BUS AND DRIVER SCHEDULING IN URBAN MASS TRANSIT SYSTEMS

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OVERVIEW

- Introduction
- Bus scheduling
- Driver duty scheduling
- Simultaneous bus and driver scheduling

URBAN BUS TRANSPORTATION

- Provides:
  - Interesting, complex and challenging problems for Operations Research
- Because:
  - Large savings can be realized
  - A large number of resources is involved

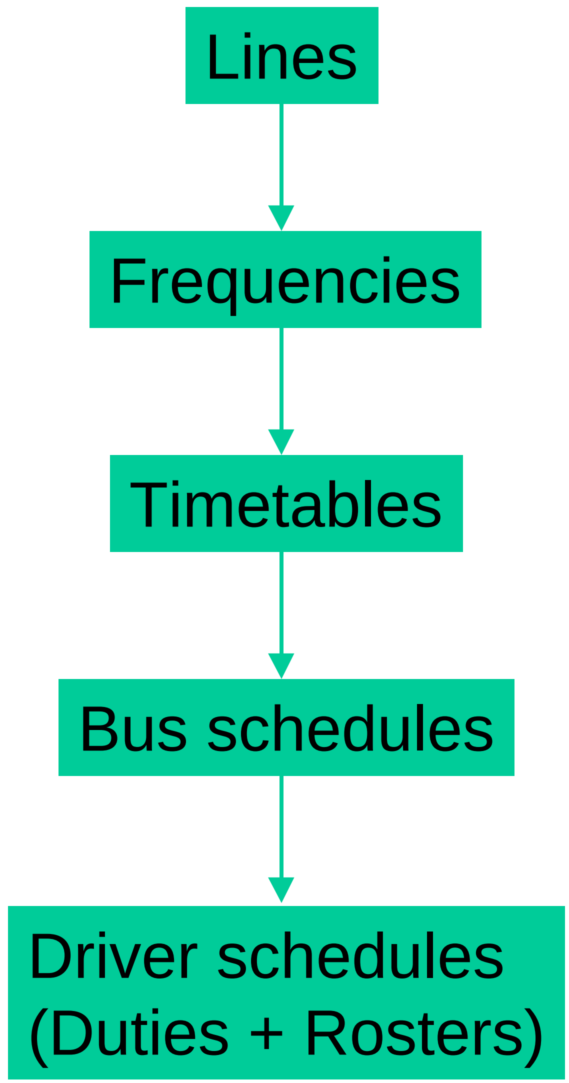
LARGE NUMBERS

	Nb Lines	Nb Buses	Nb Depots
Twin Cities	132	940	5
Montréal	206	1500	7
Paris	246	3860	23
NYC	298	4860	18

Nb of drivers ≅ 2-3 x nb of buses

Nb of daily trips ≅ 10-20 x nb of buses

OPERATIONS PLANNING PROCESS



GOAL OF THIS TALK

- Review latest approaches based on mathematical programming for
  - Bus scheduling
  - Duty scheduling
  - Simultaneous bus and duty scheduling

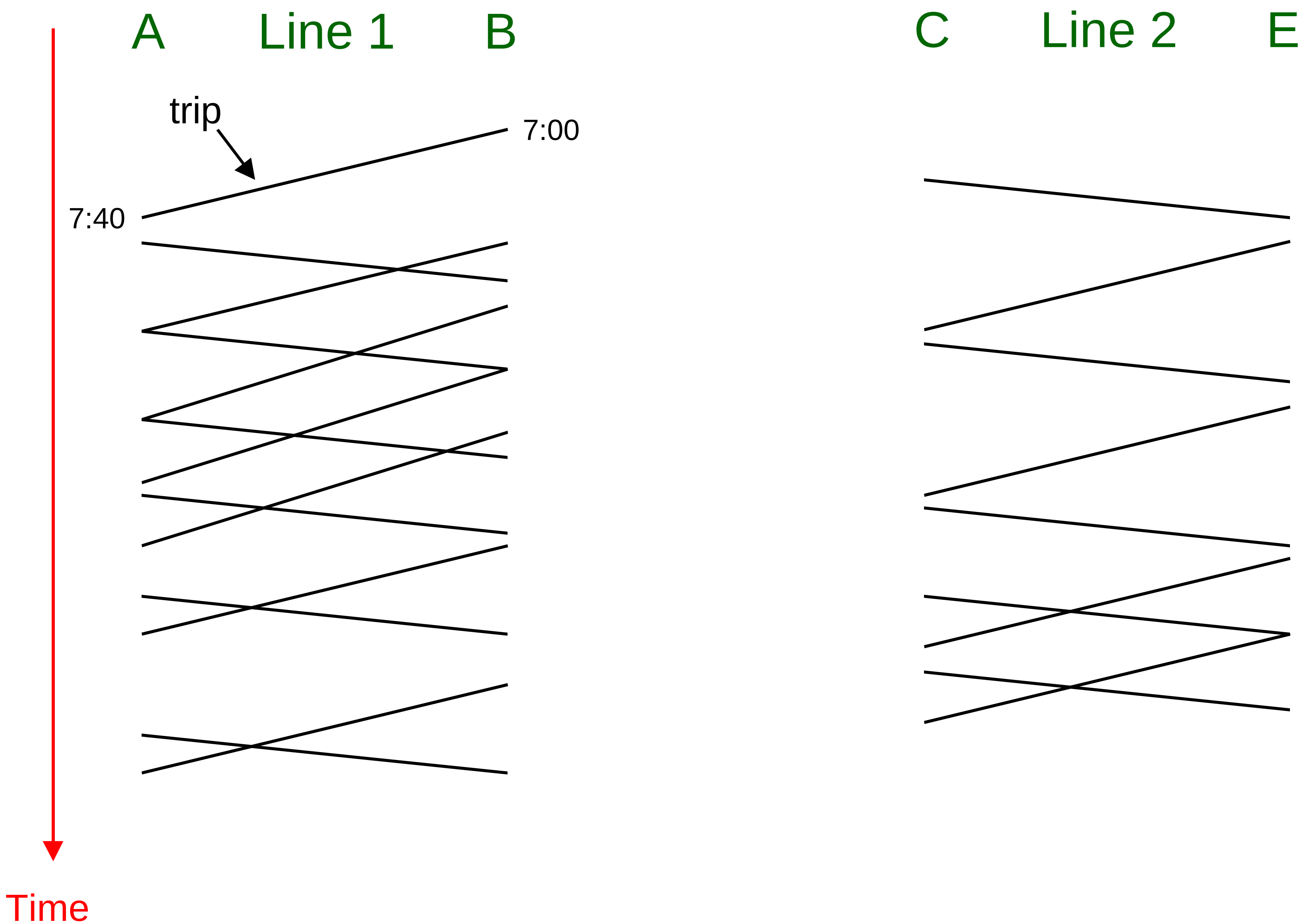
- Where do we stand with these approaches ?

BUS SCHEDULING PROBLEM DEFINITION

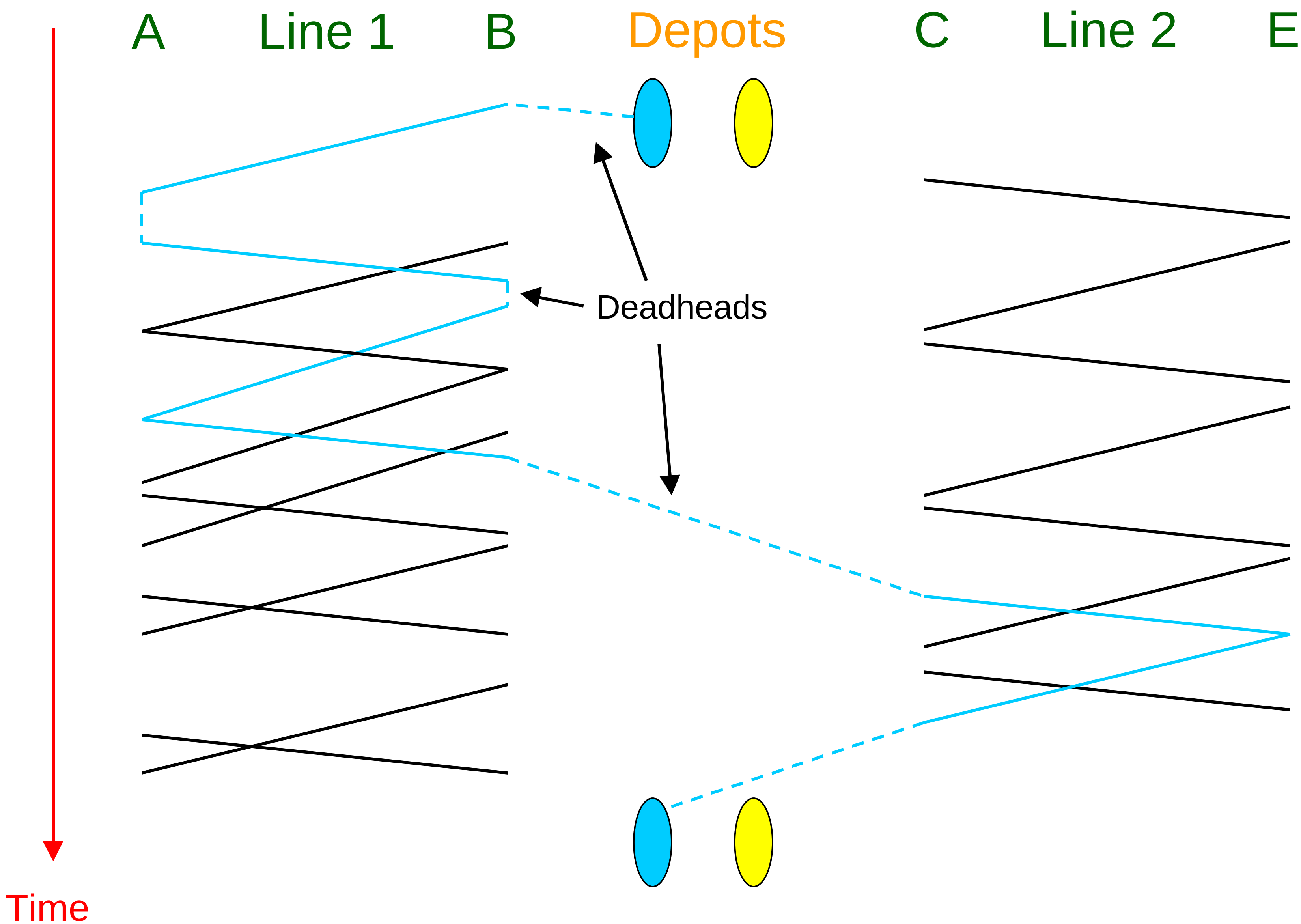
- One-day horizon

- Several depots
  - Different locations
  - Different bus types (standard, low floor, reserve lane in opposite direction, …)

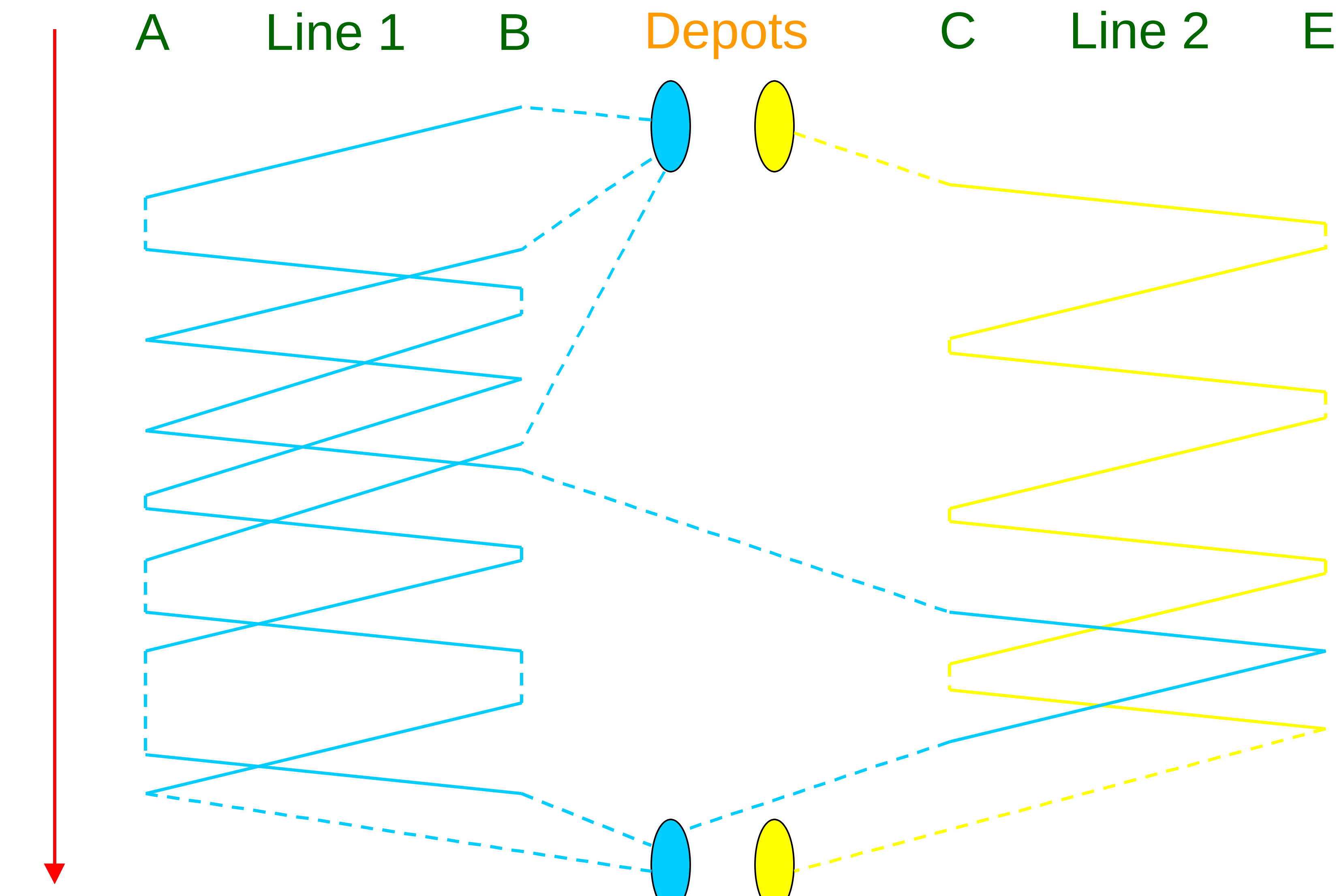
PROBLEM DEFINITION (CONT'D)



PROBLEM DEFINITION (CONT'D)



PROBLEM DEFINITION (CONT'D)



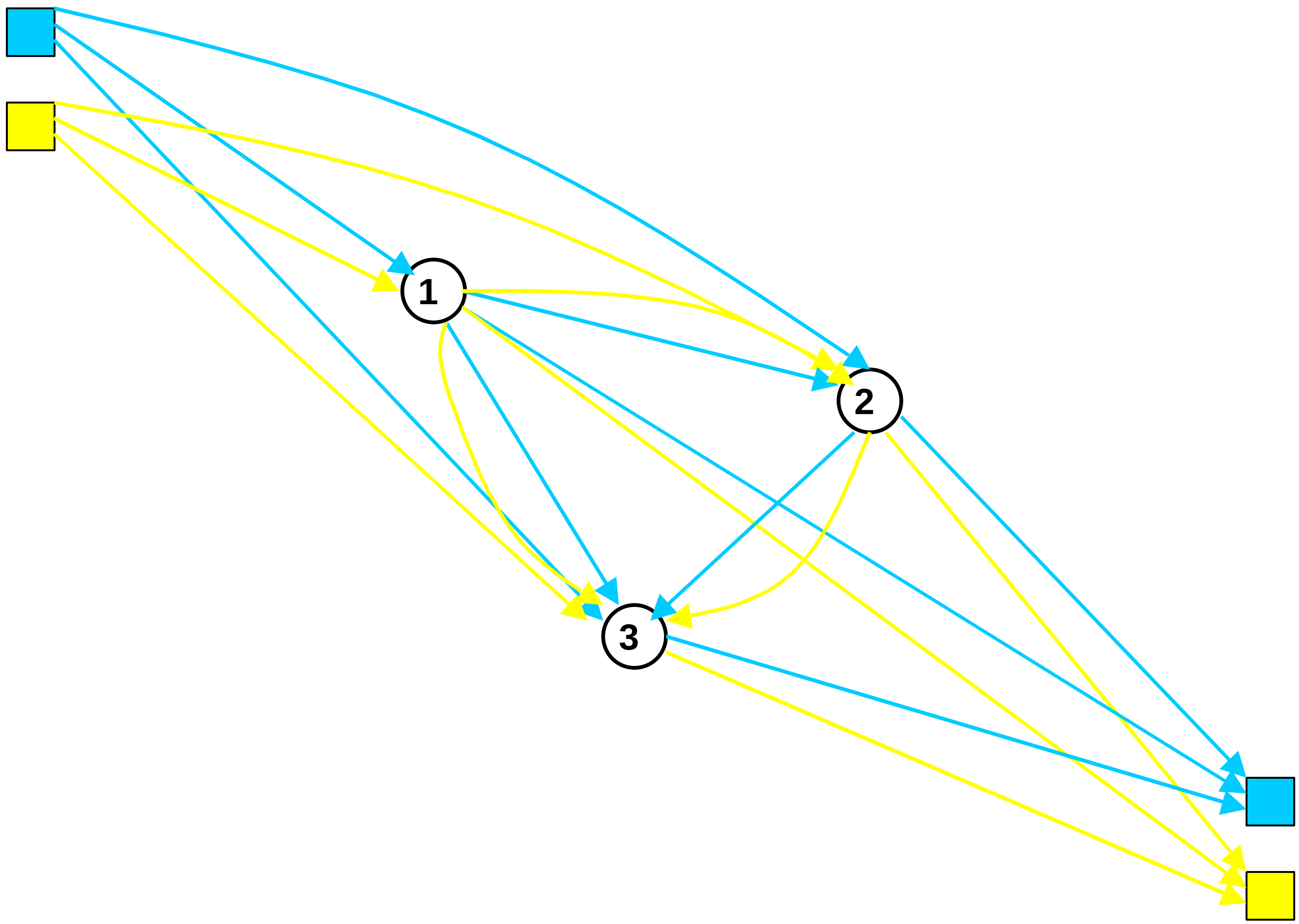
PROBLEM DEFINITION (CONT'D)

- Constraints
  - Cover all trips
  - Feasible bus routes
    - Schedule
    - Starts and ends at the same depot
  - Bus availability per depot
  - Depot-trip compatibility
  - Deadhead restrictions

PROBLEM DEFINITION (CONT'D)

- Objectives:
  - Minimize the number of buses
  - Minimize deadhead costs
    - Proportional to travel distance or time
    - Fuel, maintenance, driver wages
  - No trip costs

NETWORK STRUCTURE



SOLUTION METHODOLOGIES

- Multi-commodity + column generation
- Set partitioning + branch-and-price-and-cut

A. Löbel (1998)

Vehicle scheduling in public transit and lagrangean pricing

Management Science 44

MULTI-COMMODITY MODEL

$$X_{ij}^k = \begin{cases} 1 & \text{if arc } (i,j) \text{ from depot } k \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Minimize} \quad \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij} X_{ij}^k$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{(t,j) \in A^k} X_{tj}^k = 1, \quad \forall\, t \in T$$

$$\sum_{(o^k,j) \in A^k} X_{o^k,j}^k \leq v^k, \quad \forall\, k \in K$$

$$\sum_{(t,j) \in A^k} X_{tj}^k - \sum_{(j,t) \in A^k} X_{jt}^k = 0, \quad \forall\, t \in T, k \in K$$

$$X_{i,j}^k \in \{0,1\}, \quad \forall\, k \in K, (i,j) \in A^k$$

COLUMN GENERATION

- On the multi-commodity formulation

- Two pricing strategies
  - Lagrangean pricing
  - Standard

- LP solution is often integer
  - If not, rounding procedure

LAGRANGEAN PRICING

- For fixed dual variables, solve
  - Lagrangean relaxation 1
    - Relax trip covering constraints
    - Obtain a minimum cost flow problem
  - Lagrangean relaxation 2
    - Relax flow conservation and depot capacity constraints
    - Add

$$\sum_{k \in K} \sum_{(j,t) \in A^k} X_{jt}^k = 1, \quad \forall \, t \in T$$

    - Obtain a simple problem that can be solved by inspection

RESULTS

- Real-world instances

Depots	Trips	Depots/trip	CPU (hrs.)
4	3,413	1.7	0.7
9	2,424	4.9	12
12	8,563	2.2	14
49	24,906	1.6	10*

* Not to optimality

A. Hadjar, O. Marcotte, F. Soumis (2001)

A branch-and-cut approach for the multiple depot vehicle scheduling problem

Les Cahiers du GERAD, G-2001-25

SET PARTITIONING MODEL

$$\theta_p^k = \begin{cases} 1 \text{ if route } p \text{ from depot } k \text{ is chosen} \\ 0 \text{ otherwise} \end{cases}$$

$$\text{Minimize} \quad \sum_{k \in K} \sum_{p \in \Omega^k} c_p \theta_p^k$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{p \in \Omega^k} a_{tp} \theta_p^k = 1, \quad \forall\, t \in T$$

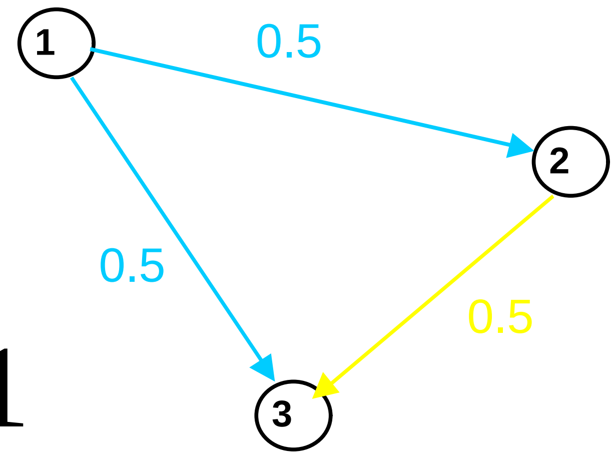
$$\sum_{p \in \Omega^k} \theta_p^k \le v^k, \quad \forall\, k \in K$$

$$\theta_p^k \in \{0, 1\}, \quad \forall\, k \in K, p \in \Omega^k$$

BRANCH-AND-PRICE-AND-CUT

- Arc elimination throughout the search tree
  - Heuristic feasible initial solution
  - Reduced cost test $\bar{c}_{ij}^{k} \geq z_{IP}^{cur} - \pi^{\top} b$

- Odd cycle cuts (facets)



$$X_{12}^{b} + X_{13}^{b} + X_{23}^{y} \leq 1$$

RESULTS

- Randomly generated instances

Depots	Trips	Depots/trip	CPU (hrs.)
2	900	2	2.5
4	500	4	3
6	500	6	5

VARIANTS

- Fueling constraints

- Departure time windows
  - Desaulniers, Lavigne, and Soumis (1998)
  - Bianco, Mingozzi, Ricciardelli (1995)

- Departure time windows and waiting costs
  - Desaulniers, Lavigne, and Soumis (1998)

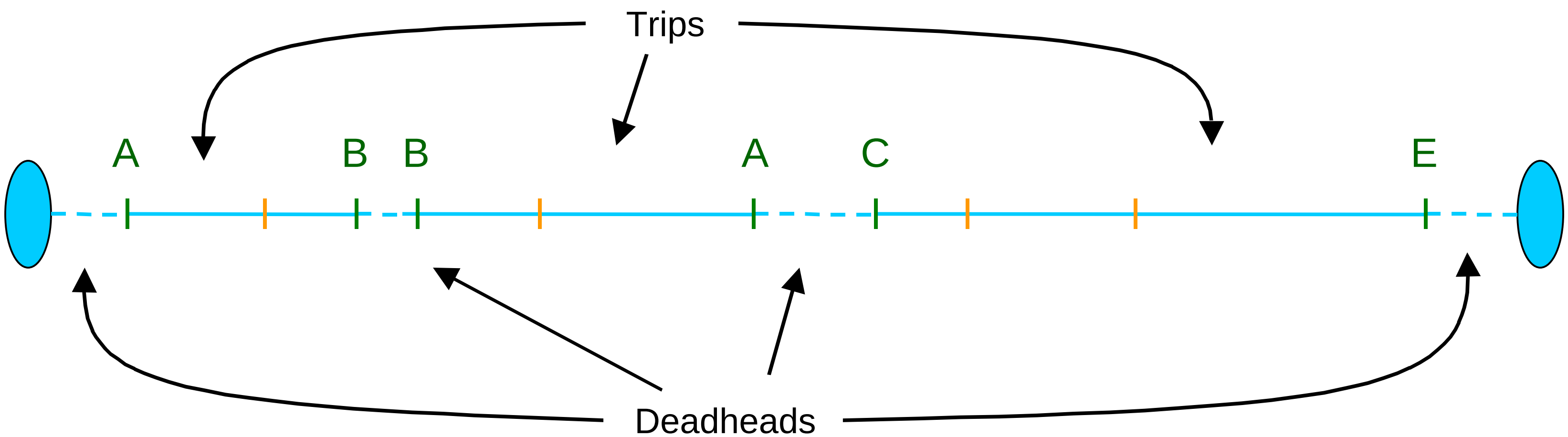
DUTY SCHEDULING PROBLEM DEFINITION

- One-day horizon
- One depot

- Bus blocks are known
- Bus deadheads are known

PROBLEM DEFINITION (CONT'D)

One bus block



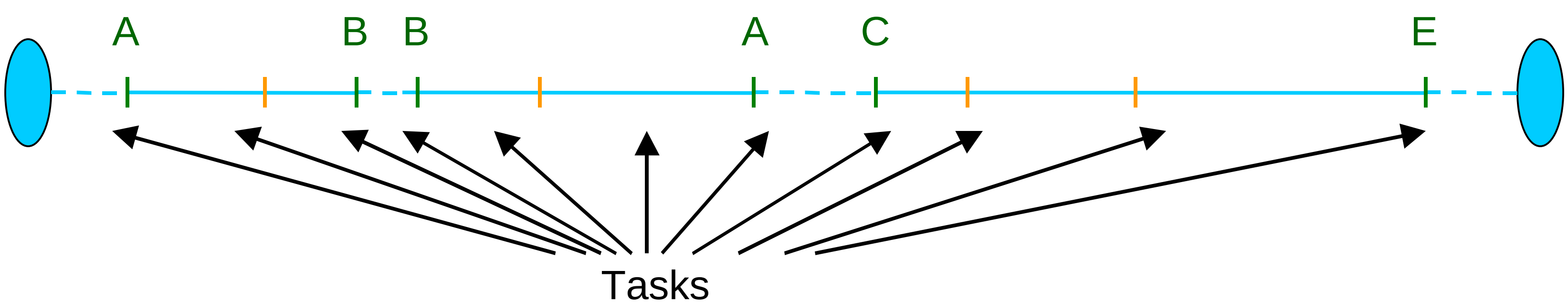
Relief point: location where a change of driver can occur

Relief points: all termini

additional locations

PROBLEM DEFINITION (CONT'D)

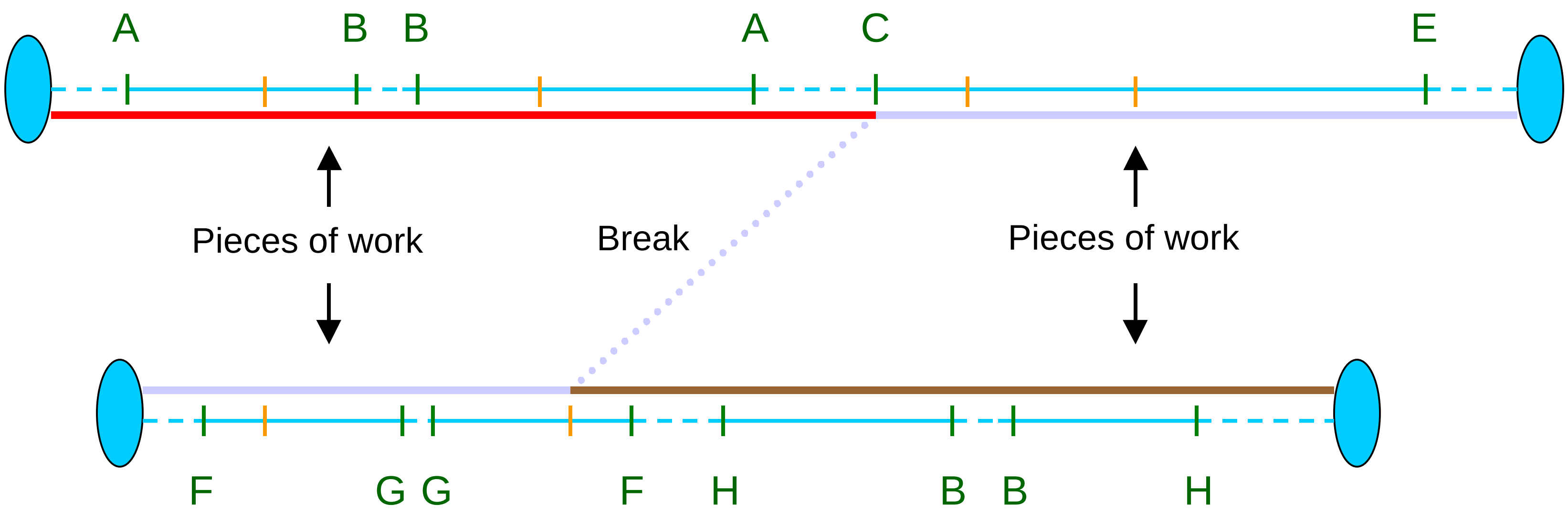
One bus block



Task: indivisible portion of work between two consecutive relief points along a bus block

Trip tasks and deadhead tasks

PROBLEM DEFINITION (CONT'D)



Duty: Sequence of tasks assigned to a driver

Piece of work: Subsequence of tasks on the same block

Duty type: Depends on work regulations

PROBLEM DEFINITION (CONT'D)

- Constraints
  - Cover all tasks (continuous attendance)
  - Feasible duties
    - Schedule
    - Work regulations for each duty type
      - Number of pieces
      - Min and max piece duration
      - Min and max break duration
      - Min and max working time
      - Valid start time interval
  - Min number of duties of each type

PROBLEM DEFINITION (CONT'D)

- Objectives
  - Minimize the number of duties
  - Minimize total wages

SET PARTITIONING MODEL

$$Y_d^u = \begin{cases} 1 & \text{if duty } d \text{ of type } u \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

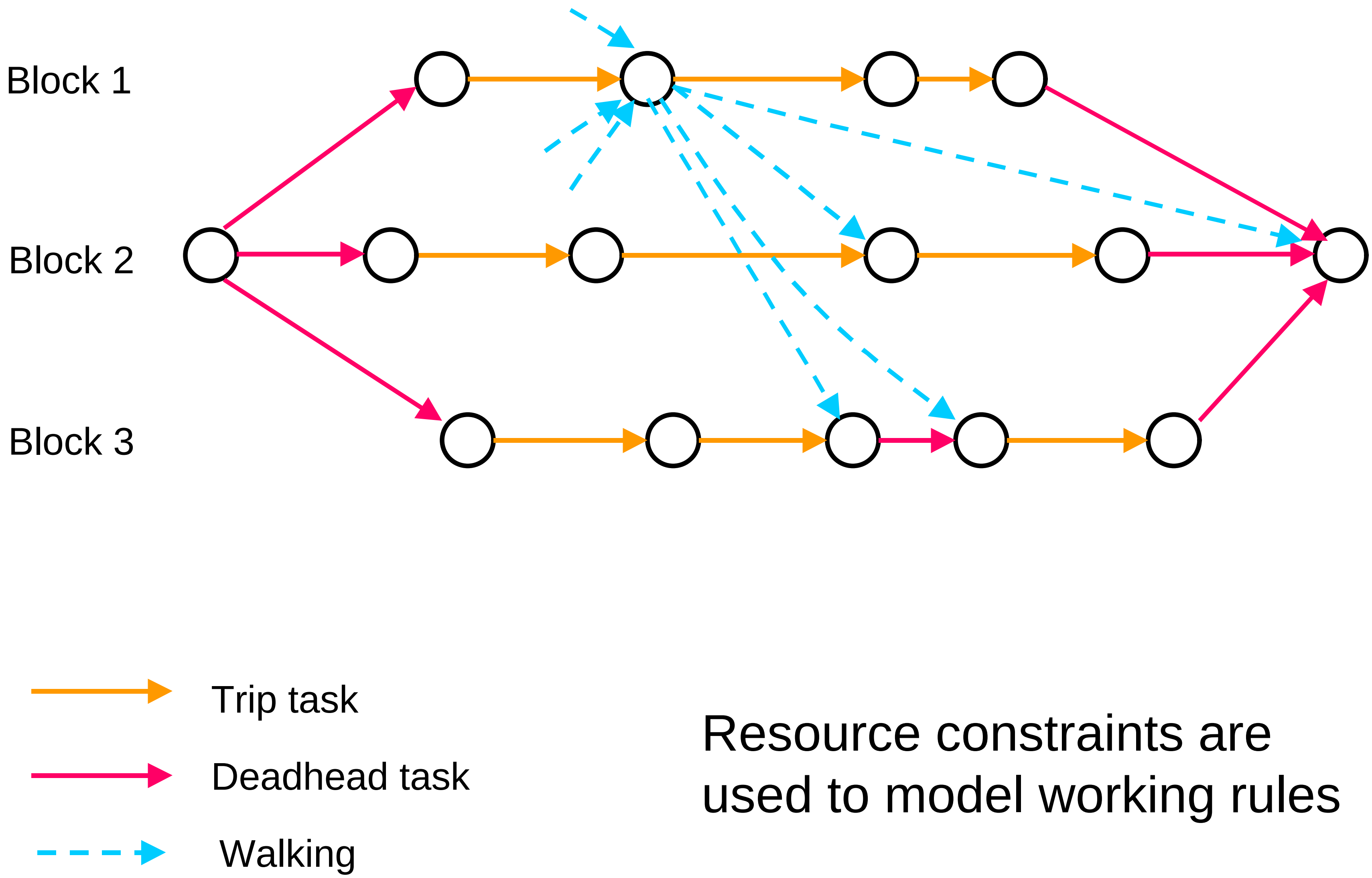
$$\text{Minimize} \quad \sum_{u \in U} \sum_{d \in \Delta^u} c_d Y_d^u$$

$$\text{s.t.} \quad \sum_{u \in U} \sum_{d \in \Delta^u} a_{sd} Y_d^u = 1, \quad \forall\, s \in S$$

$$\sum_{d \in \Delta^u} Y_d^u \geq n_u, \quad \forall\, u \in U$$

$$Y_d^u \in \{0, 1\}, \quad \forall\, u \in U, d \in \Delta^u$$

NETWORK STRUCTURE



Resource constraints are used to model working rules

SOLUTION METHODOLOGY

- Heuristic branch-and-price

R. Borndörfer, M. Grötschel, A. Löbel (2001)

Scheduling duties by adaptive column generation

ZIB-Report 01-02
Konrad-Zuse-Zentrum für Informationstchnik, Berlin

HEURISTIC BRANCH-AND-PRICE

- Master problem solved by Lagrangean relaxation

- Constrained shortest path subproblem solved by
  - Backward depth-first search enumeration algorithm
  - Lagrangean lower bounds are used to eliminate possibilities

HEURISTIC BRANCH-AND-PRICE

- Depth-first branch-and-bound
  - At each node, 20 candidate columns are selected
  - Probing is performed for each candidate
  - One variable is fixed at each node
  - No new columns are generated if the decision made does not deteriorate too much the objective function value

RESULTS

- 1065 tasks
- 3 duty types
- 1h20 of CPU time
- Reduction in number of duties from 73 to 63

SIMULTANEOUS BUS AND DUTY SCHEDULING – PROBLEM DEFINITION

- One-day horizon
- One depot

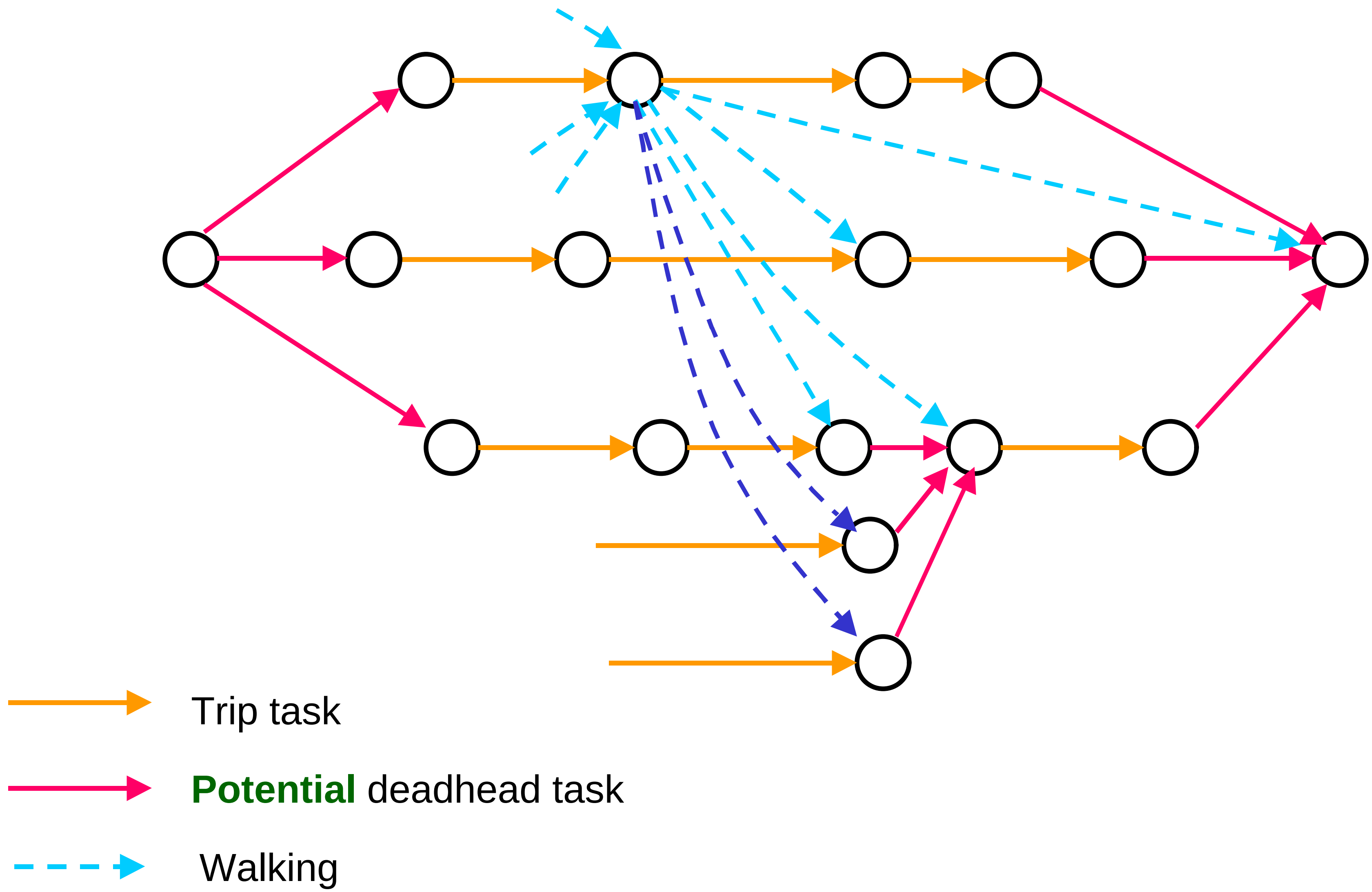
- Bus blocks are unknown
  - ⟹ Bus deadheads are unknown

PROBLEM DEFINITION (CONT'D)

- Find
  - Feasible bus blocks
  - Feasible duties

- Such that
  - Each trip is covered by a bus
  - Each trip task is covered by a driver
  - Each selected deadhead task is covered by a driver

BUS AND DUTY SCHEDULING NETWORK STRUCTURE



Trip task

Potential deadhead task

Walking

SOLUTION METHODOLOGIES

- Mixed set partitioning / flow model
  + column generation / heuristic

- Set partitioning + branch-and-price

R. Freling, D. Huisman,
A.P.M. Wagelmans (2000)

Models and algorithms for integration of vehicle and crew scheduling

Econometric Institute Report EI2000-10/A
Erasmus University, Rotterdam

MIXED SET PARTITIONING / FLOW MODEL

$$\text{Minimize} \quad \sum_{(i,j)\in A} c_{ij} X_{ij} + \sum_{u\in U} \sum_{d\in\Delta^u} c_d Y_d^u$$

$$\text{s.t.} \quad \sum_{(t,j)\in A} X_{tj} = 1, \quad \forall\, t \in T$$

$$\sum_{(i,t)\in A} X_{it} = 1, \quad \forall\, t \in T$$

$$\sum_{u\in U} \sum_{d\in\Delta^u} a_{sd} Y_d^u = 1, \quad \forall\, s \in S^T$$

$$\sum_{u\in U} \sum_{d\in\Delta^u} b_{ijd} Y_d^u - X_{ij} = 0, \quad \forall\, (i,j) \in A$$

$$Y_d^u \in \{0,1\}, \quad \forall\, u \in U, d \in \Delta^u$$

$$X_{ij} \in \{0,1\}, \quad \forall\, (i,j) \in A$$

COLUMN GENERATION / HEURISTIC

- LP relaxation is solved by column generation
  - Master problem is solved by Lagrangean relaxation
    - Relax all driver-related constraints
    - Obtain a single-depot bus scheduling problem
  - Pricing problem is solved in two phases
    - Solve an all-pairs shortest path problem to generate pieces of work
    - Combine these pieces to form negative reduced cost feasible duties

COLUMN GENERATION / HEURISTIC

- Once the LP relaxation is solved
  - Fix the bus blocks as computed in the last master problem
  - Solve a duty scheduling problem by column generation

RESULTS

- Real-world instances

Trips	Trip tasks	Gap (%)	CPU (hrs.)
113	113	0.0	0.33
148	148	2.9	1.5
238	238	3.6	67.8

K. Haase, G. Desaulniers, J. Desrosiers (2001)

Simultaneous vehicle and crew scheduling in urban mass transit systems

Transportation Science 35

SET PARTITIONING MODEL

$$\text{Minimize} \quad cB + \sum_{u \in U} \sum_{d \in \Delta^u} c_d Y_d^u$$

$$\text{s.t.} \quad \sum_{u \in U} \sum_{d \in \Delta^u} b_{td}^{\text{DH}} Y_d^u = 1, \quad \forall\, t \in T$$

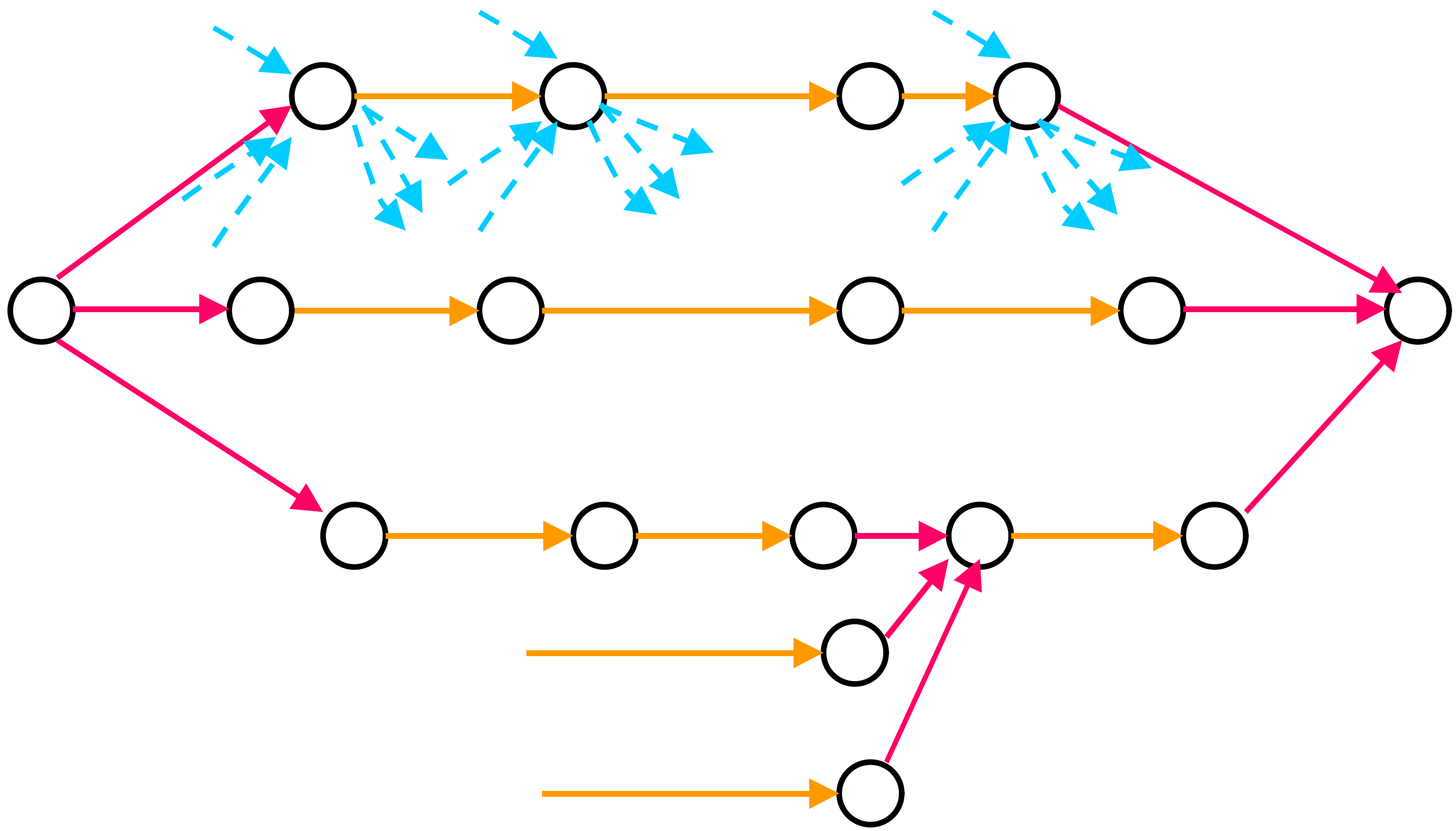
$$\sum_{u \in U} \sum_{d \in \Delta^u} \left(b_{sd}^{\text{WI}} - b_{sd}^{\text{WO}} \right) Y_d^u = 0, \quad \forall\, s \in S^T$$

$$\sum_{u \in U} \sum_{d \in \Delta^u} \left(e_{td}^{\text{WI}} - e_{td}^{\text{WO}} \right) Y_d^u = 0, \quad \forall\, t \in T$$

$$\sum_{u \in U} \sum_{d \in \Delta^u} q_{hd} Y_d^u \le B, \quad \forall\, h \in H$$

$$Y_d^u \in \{0, 1\}, \quad \forall\, u \in U, d \in \Delta^u$$

BUS AND DUTY SCHEDULING NETWORK STRUCTURE



Trip task

Potential deadhead task

Walking

EXACT BRANCH-AND-PRICE

- Bus constraints are generated dynamically
- Multiple subproblems
  - One per duty type and possible start duty time
  - Partial pricing
- Exact branching strategies

HEURISTIC BRANCH-AND-PRICE

- Early LP termination
- Depth-first search without backtracking
- Multiple branching decisions on columns

EXACT RESULTS

- Real-world instances
- Minimize number of duties

Trips	Trip tasks	Gap (%)	CPU (min.)
85	170	7.1	0.7
143	286	0	5.9
177	354	0	43.1
204	408	0	192.6
262	524	0	354.7

HEURISTIC RESULTS

- Real-world instances
- Minimize number of duties

Trips	Trip tasks	Gap (%)	CPU (min.)
177	354	4.1	5.3
204	408	0	8.2
262	524	0	36.1
378	756	2.0	70.3
463	926	0.2	199.2

VARIANTS

- Parkings
- Multiple depots
  - Desaulniers (2001), TRISTAN
  - Huisman (2002), IFORS

FUTURE RESEARCH ON SIMULTANEOUS BUS AND DUTY SCHEDULING

- Reducing solution times
  - For master problem
    - Dual variable stabilization
  - For constrained shortest path subproblems
    - Generate pieces of work instead of duties
- Target duty working time (in progress)
- Multiple depots (started)
- Integrate timetabling aspects

CONCLUSION

- Bus scheduling
  - Optimal or close-to-optimal solutions for large to very large instances
- Duty scheduling
  - Good solutions for medium to large instances
- Simultaneous bus and duty scheduling
  - Optimal or close-to-optimal solutions for small to medium instances

GILBERT'S MEASURE

Accuracy:	very high
Speed:	low to medium
Simplicity:	very low
Flexibility:	medium to high

THANK YOU !