On Performance Chasing, Mutual Fund Tournaments, and Managerial Incentives

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Abstract

Why do mutual fund investors chase past winner funds despite the absence of performance persistence among such funds? In this paper we adopt a "tournament" framework to analyze the incentives of two fund managers, with unequal performance at an interim stage, who compete for investor cash flows. Our model is characterized by an absence of differential ability among competing fund managers. We show that in equilibrium (a) it is optimal for the fund manager who is trailing behind at the interim stage (i.e., the interim loser) to increase the *idiosyncratic* risk of her portfolio, and (b) risk averse investors anticipate the incentives facing losing fund managers and rationally chase winners. Our analysis yields a number of testable predictions. In particular, we show that the increase in the idiosyncratic risk of the interim loser manager's portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). Furthermore, we show that the ex-ante utility of long-term fund investors is decreasing in the strength of the tournament effect. Our results have implications for several aspects of fund design including the optimal fund entry/exit policy, and choice of organizational form (i.e., closed-end vs. open-end).

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The tendency to chase past performance is one of the best known facts regarding the behavior of mutual fund investors.[1] However, investors are routinely warned that "past performance is no guarantee of future results" and there is little evidence to suggest that mutual fund performance persists.[2] What then explains the performance chasing behavior of fund investors? In this paper we provide a potential explanation for such investor behavior based on an analysis of the incentives faced by the fund managers. We show that, in light of the managerial incentives, performance chasing represents an optimal investor response, even if fund managers do not possess superior ability and there is no persistence in fund performance.

Our analysis builds on the observation that funds with superior relative performance attract disproportionately large cash inflows in the future. In this sense the mutual fund industry resembles a tournament in which players are competing with each other for investor cash flows. As shown by Brown, Harlow, and Starks (1996), the tournament nature of the mutual fund industry provides adverse incentives to losing fund managers. Specifically, they find that funds that trail at the half-year mark tend to subsequently increase the volatility of their portfolios in an attempt to catch up. Similarly, Chevalier and Ellison (1997) study the relationship between fund flows and past performance of mutual funds and conclude that the flow-performance relationship provides incentives for the funds to alter the risk of their portfolios at the interim stage.[3] In this study we adopt a tournament framework to analyze the incentives of fund managers and investors with a view to better understand the process of capital allocation to intermediaries like mutual funds.

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[1] A number of studies have documented a strong positive relation between a fund's past performance and its future fund inflows. See, for example, Ippolito (1992), Chevalier and Ellison (1997), Sirri and Tufano (1998), and Sapp and Tiwari (2004).

[2] Evidence of performance persistence has been documented by Grinblatt and Titman (1992), Hendricks, Patel and Zeckhauser (1993), Ibbotson and Goetzmann (1994), Brown and Goetzmann (1995), Malkiel (1995), and Elton, Gruber and Blake (1996), among others. However, Carhart (1997) shows that such persistence is explained by common factors in stock returns, rather than by managerial skill.

[3] See, also, Koski and Pontiff (1999). In related work, Busse (2001) analyzes daily returns for a sample of 230 equity funds over the period 1985–1995, but fails to find support for the hypothesis that fund managers actively alter the risk of

We analyze a model in which two risk-neutral fund managers with unequal performances at an interim stage, compete for investor cash flows. Given the unequal performances of the managers at the interim stage, we may think of one manager as an interim winner, and the other as the interim loser. Neither manager possesses superior information. Consistent with standard industry practice, managers' compensation is assumed to be a fixed proportion of the assets under management. Each manager has the choice of investing in a market index in addition to a security that represents pure idiosyncratic risk. Our focus is on the portfolio choices made by the two managers after they observe their interim performances. In the interests of expositional clarity, we begin by analyzing a single-period version of the model and then extend it to a multi-period setting which allows us to analyze the optimal response of risk averse investors to the incentives facing the fund managers. In this context we consider two groups of investors, namely, long-term or passive investors, and short-term or active investors.

We show that there exists an equilibrium in which (a) it is optimal for the fund manager who is trailing behind at the interim stage (i.e., the interim loser) to increase the *idiosyncratic* risk of her portfolio, and (b) risk averse investors anticipate the incentives facing losing fund managers and rationally chase winners.[4] Our analysis yields a number of testable predictions. In particular, we show that the increase in the idiosyncratic risk of the interim loser manager's portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). Furthermore, we show that the ex-ante utility of long-term fund investors is decreasing in the strength of the tournament effect. While a full welfare analysis of the costs and benefits of the implicit incentives generated by the tournament effect is beyond the scope of the present paper, our results point to several directions in which

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their portfolios in response to past performance. However, he recognizes that "uncovering a more complex behavior patter should be a fruitful area for future research" (p. 73).

[4] In a model with decreasing returns to scale in active portfolio management, and managers with differential ability, Berk and Green (2002) show that the response of fund flows to past performance reflects a competitive allocation of capital to the fund industry. In contrast, we focus on the incentive effects of the tournament framework (in the absence of differential information and moral hazard) to analyze the flow-performance relationship.

this research may be extended with potential implications for the optimal design of fund entry/exit policies, among other features.[5]

Our study contributes to a growing literature that examines managerial incentives and investor behavior in the mutual fund industry. On the theoretical front, a number of recent studies analyze the incentives facing mutual fund managers in a "tournament" framework (see, e.g., Goriaev, Palomino, and Prat (2003), and Taylor (2003)). Our model is closest in spirit to that of Taylor (2003) who develops a two-period model of a mutual fund tournament in which two fund managers with unequal midyear performances compete for new cash inflows.[6] Taylor derives a mixed-strategy equilibrium for the case when both funds are actively managed. The pure strategies that are the basis for the mixed-strategies are those of the extreme form, i.e. to invest fully in risky assets or to invest fully in the risk-free asset. In contrast, we allow the fund managers the option to take on idiosyncratic risk in addition to investing in the market portfolio and the risk free asset, and focus on the pure strategies equilibrium. A key feature of the earlier studies is that they take the performance chasing behavior of investors as given, i.e., they assume the existence of a tournament effect. In contrast, our model has a dynamic structure where investors make strategic decisions when choosing a particular fund. In this sense we endogenize the investor's decision regarding the choice of a fund to invest in. Such a framework allows us to provide a rationale for the existence of the tournament effect in the mutual fund industry. Furthermore, our model provides a rationale for performance chasing by investors in a setting where managers do not differ in terms of talent, efficiency, or in terms of superior information.

The rest of the paper is organized as follows. Section I presents a one-period version of our model while Section II extends the analysis to a multi-period version of the basic model. Section III provides concluding remarks. All technical proofs are relegated to the appendices.

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[5] Meyer and Vicker (1997) show that in a dynamic principal-agent framework, relative performance evaluation may be either welfare increasing or decreasing.

[6] Our one-period framework which includes two sub-periods corresponds to Taylor's two-period setting.

I. One Period Model

For the sake of clarity and to enable comparison with recent theoretical studies in the area, we begin our analysis by considering a one-period model in which we explore the nature of managerial incentives and implications for portfolio performance when portfolio managers are faced with investors who chase recent performance. Subsequently, in Section II, we analyze a multi-period model in which we explicitly model the incentives and the resultant behavior of the fund investors.

A. Model Structure

Consider two risk neutral managers who manage a pool of assets for a single period. Let the single period be divided into two (not necessarily equal) sub-periods (indexed by dates $t = 0$ to $t = 1$, and $t = 1$ to $t = 2$), with the end of the first sub-period marking an interim evaluation stage at which managers observe their performances to date, and make portfolio choices for the second sub-period. Figure 1 contains a timeline representing the sequence of events. Let m_g and m_b be the year-to-date cumulative performances of the two managers at the end of the first sub-period (i.e., at the interim evaluation stage), with the assumption that $m_g > m_b$. Based on their relative performances in the first sub-period, we denote manager *g* (*b*) a "winning" ("losing") manager. Let the performance gap between manager *g* and manager *b* at the interim stage (date $t = 1$) be denoted by m_δ, where $m_\delta = m_g - m_b$.

A.1. Managers' compensation

We assume that both managers begin with an asset base of size *s* at date $t = 0$. At date $t = 2$, the manager with the higher average return over the two sub-periods realizes additional fund inflows equal to λ. In other words, in this part of the paper we assume that there is a group of investors, with aggregate wealth λ, who chase the winner fund. Consistent with normal practice in the industry, we assume that managers are paid a fixed percentage *k* of the assets under their management at date $t = 2$. Hence, each manager's objective is to enlarge the size of their asset base which can be achieved by earning a higher return over the single period and by attracting the performance chasing investors at the end of the period.

A.2. Investment opportunity set and asset returns

Each manager faces an identical investment opportunity that consists of a riskfree asset with return $R_f > 0$, and a market index portfolio with return R_m. In addition to the market index portfolio, each manager has the option to invest in a portfolio that offers pure idiosyncratic risk (i.e., it offers zero expected risk premium). Let R_g denote the random return realized by the idiosyncratic portfolio chosen by the winning manager and let R_b be the random return realized by the idiosyncratic portfolio chosen by the losing manager. We assume R_m, R_g, and R_b are mutually independent normal random variables, where $R_m \sim N(\mu, \sigma_m)$ and $\mu > R_f$, and $R_g, R_b \sim N(R_f, \sigma_i)$. The properties of the asset returns are public knowledge, so neither manager possesses superior knowledge.[7]

B. Analysis of the Model

We now focus on the portfolio choices made by each manager at the start of the second sub-period. Manager *g*'s portfolio allocation decision is characterized by $\alpha^g = \begin{bmatrix} \alpha_1^g \\ \alpha_2^g \end{bmatrix} \geq 0$, with $\alpha_1^g + \alpha_2^g \leq 1$, where α_1^g is the proportion of the total assets she decides to invest in the market index portfolio, α_2^g is the proportion invested in the idiosyncratic portfolio, and $1 - \left(\alpha_1^g + \alpha_2^g\right)$ in the riskfree asset. Similarly manager *b*'s decision is characterized by $\alpha^b = \begin{bmatrix} \alpha_1^b \\ \alpha_2^b \end{bmatrix} \geq 0$, with $\alpha_1^b + \alpha_2^b \leq 1$.[8] Note that the managers' end-of-period compensation is given by

$$C_g = ks\Big((1+m_g)\big(1+R_f+\alpha_1^g(R_m-R_f)+\alpha_2^g(R_g-R_f)\big)+\lambda\Pi[\alpha^g,\alpha^b]\Big), \tag{1}$$

and

$$C_b = ks\Big((1+m_b)\big(1+R_f+\alpha_1^b(R_m-R_f)+\alpha_2^b(R_b-R_f)\big)+\lambda(1-\Pi[\alpha^g,\alpha^b])\Big), \tag{2}$$

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[7] While the assumption that managers do not possess superior information, is certainly consistent with empirical evidence on the performance of actively managed funds (see, for example Jensen (1968), Gruber (1996), Grinblatt and Titman (1993), Carhart (1997), among others), it is not critical for our analysis. Our results hold as long as the two managers do not differ in their abilities.

[8] Consistent with the restrictions facing the typical mutual fund, we disallow short sales.

where

$$\Pi[\alpha^g, \alpha^b] = 1 \quad \text{iff} \ \frac{(m_g + \alpha_1^g (R_m - R_f) + \alpha_2^g (R_g - R_f))}{2} > \frac{(m_b + \alpha_1^b (R_m - R_f) + \alpha_2^b (R_b - R_f))}{2}$$

$$\Pi[\alpha^g, \alpha^b] = 0 \quad \text{otherwise.}$$

At the end of the single period, with probability 1, one of the two funds will end up with a higher average return measured over the two sub-periods. The fund with the higher average return over the two sub-periods will be the ultimate "winner" of the tournament and it will experience a positive cash flow into the fund while the "loser" fund will not get any additional cash inflow. Note that the indicator variable $\Pi[\alpha^g, \alpha^b]$ equals 1 when manager *g* ends up as the ultimate "winner" and it equals 0 when manager *b* ends up as the ultimate winner. Hence, the last term in the expressions for the compensation of the managers, reflects the inflow of funds accruing to the "winner" fund. It is straightforward to note that

$$E[\Pi] = N\left(\frac{m_\delta + (\alpha_1^g - \alpha_1^b)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + {\alpha_2^b}^2){\sigma_i}^2 + (\alpha_1^g - \alpha_1^b)^2 {\sigma_m}^2}} \right). \tag{3}$$

where N(.) denotes the standard normal distribution function. We note that the above expression represents the probability of manager *g* finishing the game as the ultimate winner.

Since the constant *k* (proportional management fee charged by the managers) and *s* (the initial size of each fund) have no bearing on a manager's strategy, we will assume in this section, for simplicity, that $ks = 1$. We now consider the expected utility of the two managers for a strategies pair $\{\alpha^g, \alpha^b\} = \left\{ \begin{bmatrix} \alpha_1^g \\ \alpha_2^g \end{bmatrix}, \begin{bmatrix} \alpha_1^b \\ \alpha_2^b \end{bmatrix} \right\}$. The winning manager's expected utility is

$$U_g(\alpha^g, \alpha^b) = (1 + m_g)(1 + R_f + \alpha_1^g (\mu - R_f)) + \lambda N\left(\frac{m_\delta + (\alpha_1^g - \alpha_1^b)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + {\alpha_2^b}^2){\sigma_i}^2 + (\alpha_1^g - \alpha_1^b)^2 {\sigma_m}^2}} \right), \tag{4}$$

and the losing manager's expected utility is

$$U_b(\alpha^g, \alpha^b) = (1 + m_b)(1 + R_f + \alpha_1^b (\mu - R_f)) + \lambda - \lambda N\left(\frac{m_\delta + (\alpha_1^g - \alpha_1^b)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + {\alpha_2^b}^2){\sigma_i}^2 + (\alpha_1^g - \alpha_1^b)^2 {\sigma_m}^2}} \right). \tag{5}$$

At date $t=1$, manager g chooses portfolio holdings α^g to maximize her expected utility (4), and manager b chooses α^b to maximize (5). The following result characterizes the portfolio choices made by the two managers at date *t=1*, in equilibrium.

PROPOSITION 1: *In equilibrium,* $\alpha_1^g \geq \alpha_1^b$*, where the equation hold only when* $\alpha_1^g = \alpha_1^b = 1$. *Moreover,* $\alpha_2^g = 0$ *and* $\alpha_1^b + \alpha_2^b = 1$.

Proof: See appendix A.

Proposition 1 establishes that, in equilibrium, (a) manager g (i.e., the winning manager at the interim stage) will invest a proportion of her portfolio in the market index that is at least as large as the proportion chosen by manager b, (b) manager g's portfolio will not involve any exposure to idiosyncratic risk, and (c) manager b will not invest in the risk free asset. We also note that the manager g will always desire a portfolio allocation to the market index that is strictly greater than the manager b's allocation to the market index when the short sale constraint is not binding. Intuitively, the manager who is ahead at the interim stage (date $t = 1$) would like to "lock in" her gains while the interim loser attempts to increase her portfolio's idiosyncratic risk in an attempt to "hit a home run" in order to catch up. Note that in our setting the interim loser fund has an incentive to increase her portfolio's idiosyncratic risk, rather than the systematic risk of the portfolio. The intuition for this result is that the only way for the loser fund to catch up to (and possibly surpass) the winner fund is to hold a portfolio that is different from that held by the winner fund. Given the winner fund manager's incentive to hold an indexed position, the loser fund manager attempts to take on additional idiosyncratic risk. Our result that the interim loser fund manager has an incentive to increase idiosyncratic risk is different from earlier studies (e.g., Taylor (2003)) that focus primarily on the losing fund manager's incentive to affect the total volatility of her portfolio.

Proposition 1 allows us to reduce each manager's portfolio decision problem a choice involving a single variable, namely her allocation to the market index (i.e., α_1^g or α_1^b), because in the equilibrium $\alpha_2^g = 0$ and $\alpha_2^b = 1 - \alpha_1^b$. The following lemma gives further description of the equilibrium in terms of the optimal portfolio allocations, α^{g^*}, and α^{b^*}.

LEMMA 1: *In equilibrium either* $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, *or* $\alpha^{b^*} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, *or both.*

Proof: See Appendix B.

We focus on the equilibrium with $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Later, we provide a sufficient condition for this to be the case. We make the following assumption that

$$(1+m_b)(\mu - R_f) \geq \lambda n \left(\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} . \tag{6}$$

The left hand side of the above inequality is the expected rate of increase of manager *b's* assets if the manager chooses to increase her allocation to the market index portfolio, and the right hand side represents the additional reward the manager expects to get for having a larger chance of winning the tournament by lowering the allocation to the market index. Intuitively, the above assumption states that the tournament effect is not so overwhelming that manager *b* would want to go to the extreme of allocating 100% of her portfolio to the idiosyncratic risky asset in order to win the tournament. Inequality (6) is equivalent to

$$\frac{\partial}{\partial \alpha_1^b} U_b \left(\alpha^g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^b = \begin{bmatrix} \alpha_1^b \\ 1-\alpha_1^b \end{bmatrix} \right) \Bigg|_{\alpha_1^b = 0^+} = (1+m_l)(\mu - R_f) - \lambda n \left(\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} > 0 .$$

Thus, the assumption represented by inequality (6) ensures that the interim loser fund has a non-zero allocation to the market index (i.e., $\alpha_1^{b^*} > 0$) in the equilibrium.

It is worthwhile to have a closer look at the first order optimality condition for the portfolio allocation of manager *b* (See Appendix C for the detailed discussion). In general, the first order condition is equivalent to

$$h(x) \equiv \ln \left(\frac{(1+m_l)(\mu - R_f)\sqrt{\sigma_i^2 + \sigma_m^2}\sqrt{2\pi}}{\lambda m_\delta} \right) + \frac{1}{2} \left(\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} x + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 - 2 \ln x = 0 \tag{7}$$

where $x = \frac{1}{1-\alpha_1^b}$. Function $h(x)$ is strictly convex, and thus has at most two roots. In the case that $h(x)$ has only one root or no root in the interval $[1,\infty)$, U_b will be a strictly increasing function with respect to the variable α_1^b. We can then conclude that, in this case, $\alpha^{b^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ will be the manager *b*'s optimal response. We thus have the following statement regarding the existence of equilibrium.

PROPOSITION 2: *In the case that* $\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} \geq C_1$, *the pair* $\left\{ \alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^{b^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ *constitutes a Nash equilibrium where* $C_1 \propto \lambda$.

(See Appendix C for the definition of C_1*)*

Proof: See Appendix C.

Proposition 2 states that if the interim performance gap, m_δ, is larger than $C_1\sqrt{\sigma_i^2 + \sigma_m^2}$, it will be optimal for manager *b* choose strategy $\alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. As a result, her chance of winning the tournament is exactly zero. Essentially this means that manager *b* withdraws from the tournament. The intuition is as follows. When the performance gap is large, the chance for manager *b* to make up such a large gap before the end of the game is so slim that she simply gives up and decides instead to focus on the reward from the linear contract. Moreover, the bound on the size of the performance gap is proportional to λ, the reward from winning the tournament in the form of the end-of-period cash inflow. Intuitively, the higher the reward to the winner of the tournament, the stronger the desire for the manager *b* to stay on in the tournament. She might, therefore, still consider participating in the tournament even when she is lagging behind significantly at the interim stage.

In the case that function $h(x)$, defined by identity (7), has two roots in the interval $[1,\infty)$, we denote the roots by x_1 and x_2 with $1 < x_1 < x_2$. We have that $h(x) > 0$ for $x \in [1, x_1) \cup (x_2, \infty)$, and that $h(x) < 0$

for $x \in (x_1, x_2)$. Thus U_l is increasing for $\alpha_1^b \in [0, 1-1/x_1) \cup (1-1/x_2, \infty)$, and decreasing for $\alpha_1^b \in [1-1/x_1, 1-1/x_2]$. Therefore,

$$\arg\max_{\alpha_1^b} U_b\left(\alpha^g = \begin{bmatrix}1\\0\end{bmatrix}, \alpha^b = \begin{bmatrix}\alpha_1^b\\1-\alpha_1^b\end{bmatrix}\right) = 1 \text{ or } 1 - \frac{1}{x_1}. \tag{8}$$

A more detailed version of above discussion regarding $h(x)$ and its relation with the first order optimality condition for the portfolio allocation of manager *b,* with some properties of the root x_1, is provided in Appendix C. With those properties, we can attain the following result regarding the equilibrium.

PROPOSITION 3: *In case that* $\alpha^{g^*} = \begin{bmatrix}1\\0\end{bmatrix}$, *and* $\alpha_1^{b^*} < 1$, *we have* $\frac{\partial}{\partial \lambda}(1-\alpha_1^{b^*}) > 0$, *and* $\frac{\partial}{\partial m_\delta} N\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\frac{1}{1-\alpha_1^{b^*}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right) > 0$.

Proof: See appendix D.

Proposition 3 implies that when $\alpha^{b*} = \begin{bmatrix}\alpha_1^{b^*}\\1-\alpha_1^{b^*}\end{bmatrix}$ is manager *b's* optimal response with $\alpha_1^{b^*} < 1$, the higher the stakes in the tournament (i.e. the larger value of λ), the greater the weight on the idiosyncratic risky asset in manager *b*'s portfolio. This proposition also points out that in the equilibrium, everything else equal, the larger the lead enjoyed by manager *g* at the interim stage of the game (date *t = 1*), the greater the probability of manager *g* finishing the game as the ultimate winner, after taking into consideration the losing manager's incentive of taking on more idiosyncratic risk to reduce this probability.

So far, most of our discussion of the equilibrium is based on the assumption that it exists. The following theorem establishes the existence of the equilibrium under the condition that the interim performance gap (i.e. m_δ) between the two managers is not too big.

THEOREM 1: *There is a positive value* C *that depends on other parameters except* m_δ*, such that if* $m_\delta < C\sqrt{\sigma_i^2+\sigma_m^2}$*, the pair* $\left\{\alpha^{g^*}=\begin{bmatrix}1\\0\end{bmatrix}, \alpha^{b^*}=\begin{bmatrix}\alpha_1^{b^*}\\1-\alpha_1^{b^*}\end{bmatrix}\right\}$ *will be an equilibrium, where* $\alpha_1^{b^*}=1-\frac{1}{x_1}$*. Furthermore,* $0<\alpha_1^{b^*}<1$*,* $\frac{\partial(1-\alpha_1^{b^*})}{\partial m_\delta}>0$ *and* $\lim_{m_\delta\to 0^+}(1-\alpha_1^{b^*})=0$*. To be exact, we have that* $1-\alpha_1^{b^*}=O(\sqrt{m_\delta})$.

Proof: See Appendix D for a more precise version of the theorem and the proof. Also notice that under the condition of the theorem, the claims made in proposition 1 and 3 are true.

COROLLARY 1: *The Sharpe Ratio of the fund b is lower than that of fund g, and it is decreasing with respect to* m_δ *and* λ .

Proof: See Appendix D.

In additional to assuring the existence of the equilibrium, this theorem also points out that manager *b'*s portfolio holding of the idiosyncratic risky asset is an increasing function of the performance gap, m_δ. That is, the larger the gap, the more the aggressive is manager *b's* portfolio allocation in respect of the magnitude of idiosyncratic risk in her portfolio. When the performance gap approaches zero, manager *b's* portfolio holding of idiosyncratic risk also approaches zero. Furthermore, the corollary to the theorem shows that the interim loser fund has a lower Sharpe ratio compared to the interim winner fund. As we show later, in the context of the multi-period version of our model, it is optimal for short-term investors to invest in the winner fund. Below we discuss some issues relating to the existence of the pure strategy equilibrium identified by us.

C. Discussion

Theorem 1 places a restriction on the magnitude of the performance gap, m_δ, between the two managers, namely, $m_\delta < C\sqrt{\sigma_i^2+\sigma_m^2}$. As pointed out in proposition 2, when the gap is too large it will be optimal for the losing manager to choose $\alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as her optimal strategy. In that case, the pair $\left\{ \alpha^{w^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^{l^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ will be an equilibrium. Therefore, the above condition which places an upper bound on the performance gap, is by no means necessary for the existence of equilibrium. The true condition for the existence of equilibrium is certainly less restrictive than the condition stated in the theorem. Having said that, we note that the equilibrium may not exist unconditionally. To see this, we take an extreme, unrealistic case as an example. Let's assume that $\mu - R_f = 0$ and $\sigma_i^{\ 2} = 0$. Under the condition that $\mu - R_f = 0$, all feasible strategies for both managers provide the same expected return. The condition that $\sigma_i^{\ 2} = 0$ reduce the model to the case where the only investment opportunity for both managers consists of a risk-free asset and the market index , as in the model examined by Taylor(2003). Now, the probability that manager *g* wins the tournament reduces to $N\left[\frac{m_\delta}{\left| \alpha_1^g - \alpha_1^b \right| \sigma_m} \right]$. The optimization problem for manager *g* is to choose α_1^g to maximize such probability, and that for manager *b* is to choose α_1^b to minimize it. For any α_1^b, manager *g* wants to set $\alpha_1^g = \alpha_1^b$. On the other hand, the losing manager wants to set α_1^b to be 1 or 0, depending on whether α_1^g or $(1-\alpha_1^g)$ is larger. Therefore, there is no pure strategy equilibrium under the condition that $\mu - R_f = 0$ and $\sigma_i^{\ 2} = 0$.

Allowing managers to have access to idiosyncratic risk, i.e. $\sigma_i^{\ 2} > 0$, helps ensure the existence of the equilibrium. Let us take for example that $\sigma_i^{\ 2} > \sigma_m^{\ 2}$, and still keep the assumption that $\mu - R_f = 0$. We have that the probability that manager *g* wins the tournament equals to $N\left[\frac{m_\delta}{\sqrt{(\alpha_2^{g\,2} + \alpha_2^{b\,2})\sigma_i^{\ 2} + (\alpha_1^g - \alpha_1^b)^2 \sigma_m^{\ 2}}} \right]$. It is then not difficult to see that the pair $\left\{ \alpha^{g^*} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \alpha^{b^*} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ forms a Nash equilibrium.

Taylor (2003) faces the problem of nonexistence of an equilibrium, which is particularly severe due to the fact that $\sigma_i^2 = 0$ in his model, as we have seen from the above two examples. He coped with the problem by focusing on the mixed strategy equilibrium. Even with the mixed strategy equilibrium, one still has to impose some ad hoc restrictions on the managers' feasible strategies in order to apply some version of the fixed point theorem to ensure the existence of equilibrium. Taylor makes the assumption that both managers are only allowed to mix the two extreme pure strategies, that is, either to invest all assets in the market index or to invest all assets in the risk free asset. Under this condition, Taylor is able to derive the equilibrium. Even with the existence of equilibrium assured, there is a potential problem of the mixed strategy equilibrium in this case, i.e., its stability. If we start with the assumption that the manager *g* chooses on her part the mixed strategy as specified in the equilibrium, then manager *b* will be indifferent to all of her feasible strategies. This is essentially the condition of the mixed strategy equilibrium. Such a condition makes the chance slim that manager *b* will happen to choose on her part the mixed strategy as specified in the equilibrium. The same is true when we reverse the role of the two managers in the above argument.[9] The pure strategy equilibrium stated in Theorem 1 does not suffer from this problem, because for each manager, it is optimal for her to take only the strategy stated in the equilibrium given that her counterpart does so.

II. Multi-Period Model

We now extend our basic model to a multi-period setting in which the incentives of mutual fund shareholders, as well as the incentives of managers, are explicitly analyzed. The multi-period setting allows us to examine the reasons for the existence of a tournament effect.

A. Model Structure

Consider two risk neutral managers who manage a pool of assets for three periods. Let the three periods be indexed by $s = 1, 2, 3$, and they correspond to the time intervals $t \in [0,1)$, $t \in [1,2)$, and $t \in [2,3]$

[9] The interpretation that each manager is a representative of a population of the same type of managers will not resolve this problem.

respectively. At the beginning of the first period, there is a continuum of investors, indexed by $j \in [0,2]$.[10] The investment horizon of these investors, who may be viewed as passive investors, is three periods. The passive investors pick a fund to invest in at the beginning of the first period. They then stay with the fund throughout the three periods, and take the payoff at the end of the third period.[11] At the beginning of the second and the third periods, new investors with a single period investment horizon, and with measure λ for each period, choose to invest in one of the funds.[12] These investors, who may be viewed as active investors, select one of the two funds at the beginning of their investing period and take the payoff at the end of that period. They are indexed by $j \in (2, 2+\lambda]$ for investors participating in period 2, and $j \in [3, 3+\lambda]$ for investors participating in period 3. We further assume that investors have log utility, i.e., $U_p(C) = \log(C)$.[13] At the beginning of period two and period three (i.e., $t = 1$ and $t = 2$), both managers' performances to date are publicly observed by all the players in the game, i.e., all investors and both managers. Each player in the game makes strategic decisions based on the information available to date to maximize her expected utility.

A.1. Distribution of payoffs among managers and investors

At the end of each period, managers and investors are rewarded according to the following scheme. As in the one-period model, managers are paid by a fixed percentage k of the assets currently under their management. The residual assets of each fund are then distributed evenly among all the investors who invest in the fund at the beginning of the period. In the case of a long-term, passive investor, her portion of the residual assets at the end of period one or period two, are invested in the fund for the coming period on her behalf.

[10] We choose to normalize the measure of these investors to be 2 instead of 1 so that each fund will have one unit of asset under management at the beginning of the game.

[11] We can either assume that there is a sufficiently high cost to switch fund during the game or at least there are some investors who don't switch for other reasons. Under either interpretation, such an assumption is consistent with empirical findings.

[12] It will not change any of our results if we assume that some short term investors also invest in the funds at the beginning of the first period.

A.2. Investment opportunity set and asset returns

At the beginning of period s ($s = 1,2,3$), manager i ($i=1,2$) faces the investment opportunity set that consists of a riskfree asset with return $R_f > 0$, the market index portfolio with return $R_{m,s}$, and an idiosyncratic risky portfolio with return $R_{i,s}$. There are no changes in the investment opportunity set across periods. We assume that $R_{m,s}$ and $R_{i,s}$ are mutually independent lognormal random variables. More specifically, we assume that throughout the whole duration of the game, the four assets evolve as follows:

$$dR_{f,t} = R_{f,t} \cdot r_f dt;$$

$$dR_{m,t} = R_{f,t} \cdot \left(\mu_m dt + \sigma_m dB_m\right);$$

$$dR_{i,t} = R_{i,t} \cdot \left(r_f dt + \sigma_i dB_i\right) \quad \text{for } i = 1,2,$$

where $B_{m,t}$, $B_{1,t}$ and $B_{2,t}$ represent three independent standard Brownian motion processes. Notice that the drift term of $dR_{i,t}$ is the same as the instantaneous risk free rate. That is, there is no risk premium for holding the idiosyncratic risky asset. We assume that $\mu_m - r_f \geq \sigma_m{}^2$, and $\sigma_1 = \sigma_2$.[14] Each manager at the beginning of each period decides her instantaneous holding on each of the three assets that are available for her. Manager i's decision for period s is formalized by $\begin{bmatrix} \alpha_{1,s}^i \\ \alpha_{2,s}^i \end{bmatrix} \geq 0$, $\alpha_{1,s}^i + \alpha_{2,s}^i \leq 1$, where $\alpha_{1,s}^i$ is the portion of the total asset she decides to invest in the market index portfolio, $\alpha_{2,s}^i$ is the portion in the idiosyncratic risky portfolio, and the rest, $(1 - \alpha_{1,s}^i - \alpha_{2,s}^i)$, in the risk free asset. The manager then holds such weights constant throughout each period, and only adjusts the weights at the beginning of the next period. Manager i's wealth process in period s is given by

$$\begin{aligned} \frac{dW_t^i}{W_t^i} &= r_f dt + \alpha_{1,s}^i \left(\frac{dR_{m,t}}{R_{m,t}} - r_f dt \right) + \alpha_{2,s}^i \left(\frac{dR_{i,t}}{R_{i,t}} - r_f dt \right) \\ &= (r_f + \alpha_{1,1}^i (\mu_m - r_f))dt + \alpha_{1,s}^i \sigma_m dB_{m,t} + \alpha_{2,s}^i \sigma_i dB_{i,t}, \quad \text{for } t \in [s-1, s) \end{aligned} \tag{9}$$

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[13] While the assumption of log utility is made for reasons of tractability, our main results are not dependent on such an assumption.

[14] Since investors have log utility, this assumption is satisfied in the equilibrium of a CAPM model.

What differentiates the above expression from the typical wealth process often seen in continuous time portfolio decision problems is that the instantaneous portfolio weights, $\alpha_{j,s}^{i}$ $(i=1,2;\ j=1,2)$, are piecewise constant. Such an assumption demands some explanation. First, we make this assumption because we abstract away private information and other reasons for managers to change their instantaneous holdings. Our earlier exploration of the one period model follows a more traditional method to abstract away the reason for managers to trade actively, namely, to assume that managers follow buy and hold strategies. The buy and hold strategies work well with the assumption of normality of asset returns. However, normality doesn't serve us well in the multi-period model due to the fact that it can lead to negative outcomes. In the multi-period model, a negative return for the fund could result in a negative asset base for the manager to start the next period, which is unacceptable for the model. With normality substituted by log-normality, the buy and hold strategies are not as tractable. The assumption we made above regarding managers' investment opportunity set allows us to avoid the unacceptable properties of possible negative returns and at the same time keep the tractability. Moreover, our assumption does not seem to be any less natural than the traditional buy and hold modeling assumption, given the fact that in reality managers do balance their portfolio holdings from time to time.

In our model, neither manager possesses superior information. To introduce the random event of one manager outperforming the other in the first period, we assume that there are mean zero noise variables c_i $(i=1,2)$ subject to random shocks, that affect the performance of manager i in period 1.[15] The wealth process of manager i for $t \in [0,1)$ is modified to be

$$\begin{aligned}\frac{dW_t^i}{W_t^i} &= (r_f - c_i)dt + \alpha_{1,1}^i\left(\frac{dR_{m,t}}{R_{m,t}} - r_f dt\right) + \alpha_{2,1}^i\left(\frac{dR_{i,t}}{R_{i,t}} - r_f dt\right) \\ &= \left((r_f - c_i) + \alpha_{1,1}^i(\mu_m - r_f)\right)dt + \alpha_{1,1}^i\sigma_m dB_{m,t} + \alpha_{2,1}^i\sigma_i dB_{i,t}\end{aligned} \qquad (10)$$

We assume that c_i (*i=1,2*) are random variables with independent identical continuous distributions of compact support $[-c,c]$. We assume that the magnitude of c is not large so that at the end of the first period, the manager that happens to have faced the larger cost is still well in the game. Loosely speaking, this means that the manager who is trailing at the end of the first period will have a reasonable chance of winning the cash inflow at the end of the second period by adjusting her portfolio strategies.

B. Analysis of the game

Since managers are paid a fixed percentage of the assets under their management at dates $t = 1, 2, \text{and } 3$, each manager's objective is to enlarge the size of her fund's assets in order to earn higher compensation. In addition to the absolute growth of the assets through investments, the funds' asset can also be enlarged by attracting new investors into the fund. Therefore, when a manager makes portfolio decisions, she has to take into consideration the investors' behavior along with the optimal strategy of the competing manager. We consider a possible equilibrium in which active investors chase the fund with the superior to-date performance. We specify the investors' strategies as follows:

(J1) At the beginning of the first period, investors $j \in [0,2]$ are indifferent between the two managers. One half of the investors invest with one manager and the other half invest with the other manager.

(J2) At the beginning of the second period, investors $j \in (2, 2+\lambda]$ will all invest in the fund which performed better in the first period (performance chasing).

(J3) At the beginning of the third period, investors $j \in [3, 3+\lambda]$ will all invest in the fund whose average performance in the first two periods is better.

It is clear that the fund with higher cumulative return in the first period should be viewed as the to-date performance winner fund at date $t = 1$. We index, ex post, this performance winner fund by *g* (good), and the other fund by *b* (bad). A manager is identified with the fund that she manages. At the beginning of the third period (i.e., $t = 2$), the fund with the higher average instantaneous returns of the first two periods is regarded by the investors as the to-date performance winner fund at date $t = 2$. This is equivalent to saying that those investors who, by chance, invest at date $t = 0$ in this to-date performance winner fund

[15] The random shocks c_i may be viewed as the average instantaneous operational costs faced by fund *i* in period 1.

(at date $t=2$) will enjoy higher ex post growth of their investment in the first two periods than the investors who invest in the competing fund.

We define $F^i = \ln W^i_{1^-}$, $S^i = \ln W^i_{2^-} - \ln W^i_1$, $T^i = \ln W^i_3 - \ln W^i_2$, where $i = g$ or b. That is, F^i, S^i and T^i represent fund i's average instantaneous return within first, second, and third period respectively. We note that, by Ito's lemma, we can express F^i, S^i and T^i as follows:

$$F^i = (r_f - c_i) + \alpha^i_{1,1}(\mu_m - r_f) - \frac{1}{2}(\alpha^i_{1,1}\sigma_m)^2 - \frac{1}{2}(\alpha^i_{2,1}\sigma_i)^2 + \alpha^i_{1,1}\sigma_m B_{m,1} + \alpha^i_{2,1}\sigma_i B_{i,1}; \tag{11}$$

$$S^i = r_f + \alpha^i_{1,2}(\mu_m - r_f) - \frac{1}{2}(\alpha^i_{1,2}\sigma_m)^2 - \frac{1}{2}(\alpha^i_{2,2}\sigma_i)^2 + \alpha^i_{1,2}\sigma_m (B_{m,2} - B_{m,1}) + \alpha^i_{2,2}\sigma_i (B_{i,2} - B_{i,1}); \tag{12}$$

$$T^i = r_f + \alpha^i_{1,3}(\mu_m - r_f) - \frac{1}{2}(\alpha^i_{1,3}\sigma_m)^2 - \frac{1}{2}(\alpha^i_{2,3}\sigma_i)^2 + \alpha^i_{1,3}\sigma_m (B_{m,3} - B_{m,2}) + \alpha^i_{2,3}\sigma_i (B_{i,3} - B_{i,2}). \tag{13}$$

We further denote the first period performance difference by F^δ, namely, $F^\delta \equiv F^g - F^b > 0$. We restate the above criterion of the to-date performance winner at date $t=2$. Fund g is the to-date performance winner at date $t=2$ if and only if $F^g + S^r > F^b + S^b$, where we ignore the probability zero event that the two sides shall be equal.

Panel A of Table 1 contains a schedule of payoffs to the managers at the end of each period. Panel B specifies the returns to investors in each period. Below, we first study managers' optimal portfolio strategies for each period, taking as given investors' investment decisions specified in (J1) – (J3). We verify the optimality of such investors' decisions afterwards.

B.1. Managers' portfolio strategies

A fund's asset follows a process that has jumps at dates $t=1$ and $t=2$ due to manager's compensation, investors' withdrawing, and potentially new investors' coming in. The processes of the funds' assets have the following boundary conditions:

$W^i_0 = 1$, for $i = g$ or b;

$W^g_1 = (1-k)W^g_{1^-} + \lambda$, and $W^b_1 = (1-k)W^b_{1^-}$;

$W^g_2 = (1-k)W^g_{2^-} + \lambda$ and $W^b_2 = (1-k)W^b_{2^-}$, if $S^b - S^g < F^\delta$,

$$W_2^g = (1-k)W_{2^-}^g \text{ and } W_2^b = (1-k)W_{2^-}^b + \lambda, \quad \text{otherwise.}$$

These boundary conditions with equations (9) and (10) fully specify the processes of the funds' assets throughout three periods.

We solve the managers' optimization problems through backward induction. The following proposition characterizes managers' strategies in the third period.

PROPOSITION 4: *In equilibrium, both managers hold the market index portfolio in the third period. That is,* ${\alpha_{1,3}^i}^* = 1$ *for* $i = 1,2$. *Moreover,* $E_2[\exp(T^i)] = \exp(\mu_m)$.[16]

Proof: Since the game ends at date $t = 3$, there is no tournament effect in this period. Each manager shall maximize her fund's last period expected return, which is: $E[\exp(T^i)] = \exp[r_f + \alpha_{1,3}^i(\mu_m - r_f)]$. Therefore, ${\alpha_{1,3}^i}^* = 1$.

Proposition 4 states that in the last period of the game, both managers will take the same portfolio strategy, namely hold the market index. We next go on to study managers' decision at the beginning of the second period.

At the beginning of the second period (date $t = 1$), manager *b* is burdened by a performance disadvantage and needs to make up the performance deficit (F^δ) by the end of the second period in order to attract additional cash flow at date $t = 2$. Manager *g* wins the third period cash flow if and only if

$$\begin{aligned} &F^\delta - (\alpha_{1,2}^b - \alpha_{1,2}^g)(\mu_m - r_f) + \frac{1}{2}({\alpha_{1,2}^b}^2 - {\alpha_{1,2}^g}^2){\sigma_m}^2 + \frac{1}{2}({\alpha_{2,2}^b}^2 - {\alpha_{2,2}^g}^2){\sigma_i}^2 \\ &> (\alpha_{1,2}^b - \alpha_{1,2}^g)\sigma_m(B_{m,2} - B_{m,1}) + \alpha_{2,2}^b\sigma_i(B_{b,2} - B_{b,1}) - \alpha_{2,2}^g\sigma_i(B_{g,2} - B_{g,1}) \end{aligned}$$

Notice that the right hand side of the above inequality is a random normal variable with distribution $N\left(0, (\alpha_{1,2}^b - \alpha_{1,2}^g)^2{\sigma_m}^2 + ({\alpha_{2,2}^b}^2 + {\alpha_{2,2}^g}^2){\sigma_i}^2\right)$. Therefore,

[16] E_t denotes the expectation conditional on the information available at date t.

$$P\left[S^b - S^g < F^\delta\right] = N\left(\frac{F^\delta - (\alpha_{1,2}^b - \alpha_{1,2}^g)(\mu_m - r_f) + \frac{1}{2}({\alpha_{1,2}^b}^2 - {\alpha_{1,2}^g}^2){\sigma_m}^2 + \frac{1}{2}({\alpha_{2,2}^b}^2 - {\alpha_{2,2}^g}^2){\sigma_i}^2}{\sqrt{(\alpha_{1,2}^b - \alpha_{1,2}^g)^2 {\sigma_m}^2 + ({\alpha_{2,2}^b}^2 + {\alpha_{2,2}^g}^2){\sigma_i}^2}}\right). \quad (14)$$

The above expression is the probability of manager *g* becoming the to-date performance winner at date $t = 2$. Comparing equation (14) with equation (3), we can see that the multi-period model preserves the tractability that we have entertained in the one period model. With expression (14) and Proposition 4, we can now express manager g's objective function at date $t = 1$ as:

$$k\left(\exp(F^\delta)W_1^b + \lambda\right)\left(1 + (1-k)\exp(\mu_m)\right)\exp\left[r_f + \alpha_{1,2}^g(\mu_m - r_f)\right] + k\lambda \exp(\mu_m) P\left[S^b - S^g < F^\delta\right]. \quad (15)$$

Similarly, manager b's objective function is[17]

$$kW_1^b\left(1 + (1-k)\exp(\mu_m)\right)\exp\left[r_f + \alpha_{1,2}^b(\mu_m - r_f)\right] + k\lambda \exp(\mu_m)\left(1 - P\left[S^b - S^g < F^\delta\right]\right). \quad (16)$$

At the beginning of the second period (date $t = 1$), manager *g* chooses her portfolio holdings to maximize expression (15), and manager *b* maximizes expression (16). In equilibrium, we have the following result.

PROPOSITION 5: *In equilibrium,* ${\alpha_{1,2}^g}^* = 1 > {\alpha_{1,2}^b}^*$, ${\alpha_{1,2}^b}^* + {\alpha_{2,2}^b}^* = 1$, $\frac{\partial {\alpha_{2,2}^b}^*}{\partial \lambda} > 0$, $\frac{\partial {\alpha_{2,2}^b}^*}{\partial F^\delta} > 0$, $\lim_{F^\delta \to 0^+} {\alpha_{2,2}^b}^* = 0$, *and* $\frac{\partial}{\partial F^\delta} P\left[S^b - S^g < F^\delta\right] < 0$. *More precisely, we have* ${\alpha_{2,2}^b}^* = O\left(\sqrt{F^\delta}\right)$. *Furthermore,* $E_1[\exp(S^g)] = \exp(\mu_m)$ *and* $E_1[\exp(S^b)] < \exp(\mu_m)$.

Proof: The proof of this proposition is similar to that of propositions 1, and 3, and Theorem 1 in the one-period model. Furthermore, the existence of the equilibrium is assured here. In addition, because ${\alpha_{1,2}^g}^* = 1 > {\alpha_{1,2}^b}^*$, we have $E_1[\exp(S^g)] = e^{\mu_m}$, and $E_1[\exp(S^b)] < e^{\mu_m}$.

Proposition 5 establishes that, in equilibrium, manager *g*'s portfolio for the second period will not involve any exposure to idiosyncratic risk, and manager *b* will not invest in the risk free asset. Intuitively, the manager who is ahead at the end of the first period (date *t = 1*) would like

[17] By comparing the expected rewards of both managers, one can see that in the setting of the multi-period model, the manager that performed worse in the first period will have a stronger incentive to win the second period tournament. This is because the losing manager's reward has a lower weight on the payment from the contract.

to "lock in" her gains while the manager who is trailing attempts to hold a portfolio that is different from that held by the winner fund in order to catch up. Given the winner fund manager's incentive to hold an indexed position, the loser fund manager attempts to take on additional idiosyncratic risk.

Proposition 5 implies that the higher the stakes in the tournament (i.e. the larger value of λ), the greater the weight on the idiosyncratic risky asset in manager *b*'s portfolio. In addition, the larger the lead enjoyed by manager *g* at the end of the first period (date *t* = *1*), the greater the probability of manager *g* becoming the to-date performance winner at date $t = 2$. Manager *b's* portfolio holding of the idiosyncratic risk asset in the second period (i.e., $\alpha_{2,2}^{b\,*}$) is an increasing function of the performance gap, m_δ. That is, the larger the gap, the more the aggressive is manager *b's* portfolio allocation in respect of the magnitude of idiosyncratic risk in her portfolio. When the performance gap approaches zero, manager *b's* portfolio holding of idiosyncratic risk also approaches zero.

The managers' portfolio strategies in the first period are described in the following proposition.

PROPOSITION 6: *Both managers will hold the market index portfolio in the first period. That is,* $\alpha_{1,1}^{i\,*} = 1$ *for* $i = 1,2$. *Ex ante, each manager will become the to-date performance winner manager at the end of the first period (date* $t = 1$*) with equal probability, namely probability ½.*

Proof: See Appendix E.

Unlike the case for manager *b* in period 2, holding idiosyncratic risky assets at date $t = 0$ will not enhance a manager's chance to win against the other manager at date $t = 1$, and tends to hurt the fund's absolute performance. Therefore, it is only optimal for both managers to hold the market index for period 1. As a result, both funds provide the same expected return for the first period and also have the same chance (i.e., each with probability one half) of ending the first period with the superior performance to date.

Having solved managers' portfolio decision problems under the specification of investors investment decisions given by (J1) – (J3), we now go on to verify the optimality of investors' investment decisions.

B.2. Investors' investment decision

Due to symmetry of the setting, investors $j \in [0,2]$ will be indifferent between investing in either of the two funds when they are making the investment decisions at the beginning of the first period. This statement is further corroborated by Proposition 6. At the start of the third period, because both managers are following the same strategy as stated in proposition 4, the investors $j \in [3, 3+\lambda]$ are again indifferent between investing in either of the two funds. Therefore, it is rational for them to choose the to-date performance winner fund to invest in.[18] In summary, we have the following proposition.

PROPOSITION 7: *Investors $j \in [0,2]$ at date $t = 0$, and investors $j \in [3, 3+\lambda]$ at date $t = 2$ are indifferent between investing in either of the two funds. It is therefore optimal for investors $j \in [0,2]$ to invest in any one of the two funds at date $t = 0$, and also optimal for investors $j \in [3, 3+\lambda]$ to invest in the to-date performance winner fund at date $t = 2$.*

We formally state and justify the optimality of the strategy for investors $j \in (2, 2+\lambda]$ in the following proposition.

PROPOSITION 8: *It is only optimal for investors $j \in (2, 2+\lambda]$ to invest with manager g.*

Proof: For investors who come in at the beginning of the second period (date $t = 1$), their expected utility of investing with manager i is $\ln(1-k) + E_1[S^i]$. By Proposition 5, we have $\alpha_{1,2}^{g}{}^{*} = 1 > \alpha_{1,2}^{b}{}^{*}$. Therefore, under the assumption $\mu_m - r_f \geq \sigma_m{}^2$,

$$E_1[S^b] = r_f + \alpha_{1,2}^b(\mu_m - r_f) - \frac{1}{2}(\alpha_{1,2}^b \sigma_m)^2 - \frac{1}{2}(\alpha_{2,2}^b \sigma_i)^2 < r_f + (\mu_m - r_f) - \frac{1}{2}\sigma_m{}^2 = E_1[S^g].$$

Thus the proposition is proved.

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[18] Another possible way to reconcile this fact is a model within which the game stops at a random stopping time. Then, at the beginning of every period, faced with the possibility that the game will go on into the next period, the manager currently behind will have the incentive to take on more idiosyncratic risk, which in turn makes it rational for investors to chase performance.

Propositions 7 and 8 together justify the optimality of all investors' investment decisions that are stated in (J1) – (J3). Therefore, the set of strategies for investors and for managers forms a Nash equilibrium. In this Nash equilibrium, we have the following result regarding the welfare of the passive investors ($j \in [0,2]$)

PROPOSITION 9: *The ex ante utility of the passive investors is a decreasing function of λ.*

Proof: For passive investors ($j \in [0,2]$), one dollar invested with manager *i* at date $t=0$ will grow to $(1-k)^3 \exp[F^i + S^i + T^i]$ at date $t=3$. By Proposition 6, the long term investors will have probability equal to ½ of investing with ex post manager *g* and an equal probability of investing with manager *b*. Therefore, the expected utility for the long term investors is given by

$$U \equiv 3\ln(1-k) + 1/2 E_0[F^g + S^g + T^g] + 1/2 E_0[F^b + S^b + T^b]$$

Substituting the portfolio allocations given by Proposition 4, 5, 6, we have

$$U \equiv 3\ln(1-k) + \frac{5}{2}(\mu_m - \frac{1}{2}\sigma_m{}^2) + \frac{1}{2}r_f + \frac{1}{2}E_0\left[\alpha_{1,2}^{b\ *}(\mu_m - r_f) - \frac{1}{2}(\alpha_{1,2}^{b\ *}\sigma_m)^2 - \frac{1}{2}((1-\alpha_{1,2}^{b\ *})\sigma_i)^2\right].$$

By Proposition 5, $\frac{\partial \alpha_{1,2}^{b\ *}}{\partial \lambda} < 0$, and therefore,

$$\frac{\partial U}{\partial \lambda} = \frac{1}{2}E_0\left[\left((\mu_m - r_f) - \alpha_{1,2}^{b\ *}\sigma_m{}^2 + (1-\alpha_{1,2}^{b\ *})\sigma_i{}^2\right)\frac{\partial \alpha_{1,2}^{b\ *}}{\partial \lambda}\right] < 0,$$

because $(\mu_m - r_f) - \alpha_{1,2}^{b\ *}\sigma_m{}^2 + (1-\alpha_{1,2}^{b\ *})\sigma_i{}^2 > (\mu_m - r_f) - \sigma_m{}^2 \geq 0$.

Proposition 9 hints at the potential costs imposed on passive investors by the behavior of the active investors. Interestingly, our analysis highlights a potential utility loss to long-term investors that is purely on account of the perverse managerial incentives created by the active trading engaged in by short-term investors.[19] In future work we intend to examine the welfare consequences of the short-term investors' actions in a richer setting.

[19] Previous studies of the impact of short-term cash flows on fund performance have focused on the liquidity costs imposed by such behavior (see, for example, Edelen (1999)) or its impact on a fund's betas (Ferson and Warther (1996)).

III. Conclusions and Implications

In this paper we provide a potential explanation for the tendency of mutual fund investors to chase past performance despite the absence of evidence in favor of performance persistence. Our analysis builds on the observation that the mutual fund industry resembles a tournament in which players are competing with each other for investor cash flows. We show that, in light of the managerial incentives generated by the tournament nature of the industry, performance chasing is optimal, even if fund managers do not possess superior ability and there is no persistence in fund performance. Additionally, our study contributes to the literature on mutual fund tournaments by providing a rationale for the existence of a tournament effect in the fund industry.

We analyze a model in which two risk-neutral fund managers with unequal performances at an interim stage, compete for investor cash flows. Neither manager possesses superior information. Our focus is on the portfolio choices made by the two managers after they observe their interim performances. We also analyze the optimal response of risk averse investors to the incentives facing the fund managers. In this context we consider two groups of investors, namely, long-term or passive investors, and short-term or active investors.

We show that there exists an equilibrium in which (a) it is optimal for the fund manager whose performance lags behind at the interim stage (i.e., the interim loser) to increase the *idiosyncratic* risk of her portfolio, and (b) risk averse investors anticipate the incentives facing losing fund managers and rationally chase winners. Our analysis affords a number of testable predictions. Specifically, our results suggest that (a) the increase in the idiosyncratic risk of the interim loser manager's portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect), and (b) the ex-ante utility of long-term fund investors is decreasing in the strength of the tournament effect.

While a full welfare analysis of the costs and benefits of the implicit incentives generated by the tournament effect is beyond the scope of the present paper, our results hint at several directions in which this research may be extended with potential implications for the optimal design of fund entry/exit policies, and fund organizational form (i.e., closed-end v/s open-end), among other design features. For example, it would be interesting to extend our framework to a richer model that encompasses the twin features of (a) costly information acquisition by fund managers, and (b) tournament incentives. Such a setting will allow us to more fully explore the relative benefits (e.g., the incentive to acquire information) and the costs (such as those identified here) of the tournament nature of the fund industry. We leave these tasks to future research.

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Appendix A

Proof of Proposition 1: We prove the proposition in 4 steps.

1. *For any losing manager's feasible strategy,* $\alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, *the winning manager's strategy* $\begin{bmatrix} \theta_1 \\ 0 \end{bmatrix}$ *strictly dominate any one of her feasible strategies* $\begin{bmatrix} \alpha_1^g \\ \alpha_2^g \end{bmatrix}$ *with* $\alpha_1^g < \theta_1$, *and any one of her feasible strategies* $\begin{bmatrix} \theta_1 \\ \alpha_2^g \end{bmatrix}$ *with* $\alpha_2^g > 0$.

For $\alpha_1^g \le \theta_1$, we have:

$$\frac{m_\delta + (\alpha_1^g - \theta_1)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + \theta_2^2)\sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}} \le \frac{m_\delta}{\sqrt{({\alpha_2^g}^2 + \theta_2^2)\sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}} \le \frac{m_\delta}{\theta_2 \sigma_i},$$

and therefore,

$$N\left(\frac{m_\delta + (\alpha_1^g - \theta_1)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + \theta_2^2)\sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}} \right) \le N\left(\frac{m_\delta}{\theta_2 \sigma_i} \right),$$

where the equality holds only if $\alpha_1^g = \theta_1$ and $\alpha_2^g = 0$. We thus have, for $\alpha_1^g \le \theta_1$,

$$\begin{aligned}
& U_g\left(\alpha^g = \begin{bmatrix} \alpha_1^g \\ \alpha_2^g \end{bmatrix}; \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) \\
&= (1+m_g)(1+R_f + \alpha_1^g(\mu - R_f)) + \lambda N\left(\frac{m_\delta + (\alpha_1^g - \theta_1)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + \theta_2^2)\sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}} \right) \\
&\le (1+m_g)(1+R_f + \alpha_1^g(\mu - R_f)) + \lambda N\left(\frac{m_\delta}{\theta_2 \sigma_i} \right) \\
&\le (1+m_g)(1+R_f + \theta_1(\mu - R_f)) + \lambda N\left(\frac{m_\delta}{\theta_2 \sigma_i} \right) \\
&= U_g\left(\alpha^g = \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix}; \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right),
\end{aligned}$$

where the equality holds only if $\alpha_1^g = \theta_1$ and $\alpha_2^g = 0$.

2. *For any losing manager's feasible strategy,* $\alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, *the winning manager's strategy* $\begin{bmatrix} \alpha_1^g \\ 0 \end{bmatrix}$, *if* $\alpha_1^g \ge \theta_1$, *strictly dominate any one of her feasible strategies* $\begin{bmatrix} \alpha_1^g \\ a_2^g \end{bmatrix}$ *with* $\alpha_2^g > 0$.

For $\alpha_1^g \ge \theta_1$, we have:

$$\frac{m_\delta + (\alpha_1^g - \theta_1)(\mu - R_f)}{\sqrt{({\alpha_2^g}^2 + \theta_2^2)\sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}} \le \frac{m_\delta + (\alpha_1^g - \theta_1)(\mu - R_f)}{\sqrt{\theta_2^2 \sigma_i^2 + (\alpha_1^g - \theta_1)^2 \sigma_m^2}},$$

and therefore,

$$N\left(\frac{m_\delta+(\alpha_1^g-\theta_1)(\mu-R_f)}{\sqrt{({\alpha_2^g}^2+\theta_2^2)\sigma_i^2+(\alpha_1^g-\theta_1)^2\sigma_m^2}}\right)\le N\left(\frac{m_\delta+(\alpha_1^g-\theta_1)(\mu-R_f)}{\sqrt{\theta_2^2\sigma_i^2+(\alpha_1^g-\theta_1)^2\sigma_m^2}}\right),$$

where the equality holds only if $\alpha_2^g=0$. We thus have, for $\alpha_1^g\ge\theta_1$,

$$U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ \alpha_2^g\end{bmatrix};\alpha^l=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right)\le U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ 0\end{bmatrix};\alpha^b=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right),$$

where the equality holds only if $\alpha_2^g=0$.

3. We have $\frac{\partial}{\partial\alpha_1^g}U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ \alpha_2^g\end{bmatrix};\alpha^l=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right)\Bigg|_{\alpha^g=\begin{bmatrix}\theta_1\\ 0\end{bmatrix}}>0\cdot$

$$U_w\left(\alpha^w=\begin{bmatrix}\alpha_1^w\\ 0\end{bmatrix};\alpha^l=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right)=(1+m_w)(1+R_f+\alpha_1^w(\mu-R_f))+\lambda N\left(\frac{m_\delta+(\alpha_1^w-\theta_1)(\mu-R_f)}{\sqrt{\theta_2^2\sigma_i^2+(\alpha_1^w-\theta_1)^2\sigma_m^2}}\right).$$

We then have

$$\frac{\partial}{\partial\alpha_1^g}U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ \alpha_2^g\end{bmatrix};\alpha^b=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right)\Bigg|_{\alpha^g=\begin{bmatrix}\theta_1\\ 0\end{bmatrix}}=(1+m_g)(\mu-R_f)+\lambda\frac{\mu-R_f}{\theta_2\sigma_i}n\left(\frac{m_\delta}{\theta_2\sigma_i}\right)>0$$

where $n(\cdot)$ stands for the standard normal density function.

4. *For any winning manager's feasible strategy,* $\begin{bmatrix}\alpha_1^g\\ a_2^g\end{bmatrix}$, *if* $\alpha_1^g\ge\alpha_1^b-\frac{m_\delta}{(\mu-R_f)}$ *(especially if* $\alpha_1^g\ge\alpha_1^b$*), then the losing manager's strategy* $\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}$ *strictly dominates any one of her feasible strategies* $\begin{bmatrix}\alpha_1^b\\ \alpha_2^b\end{bmatrix}$ *with* $\alpha_1^b+\alpha_2^b<1$.

If $\alpha_1^b+\alpha_2^b<1$, with short sale constraint, we have ${\alpha_2^b}^2<(1-\alpha_1^b)^2$, and therefore $\sqrt{({\alpha_2^g}^2+{\alpha_2^b}^2)\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}<\sqrt{({\alpha_2^g}^2+(1-\alpha_1^b)^2)\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}$ In the case that $\alpha_1^g\ge\alpha_1^b-\frac{m_\delta}{(\mu-R_f)}$, we have $\Delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)>0$, and therefore

$$\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{({\alpha_2^g}^2+{\alpha_2^b}^2)\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}>\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{({\alpha_2^g}^2+(1-\alpha_1^b)^2)\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}.$$

$$U_b\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ \alpha_2^g\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ \alpha_2^b\end{bmatrix}\right)$$

$$=(1+m_b)(1+R_f+\alpha_1^b(\mu-R_f))+\lambda-\lambda N\left(\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{(\alpha_2^{g\,2}+\alpha_2^{b\,2})\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}\right)$$

$$<(1+m_b)(1+R_f+\alpha_1^b(\mu-R_f))+\lambda-\lambda N\left(\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{(\alpha_2^{g\,2}+(1-\alpha_1^b)^2)\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}\right)$$

$$=U_b\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ \alpha_2^g\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}\right).$$

To finish the proof, we derive the claim of the proposition from the above four proved statements. From statement 1, we have that in the equilibrium $\alpha_1^g \geq \alpha_1^b$. From 2, we have $\alpha_2^g = 0$. Then from 3, we have that $\alpha_1^g \geq \alpha_1^b$ with the equation hold only if $\alpha_1^g = \alpha_1^b = 1$.From 4, we have that $\alpha_1^b + \alpha_2^b = 1$. Thus the proposition is proved.

Appendix B

Proof of Lemma 1: Given the assumption that the equilibrium exist, we can denote the equilibrium to be $\left\{\begin{bmatrix}\alpha_1^{g^*}\\ 0\end{bmatrix},\begin{bmatrix}\alpha_1^{b^*}\\ 1-\alpha_1^{b^*}\end{bmatrix}\right\}$ because of the proposition 1.

$$U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ 0\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}\right)=(1+m_g)(1+R_f+\alpha_1^g(\mu-R_f))+\lambda N\left(\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{(1-\alpha_1^b)^2\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}\right)$$

$$U_b\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ 0\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}\right)=(1+m_b)(1+R_f+\alpha_1^b(\mu-R_f))+\lambda-\lambda N\left(\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{(1-\alpha_1^b)^2\sigma_i^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m^2}}\right)$$ Co

mbining the derivatives of the above two functions, we get that

$$\frac{\partial}{\partial\alpha_1^g}U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ 0\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}\right)-\frac{\partial}{\partial\alpha_1^b}U_b\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\ 0\end{bmatrix};\alpha^b=\begin{bmatrix}\alpha_1^b\\ 1-\alpha_1^b\end{bmatrix}\right)$$

$$=m_\delta(\mu-R_f)+\lambda n\left(\frac{m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)}{\sqrt{(1-\alpha_1^b)^2\sigma_i{}^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m{}^2}}\right)\cdot\frac{\left(m_\delta+(\alpha_1^g-\alpha_1^b)(\mu-R_f)\right)(1-\alpha_1^b)\sigma_i{}^2}{\left((1-\alpha_1^b)^2\sigma_i{}^2+(\alpha_1^g-\alpha_1^b)^2\sigma_m{}^2\right)^{3/2}}$$

$$>0$$

This shows that the first order optimality condition for the portfolio allocation of manager *g* and the first order condition for manager *b* will not be satisfied at the same time. As a consequence, in the equilibrium, the short sale constraint is binding for at least one of the two managers. If the short sale constraint is binding for manager *g,* we have, by Proposition 1, $\alpha^{g^*}=\begin{bmatrix}1\\ 0\end{bmatrix}$. If the short sale constraint is binding for manager *b*, we have either $\alpha^{b^*}=\begin{bmatrix}1\\ 0\end{bmatrix}$ or $\alpha^{b^*}=\begin{bmatrix}0\\ 1\end{bmatrix}$. In the case that $\alpha^{b^*}=\begin{bmatrix}1\\ 0\end{bmatrix}$, we have, again by proposition 1, $\alpha^{g^*}=\begin{bmatrix}1\\ 0\end{bmatrix}$. Therefore, in equilibrium, either $\alpha^{g^*}=\begin{bmatrix}1\\ 0\end{bmatrix}$, or $\alpha^{b^*}=\begin{bmatrix}0\\ 1\end{bmatrix}$, or both.

Appendix C

In general, the first order optimality condition for the portfolio allocation of manager b is

$$0=\frac{\partial}{\partial\alpha_1^b}U_b\left(\alpha^g=\begin{bmatrix}1\\0\end{bmatrix},\alpha^b=\begin{bmatrix}\alpha_1^b\\1-\alpha_1^b\end{bmatrix}\right)$$

$$=(1+m_l)(\mu-R_f)-\lambda n\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\frac{1}{1-\alpha_1^b}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\frac{1}{(1-\alpha_1^b)^2}.$$

In the equilibrium, either the above first order condition is satisfied, or, $\alpha^{b^*}=\begin{bmatrix}1\\0\end{bmatrix}$. The first order condition is equivalent to

$$(1+m_b)(\mu-R_f)=\lambda n\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x^2\text{, where } x=\frac{1}{1-\alpha_1^b}.$$

Notice that both sides are positive. We can therefore take natural log of both sides, which yields the equivalent condition:

$$h(x)\equiv\ln\left(\frac{(1+m_l)(\mu-R_f)\sqrt{\sigma_i^2+\sigma_m^2}\sqrt{2\pi}}{\lambda m_\delta}\right)+\frac{1}{2}\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2-2\ln x=0. \quad \text{(A1)}$$

It is straight forward to see that $h(x)$ and $\frac{\partial}{\partial\alpha_1^b}U_b\left(\alpha^g=\begin{bmatrix}1\\0\end{bmatrix},\alpha^b=\begin{bmatrix}\alpha_1^b\\1-\alpha_1^b\end{bmatrix}\right)$ have the same sign. Taking derivatives of $h(x)$, we have:

$$h'(x)=\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}-\frac{2}{x}\text{, and } h''(x)=\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2+\frac{2}{x^2}>0.$$

Therefore, $h(x)$ is a strictly convex function and thus have at most two roots. It is clear that $\lim_{x\to0^+}h(x)=\lim_{x\to\infty}h(x)=+\infty$. In the case that $h(x)$ has only one root or no root, by the middle value theorem of continuous function, we have $h(x)\geq0$ for $x\in[1,\infty)$. Therefore U_b will be a strictly increasing function with respect to the variable α_1^b. We can then conclude that, in this case, $\alpha^{b^*}=\begin{bmatrix}1\\0\end{bmatrix}$ will be the manager b's optimal response.

In the case that function $h(x)$ has two roots, we denote the roots by x_1 and x_2 with $1<x_1<x_2$. Because $h(x)$ is a convex function, we have $h(x)>0$ for $x\in[1,x_1)\cup(x_2,\infty)$, and $h(x)<0$ for $x\in(x_1,x_2)$. Thus U_l is increasing for $\alpha_1^b\in[0,1-1/x_1)\cup(1-1/x_2,\infty)$, and decreasing for $\alpha_1^b\in[1-1/x_1,1-1/x_2]$. Therefore,

$$\arg\max_{\alpha_1^b}U_b\left(\alpha^g=\begin{bmatrix}1\\0\end{bmatrix},\alpha^b=\begin{bmatrix}\alpha_1^b\\1-\alpha_1^b\end{bmatrix}\right)=1\text{ or }1-\frac{1}{x_1}.$$

We derive some properties regarding the equation $h(x)=0$, especially regarding x_1, and record the results in the following several lemmas. The existence of roots for $h(x)$ is given by the following lemma:

LEMMA 2: *$h(x)$ has two roots in $(0,+\infty)$ if and only if the following condition holds:*

$$\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}<C_1 \quad \text{(A2)}$$

where

$$C_1 = \frac{\lambda}{4(1+m_l)(\mu - R_f)} \cdot n \left[\frac{1}{2} \left(\sqrt{\frac{(\mu - R_f)^2}{\sigma_i^2 + \sigma_m^2} + 8} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \right] \cdot \left(\sqrt{\left(\frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 + 8} - \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2$$

In addition, $h(x)$ *has one root if and only if* $\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} = C_1$.

Proof: The only critical point of $h(x)$, located by the first order condition $h'(x) = 0$, is at

$$x^* = \frac{1}{2} \left(\frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^{-1} \left(\sqrt{\left(\frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 + 8} - \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right). \quad \text{(A3)}$$

Therefore, $h(x^*) = \min h(x)$. The ending behavior of $h(x)$ is given by:

$$\lim_{x \to +\infty} h(x) \geq \lim_{x \to +\infty} \frac{m_\delta^{\;2}}{2(\sigma_i^{\;2} + \sigma_m^{\;2})} x^2 - 2x = +\infty,$$

$$\lim_{x \to 0} h(x) \geq -2 \lim_{x \to +\infty} \ln x = +\infty.$$

Because $h(x)$ is convex, we then have that $h(x)$ has one root in $(0, +\infty)$ if and only if $h(x^*) = 0$, and it has two distinct roots in $(0, +\infty)$ if and only if $h(x^*) < 0$. Plug x^* into $h(x)$, we get the proof the lemma.

The above lemma puts some restriction on the gap between the winning manager and the losing manager. The gap, m_δ, can not be too large in order for $h(x)$ to have roots. If the condition of the lemma does not hold, $h(x)$ will always be positive. It will then lead to the conclusion that it is optimal for the losing manager to choose $\alpha^l = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as her optimal strategy. Notice that the bound of the gap in the condition is proportional to λ, the reward of winning the tournament.

Another way to express the bound in the above lemma is to say that there is at least one feasible portfolio holding for the losing manager at which this manager would prefer to increase her holding on idiosyncratic risk. Increasing the holding on idiosyncratic risk affect the losing manager's utility through two ways. First, for the losing manager to hold more idiosyncratic risk will increase the variance of the difference between her portfolio and the winning manager's portfolio, bring more noise into the outcome of the tournament, and therefore has the potential of increasing the chance for the losing manager to make up the gap and win the tournament. Second, increase on the holding of it means lower the holding on the market index and lower the expected return. Low expected return has in itself two consequences on the losing manager's utility. It will lower the expected the manager's expected reward from the explicit linear contract, and it will lower the chance of winning the tournament for the losing manager. The losing manager has to take all these three factors into consideration to reach the portfolio decision.

For the rest of the section, we will maintain the assumption that the condition of lemma 2 holds. That is, we assume that $h(x)$ has two roots. It will be sufficient for that condition to hold if we assume that the tournament has some effect on at least one of the managers portfolio decision. We will maintain the notation x_1 to denote the smaller one of the two roots of $h(x)$.

Proof of Proposition 2: It follows immediately from Lemma 2 that, under the condition of the proposition, $\alpha^{b^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is optimal for manager *b* given $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The optimality of $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for manager *g* is assured by Proposition 1 (see Appendix A). Thus $\left\{ \alpha^{w^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^{b^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ is a Nash equilibrium.

LEMMA 3: $\dfrac{\partial x_1}{\partial \lambda} = \dfrac{1}{h'(x_1)\lambda} < 0$

Proof: Since x_1 be the smaller one of the two roots of the convex function $h(x)$. Therefore, $h(x_1) = 0$ and $h'(x_1) < 0$. From $h(x_1) = 0$,

$$\ln\left((1+m_l)(\mu - R_f)\sqrt{\sigma_i^2 + \sigma_m^2}\sqrt{2\pi}\right) - \ln\lambda - \ln m_\delta + \frac{1}{2}\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2 - 2\ln x_1 = 0$$

Differentiating the above equation, we get $\dfrac{\partial x_1}{\partial \lambda} = \dfrac{1}{h'(x_1)\lambda} < 0$. Thus the lemma is proved.

LEMMA 4: $\dfrac{\partial [m_\delta x_1]}{\partial m_\delta} > 0$*, and* $\lim_{m_\delta \to 0^+} m_\delta x_1 = 0$*. Moreover, we have* $\lim_{m_\delta \to 0^+} x_1 = +\infty$*, and to be more precise, we have* $x_1 = O(m_\delta^{-1/2})$*.*[20]

Proof: By $h'(x_1) < 0$, we have $\left(\dfrac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \dfrac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\dfrac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}} - \dfrac{2}{x_1} < 0.$

Differentiate equation $h(x_1) = 0$, we get that

$$\frac{\partial x_1}{\partial m_\delta} = \frac{\left[\dfrac{1}{m_\delta} - \left(\dfrac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \dfrac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\dfrac{x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]}{\left[\left(\dfrac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \dfrac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\dfrac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}} - \dfrac{2}{x_1}\right]}$$

Therefore,

$$\frac{\partial [m_\delta x_1]}{\partial m_\delta} = x_1 + m_\delta \frac{\partial x_1}{\partial m_\delta}$$

$$= -\left[\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}} - \frac{2}{x_1}\right]^{-1} = -1/h'(x_1) > 0$$

We thus have that $m_\delta x_1$ is a increasing function of m_δ. Moreover, $m_\delta x_1 > 0$. Therefore, $\lim_{m_\delta \to 0^+} m_\delta x_1$ exists, is bounded and nonnegative.

An equivalent equation of $h(x_1) = 0$ is:

$$(1+m_l)(\mu - R_f) = \lambda n\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1 + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1^2 \tag{A4}$$

and this is further equivalent to

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[20] The notation *O* means that both x_1 and $m_\delta^{-1/2}$ approach infinity with the same order of speed when $m_\delta \to 0^+$.

$$(1+m_l)(\mu-R_f)m_\delta=\lambda n\left(\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{(m_\delta x_1)^2}{\sqrt{\sigma_i^2+\sigma_m^2}}.$$

Take limit of both sides of the above equality by let $m_\delta \to 0^+$:

$$0=\lambda n\left(\frac{\lim_{m_\delta\to 0^+} m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{\left(\lim_{m_\delta\to 0^+} m_\delta x_1\right)^2}{\sqrt{\sigma_i^2+\sigma_m^2}},$$

which implies that $\lim_{m_\delta\to 0^+} m_\delta x_1=0$. Go back to equation (A2). We have

$$\lim_{m_\delta\to 0^+} m_\delta {x_1}^2=\frac{1}{\lambda}(1+m_l)(\mu-R_f)\sqrt{\sigma_i^2+\sigma_m^2}\cdot\left[n\left(\frac{\lim_{m_\delta\to 0^+} m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\right]^{-1}$$

$$=\frac{1}{\lambda}(1+m_l)(\mu-R_f)\sqrt{\sigma_i^2+\sigma_m^2}\cdot\left[n\left(\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\right]^{-1}$$

The right hand side of the above identity is positive. Therefore, we have that $\lim_{m_\delta\to 0^+} x_1=+\infty$, and to be more precise, $x_1=O({m_\delta}^{-1/2})$. Thus the lemma is proved.

LEMMA 5: $\frac{\partial x_1}{\partial m_\delta}<0$ *for sufficiently small* m_δ. *In fact, it is true for any*

$$m_\delta\in\left(0,\ \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1,C_2)\right), \qquad (A5)$$

where C_1 *is given as in lemma 2, and*

$$C_2=\frac{\lambda}{4(1+m_l)(\mu-R_f)}\cdot n\left[\frac{1}{2}\left(\sqrt{\frac{(\mu-R_f)^2}{\sigma_i^2+\sigma_m^2}+4}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\right]$$
$$\cdot\left(\sqrt{\left(\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2+4}-\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2$$

Proof: In the derivation of lemma 3, we see

$$\frac{\partial x_1}{\partial m_\delta}=\frac{\left[\frac{1}{m_\delta}-\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}x_1+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]}{h'(x_1)}.$$

Because $h'(x_1)<0$, we have $\frac{\partial x_1}{\partial m_\delta}<0$ if and only if

$$1>\left(\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}+\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}.$$

The right hand side is a quadratic form of $\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}$. It is straight to solve the above inequality. Given $\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}>0$, we have $\frac{\partial x_1}{\partial m_\delta}<0$ if and only if

$$0<\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}<\frac{1}{2}\left(\sqrt{\frac{(\mu-R_f)^2}{\sigma_i^2+\sigma_m^2}+4}-\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right). \tag{A6}$$

By lemma 3, we have $\lim_{m_\delta\to 0^+} m_\delta x_1 = 0$. Therefore, the above inequalities will be satisfied for sufficiently small m_δ. To answer the question of how small is sufficiently small, we evoke the equation $h(x_1)=0$. With this equation, we can see that the necessary condition for

$$\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}=\frac{1}{2}\left(\sqrt{\frac{(\mu-R_f)^2}{\sigma_i^2+\sigma_m^2}+4}-\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)$$

is $\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}=C_2$. Therefore, for any $m_\delta \in \left(0, C_2\sqrt{\sigma_i^2+\sigma_m^2}\right)$, we always have

$$\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}\neq\frac{1}{2}\left(\sqrt{\frac{(\mu-R_f)^2}{\sigma_i^2+\sigma_m^2}+4}-\frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right).$$

This with the fact that inequalities (A6) is satisfied by sufficiently small m_δ together implies that inequalities (A6) is satisfied by any $m_\delta \in \left(0, C_2\sqrt{\sigma_i^2+\sigma_m^2}\right)$, where the fact that $\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}$ is a continuous function of m_δ is used. Thus the lemma is proved.

Appendix D

Proof of Proposition 3: In case that $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\alpha_1^{b^*} < 1$, the condition in lemma 2 is satisfied. Therefore, Lemma 3 and lemma 4 imply that $\alpha_1^{b^*} = 1 - 1/x_1$, and therefore, $\frac{\partial(1-\alpha_1^{b^*})}{\partial\lambda} = \frac{\partial}{\partial\lambda}(1/x_1) = -\frac{1}{x_1^{\,2}}\frac{\partial x_1}{\partial\lambda} > 0$, because of lemma 3. This proves the first part of the proposition. Taking the derivative and using lemma 4,

$$\frac{\partial}{\partial m_\delta} N\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\frac{1}{1-\alpha_1^{b^*}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right) = n\left(\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}}\frac{1}{1-\alpha_1^{b^*}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right) \times \frac{\partial(m_\delta x_1)}{m_\delta} > 0.$$

Thus the proposition is proved.

LEMMA 6*: If* $\alpha^{g^*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$*, then for sufficiently small* m_δ *we have* $\alpha_1^{b^*} < 1$*. In fact, it is true for any* $m_\delta \in \left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1, C_2, C_3)\right)$*, where* $C_3 = \tilde{m}_\delta / \sqrt{\sigma_i^2+\sigma_m^2}$ *with* $\tilde{m}_\delta$ *being a root in* $\left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1, C_2)\right)$*. Further,*

$$(1+m_b)(\mu - R_f) = \lambda\cdot\left(1 - N\left[\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\right)x_1 \qquad \text{(A7)}$$

if there such a root, and $C_3 = +\infty$ *otherwise.*

Proof: Notice that

$$\frac{\partial}{\partial m_\delta}\left(1 - N\left[\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\right)x_1$$

$$= -n\left[\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\frac{x_1^{\,2}}{\sqrt{\sigma_i^2+\sigma_m^2}_{\,1}} + \left(1 - N\left[\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\right)\frac{\partial x_1}{\partial m_\delta}$$

$$< 0$$

for m_δ in $\left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1, C_2)\right)$, because the first term is clearly less than zero, and the second term is less than zero because of lemma 4. Therefore, the right hand side expression of equation (A7) as a function of m_δ is decreasing in $\left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1, C_2)\right)$. Thus equation (A7) has at most one root in the interval. And if there is a root, any m_δ less than the root will satisfied the inequality with left hand side of (A7) strictly less than the right hand side. Furthermore, we note the fact that

$$\lim_{m_\delta\to 0^+}\lambda\cdot\left(1 - N\left[\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\right)x_1 = \lambda\cdot\left(1 - N\left[\frac{\lim_{m_\delta\to 0^+} m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right]\right)\lim_{m_\delta\to 0^+} x_1 = +\infty$$

Therefore, in the case that equation (A7) has no root in the interval, we will always have the left hand side of (A7) strictly less than the right hand side. Here the continuity of the right hand side of (4) as a function of m_δ is used.

In summary, we have that the left hand side of (A7) is strictly less than the right hand side if $m_\delta \in \left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot\min(C_1, C_2, C_3)\right)$. Earlier discussion about the first order condition of the losing manager's optimization problem leads us to conclude that

$\arg\max_{\alpha_1^b} U_b\left(\alpha^g = \begin{bmatrix}1\\0\end{bmatrix}, \alpha^b = \begin{bmatrix}\alpha_1^b\\1-\alpha_1^b\end{bmatrix}\right) = 1 \text{ or } 1-\frac{1}{x_1}$. By comparing the value of U_b at $\alpha_1^b = 1$ with the value of U_b at $\alpha_1^b = 1-1/x_1$, we have the necessary and sufficient condition for $\alpha_1^{b^*} < 1$ to be that the condition (1) in lemma 2 holds and that at the same time the left hand side of (A7) is strictly less than the right hand side. For each $m_\delta \in \left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot \min(C_1, C_2, C_3)\right)$, both the above conditions are satisfied. Thus the lemma is proved.

Based on the earlier lemmas, we derive the following sufficient condition for the existence of the equilibrium.

THEOREM 1: *Let* $\alpha_1^{b^*} = 1-1/x_1$. *The pair* $\left\{\alpha^{g^*} = \begin{bmatrix}1\\0\end{bmatrix}, \alpha^{b^*} = \begin{bmatrix}\alpha_1^{b^*}\\1-\alpha_1^{b^*}\end{bmatrix}\right\}$ *will be an equilibrium for any*

$$m_\delta \in \left(0, \sqrt{\sigma_i^2+\sigma_m^2}\cdot \min(C_1, C_2, C_3, C_4)\right), \tag{A8}$$

where $C_4 = \frac{\lambda(\mu-R_f)\sigma_i^{\,2}}{(1+m_l)\sigma_m^{\,2}}\cdot n\left[\frac{(\mu-R_f)\sqrt{\sigma_i^2+\sigma_m^2}}{\sigma_m^{\,2}}\right]$.

Furthermore, $\frac{\partial(1-\alpha_1^{b^*})}{\partial m_\delta} > 0$ *and* $\lim_{m_\delta\to 0^+}(1-\alpha_1^{b^*}) = 0$. *To be more precise, we have that* $1-\alpha_1^{b^*} = O(\sqrt{m_\delta})$.

Proof: The optimality of the losing manager's strategy $\alpha^{b^*} = \begin{bmatrix}1-1/x_1\\1/x_1\end{bmatrix}$ has already been established in lemma 6. We now prove the optimality of the winning manager's strategy $\alpha^{g^*} = \begin{bmatrix}1\\0\end{bmatrix}$ under the condition of the theorem. We evoke the equation $h(x_1) = 0$, which is equivalent to

$$\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}} = (1+m_l)(\mu-R_f)\lambda n\left(\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}} + \frac{(\mu-R_f)}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)\left(\frac{m_\delta x_1}{\sqrt{\sigma_i^2+\sigma_m^2}}\right)^2.$$

With this equation, we can see that the necessary condition for $(\mu-R_f)\sigma_i^2 - m_\delta x_1\sigma_m^2 = 0$ is $\frac{m_\delta}{\sqrt{\sigma_i^2+\sigma_m^2}} = C_4$. Therefore, there is no root for $(\mu-R_f)\sigma_i^2 - m_\delta x_1\sigma_m^2 = 0$ with $m_\delta \in \left(0, C_4\sqrt{\sigma_i^2+\sigma_m^2}\right)$. This with the fact that $\lim_{m_\delta\to 0^+} m_\delta x_1 = 0$ implies that $(\mu-R_f)\sigma_i^2 - m_\delta x_1\sigma_m^2 > 0$ for $m_\delta \in \left(0, C_4\sqrt{\sigma_i^2+\sigma_m^2}\right)$. We next compute the derivative of U_g with respect to α_1^g.

$$\frac{\partial}{\partial \alpha_1^g} U_g\left(\alpha^g=\begin{bmatrix}\alpha_1^g\\0\end{bmatrix};\alpha^b=\begin{bmatrix}1-1/x_1\\1/x_1\end{bmatrix}\right)$$

$$=(1+m_b+m_\delta)(\mu-R_f)+\lambda n\left(\frac{m_\delta x_1+(x_1\alpha_1^g-x_1+1)(\mu-R_f)}{\sqrt{\sigma_i^2+(x_1\alpha_1^g-x_1+1)^2\sigma_m^2}}\right)$$

$$\frac{x_1(\mu-R_f)\left(\sigma_i^2+(x_1\alpha_1^g-x_1+1)^2\sigma_m^2\right)-x_1\left(m_\delta x_1+(x_1\alpha_1^g-x_1+1)(\mu-R_f)\right)(x_1\alpha_1^g-x_1+1)\sigma_m^2}{\left(\sigma_i^2+(x_1\alpha_1^g-x_1+1)^2\sigma_m^2\right)^{3/2}}$$

$$=(1+m_b+m_\delta)(\mu-R_f)+\lambda n\left(\frac{m_\delta x_1+(x_1\alpha_1^g-x_1+1)(\mu-R_f)}{\sqrt{\sigma_i^2+(x_1\alpha_1^g-x_1+1)^2\sigma_m^2}}\right)$$

$$\frac{x_1\left[(\mu-R_f)\sigma_i^2-m_\delta x_1{}^2(\alpha_1^g-1+1/x_1)\sigma_m^2\right]}{\left(\sigma_i^2+(x_1\alpha_1^g-x_1+1)^2\sigma_m^2\right)^{3/2}}$$

$$>0$$

The last inequality is true for all $\alpha_1^g\in[\alpha_1^l,1]$, and for $m_\delta\in\left(0,\,C_4\sqrt{\sigma_i^2+\sigma_m^2}\right)$. Therefore, $\alpha^{g^*}=\begin{bmatrix}1\\0\end{bmatrix}$ is optimal for the winning manager. Furthermore, $\frac{\partial(1-\alpha_1^{b^*})}{\partial m_\delta}=-\frac{1}{x_1{}^2}\frac{\partial x_1}{\partial m_\delta}<0$ by lemma 5, and $\lim_{m_\delta\to 0^+}(1-\alpha_1^{b^*})=\lim_{m_\delta\to 0^+}\frac{1}{x_1}=0$ by lemma 4. Also by lemma 4, $1-\alpha_1^{b^*}=O(\sqrt{m_\delta})$.

Thus we have proved the theorem.

Proof of Corollary 1: The Sharpe Ratio of fund *i*, denoted by SR^i is:

$$SR^i=\frac{\alpha_1^i(\mu-R_f)}{\sqrt{(\alpha_1^i\sigma_m)^2+((1-\alpha_1^i)\sigma_i)^2}}.$$

Taking the derivative of SR^i with respect to variable α_1^i and after simplification, we have

$$\frac{\partial}{\partial\alpha_1^i}(SR^i)=\frac{(\mu-R_f)\sigma_i{}^2}{\left(\sqrt{(\alpha_1^i\sigma_m)^2+((1-\alpha_1^i)\sigma_i)^2}\right)^3}\cdot(1-\alpha_1^i).$$

Therefore, $\frac{\partial}{\partial\alpha_1^i}(SR^i)>0$ for $\alpha_1^i<1$. From Theorem 1, in the equilibrium, we have $\alpha_1^{g^*}=1$ and $\alpha_1^{b^*}<1$, and thus $SR^b<SR^g$. Moreover, we have that $\frac{\partial}{\partial m_\delta}(SR^b)=\frac{\partial}{\partial\alpha_1^b}(SR^b)\cdot\frac{\partial\alpha_1^{b^*}}{\partial m_\delta}<0$ from Theorem 1, and $\frac{\partial}{\partial\lambda}(SR^b)=\frac{\partial}{\partial\alpha_1^b}(SR^b)\cdot\frac{\partial\alpha_1^{b^*}}{\partial\lambda}<0$ from Proposition 3.

Appendix E

Proof of Proposition 6: We have to prove the optimality of the strategy $\begin{bmatrix}\alpha_{1,1}^1 \\ \alpha_{2,1}^1\end{bmatrix} = \begin{bmatrix}1 \\ 0\end{bmatrix}$ for manager 1, given manager 2's strategy for the first period $\begin{bmatrix}\alpha_{1,1}^2 \\ \alpha_{2,1}^2\end{bmatrix} = \begin{bmatrix}1 \\ 0\end{bmatrix}$. Assume that Manager 1 adopts the strategy $\begin{bmatrix}\alpha_{1,1}^1 \\ \alpha_{2,1}^1\end{bmatrix} = \begin{bmatrix}\xi_1 \\ \xi_2\end{bmatrix}$. She outperforms manager *2* if and only if $F^1 > F^2$. That is,

$$(c_2 - c_1) + (\xi_1 - 1)(\mu_m - r_f) - \frac{1}{2}(\xi_1^{\,2} - 1)\sigma_m^{\,2} - \frac{1}{2}\xi_2^{\,2}\sigma_i^{\,2} + (\xi_1 - 1)\sigma_m B_{m,1} + \xi_1\sigma_i B_{1,1} > 0 \cdot$$

Note that $(\xi_1 - 1)(\mu_m - r_f) - \frac{1}{2}(\xi_1^{\,2} - 1)\sigma_m^{\,2} - \frac{1}{2}\xi_2^{\,2}\sigma_i^{\,2} \le 0$ with the equality holding only when $\xi_1 = 1$ and $\xi_2 = 0$. The term $(c_2 - c_1) + (\xi_1 - 1)\sigma_m B_{m,1} + \xi_1\sigma_i B_{1,1}$ is a random variable with symmetric distribution centered on zero. Therefore, $P[F^1 > F^2] \le 1/2$ with the equality holding only when $\xi_1 = 1$. Therefore, by setting $\xi_1 = 1$, manager 1 maximizes her chance of outperforming manager 2. The proposition now follows by noticing that setting $\xi_1 = 1$ also maximizes manager 1's expected reward from absolute growth of the assets within the first period.

Figure 1: One Period Model

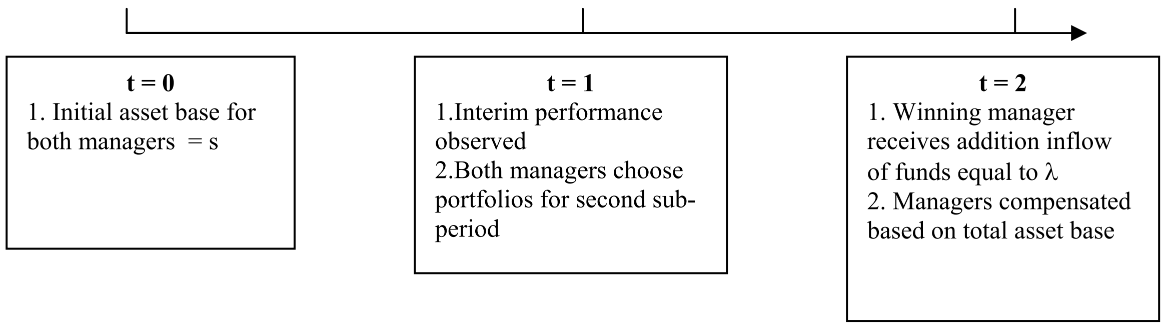

Table 1

A. Payoffs to managers at the end of each period:

	Period 1	Period 2	period 3	
Manager g	$k\exp(F^g)$	$k((1-k)\exp(F^g)+\lambda)$ $\exp(S^g)$	If $S^b - S^g < F^\delta$	$k[(1-k)((1-k)\exp(F^g)+\lambda)\exp(S^g)+\lambda]$ $\exp(T^g)$
			Otherwise	$k(1-k)((1-k)\exp(F^g)+\lambda)\exp(S^g+T^g)$
Manager b	$k\exp(F^b)$	$k(1-k)\exp(F^b+S^b)$	If $S^b - S^g < F^\delta$	$k(1-k)^2\exp(F^b+S^b+T^b)$
			Otherwise	$k[(1-k)^2\exp(F^b+S^b)+\lambda]\exp(T^b)$

B. Returns to the investors in each period:

	Return in period 1	Return in period 2	Return in period 3
Invest with manager g	$(1-k)\exp(F^g)$	$(1-k)\exp(S^g)$	$(1-k)\exp(T^g)$
Invest with manager b	$(1-k)\exp(F^b)$	$(1-k)\exp(S^b)$	$(1-k)\exp(T^b)$