Aircraft trajectory optimization

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The problem
Aircraft simulation models
Performance modeling
Methods for solution
Interactive use and remote computing
Flight testing
Future developments
Aircraft simulation models

Exist for everything: Structures, Aerodynamics, Controls, Subsystems

Different models for different purposes

Ordinary and partial differential equations

Sometimes also with integral terms (memory)

Some are interactive others run in batch
Flight simulator

Rigid body six-degrees of freedom (position and orientation)
Aerodynamic force modeling is (very) complex
Flight testing and wind tunnels are the main sources
System of nonlinear ODEs: $f(\dot{x}, x, u) = 0$
12 primary state variables in $x$
Total number for JAS-39 Gripen is about 2500
Stick, pedals, throttle and many more define $u$
Simulators typically run at 60-240 Hz
Trajectory optimization

Compute optimal trajectory for minimum time, minimum fuel etc
Performance data from Saab
Aircraft equations of motion are discretized
Numerical optimization finds optimal control for a given mission
Optimal acceleration for J35 Draken

Minimize time

Initial condition: level flight, Mach 0.8, altitude 8 km

Final condition: level flight, Mach 1.2, altitude 8 km
Numerical results

Acceleration M=0.8 to M=1.2 at 8 km

Nondimensional time

Nondimensional altitude

Nondimensional time

Mach

Nondimensional time
Flight test, Dan Eriksson (2.div F10)

Reference level acceleration $M=0.8$ to $M=1.2$ at 8 km

Optimal acceleration $M=0.8$ at 8 km to $M=1.2$ at 8 km
Web version for SK60

- Familiar user interface
- No manuals
- No distribution/installation
- Easy to upgrade and modify
- Low cost, Linux PC
- Efficient, solution in 5-10 s
- Graphics and listing presented to the user
Optimal trajectories for SK60 with your WAP-phone

- WAP compatible cellular phone
- Input data is sent to a server at KTH
- The trajectory is computed
- The nonlinear optimization problem is solved
- Time 5-10 s
- The pilot instruction is shown on the phone display
Field testing with the Swedish Air Force

Flight testing
Minimum time to climb
Minimum fuel to climb
Minimum time base to base
Minimum fuel base to base
Evaluation using GPS
Minimum time to climb strategy

Initial condition
Fuel 90 %
Altitude 0.2 km
Indicated airspeed 350 km/h

Final condition
Altitude 9 km
Mach 0.68
Minimum time to climb

**Initial condition**
Altitude 0.2 km
Indicated airspeed 350 km/h

**Final condition**
Altitude 9 km
Mach 0.68
Mission evaluation

- J35/SK60 has no recording of flight data
- GPS only alternative (cost)
- Use optimization in reverse
- Measure some states with GPS
- Weather and winds from the weather service
- Estimate remaining states and controls with a simulation model
GPS-based evaluation system

Swepos

GPS #1

GPS #2

Markbaserad referensstation

Utvärdering i Matlab

Flyg
GPS requirements

- Handheld
- Internal (full) data storage, at least 1 hour
- Differential correction (DGPS) after flight
- Local base station or SWEPOS
- Software

Late -98: Trimble GeoExplorer II (III), Garmin 100 SRVY II
Flight manual climb for the SK60

Numerical model (- - -), GPS measurement (——)

![Graphs showing altitude, Mach number, and speed vs. time for the SK60 flight manual climb.]
Postprocessing of GPS data

- The trajectory is known \((x(t),y(t),z(t))\)
- Velocities and accelerations are estimated
- Weather model
- Simulation model
- Estimate:
  - Angle-of-attack
  - Attitude
  - Load factor
  - Thrust setting
  - Fuel burn
Visualization

Matlab

OpenGL
Developments

- Experiments with integrated GPS/INS
- Refined methods for the inverse problem
- Flight test data used for model verification
- Improved visualization
- Demonstrator for mission planning/evaluation for the SK60
- Collaboration with the Swedish Air Force
Future developments

- Modeling of weather and air space restrictions
- The pilot interface
- Autopilot for automatic path following
- Closed loop control
- Nonlinear optimization in close to real time
- Simple and robust methods for approximate solution
- Complex model and efficient methods for refined solution
- Modeling, numerical analysis, optimization and flight testing
Remote computation

Pilot request

Trajectory data

SNOPT running
Reverse processing improves the model

Unknown parameters $p$, e.g. $C_L\alpha$, in $\dot{x} = f(x, u, p)$

Measure state $\hat{x}(t)$ and control $\hat{u}(t)$ in flight testing

Solving $\dot{x} = f(x, u, p)$ gives $x(t, p)$ and $u(t, p)$

Find $p$ that minimizes the differences $x(t, p) - \hat{x}(t)$ and $u(t, p) - \hat{u}(t)$