2 get priority over Route 0
- Stability condition:

$$\rho_0 < (1-\rho_1)(1-\rho_2)$$

versus

$$\rho_0 < \min(1-\rho_1, 1-\rho_2)$$

What if we use TCP for resource allocation?



$$\max_{\{x_r\}} \sum_r n_r U_r(x_r)$$

$$n_0 x_0 + n_1 x_1 \leq c_A$$

$$n_0 x_0 + n_1 x_1 \leq c_B$$

A Class of Utility Functions

- Mo and Walrand:

$$U_r(x_r) = w_r \frac{x_r^{1-\alpha}}{1-\alpha}, \qquad \alpha > 0$$

- α !1, max-main fairness
- α! 2, TCP-Reno
- α! 1, proportional fairness
- α! 0, maximize total throughput

Connection-Level Fluid Model

$$\dot{n}_r = \lambda_r - n_r x_r \mu_r$$

- Rate which number of files increases is the arrival rate minus the departure rate

- Stochastic stability: Network is stable if the number of files in the fluid model goes to zero in finite time

Proof of Stability

$$V(n) = \sum_r k_r n_r^2$$

- Using properties of concave functions, show that

$$\dot{V} = \sum_r 2 k_r n_r \dot{n}_r \leq -\epsilon\sqrt{V}$$

when the necessary condition for stability is satisfied

Some Open Problems

- Global stability for networks
- Getting to the equilibrium (slow start)
- Connection-level stability for general file-size distributions