A method of joint analysis of seismic regimes is proposed for recognition collective behavior phenomena of seismicity in a group of areas that form a large seismically active region. The method is based on the robust multidimensional wavelet analysis of square root values of earthquake energies released in each of the areas (cumulative Benioff curves proportional to the values of elastic stresses accumulated and released in an earthquake source). This method is a further development of the method of wavelet-aggregated signals previously proposed by the author to analyze multidimensional time series of geophysical monitoring. It is based on robust multidimensional analysis of canonical and principal components of wavelet coefficients. The compactness of the basis functions involved in the signal expansion makes it possible to analyze not only Gaussian but also essentially non-stationary time series, which allows the application of non-parametric methods of analysis of multidimensional time series to non-Gaussian signals, including series obtained from seismic catalogs. The result gives a hierarchical pattern of arising of correlation peaks on different time intervals and for different time scales which could be used for extracting time segments of seismic history of the region and for searching strong earthquakes precursors.
1-st stage of wavelet robust aggregation: seeking canonical components

\( \alpha \) - detail level number, \( \tau \) - time corresponding to the center of wavelet basis function of the \( \alpha \)-th level

\[
\phi_k(\alpha, \tau) = \sum_{j \neq k} \gamma_j^{(k)}(\alpha) C_j(\alpha, \tau)
\]

- canonical wavelet coefficients

where \( \gamma_j^{(k)}(\alpha) : \sum_{\tau} |C_k(\alpha, \tau) - \sum_{j \neq k} \gamma_j C_j(\alpha, \tau)| \rightarrow \min_{\gamma} \)

\[
\mu(\alpha) = \prod_{k} |\nu_k(\alpha)|, \quad 0 \leq \mu(\alpha) \leq 1
\]

- robust wavelet coherence measure

where \( \nu_k(\alpha) = \text{Rob.Corr.Coeff.}(\phi_k(\alpha, \tau), C_k(\alpha, \tau)) \)
2-nd stage of robust wavelet aggregation: seeking for robust principal component of the set of robust canonical components

\[ \sum_j \beta_j(\alpha) \phi_j(\alpha, \tau) = \eta(\alpha, \tau) \]

\[ \beta_j(\alpha) \text{ - from the problem:} \]
\[ \sum_\tau |\eta(\alpha, \tau)| \rightarrow \max_\beta , \]
\[ |\beta(\alpha)|^2 = 1 \]

- \( Y(t) \) - robust wavelet aggregated signal of the initial multiple time series
Geophysical monitoring time series in North-East China before and after TangShan earthquake (1976, 28 July 1976, M=7.8) and its wavelet-aggregated signal
Change of the form of the right-hand final interval of the wavelet-aggregated signal in dependence of the length $N$ of processed samples of initial time series. The value of $N$ approaches the moment of Tangshan earthquake, $M=7.8$, July 28, 1976, 1,671-th day from the beginning of 1972.
Different variants of the aggregated signal in dependence on used wavelet, length of time window = 700.

- Haar
- Daub 04
- Daub 12
- Daub 20
Hypocenters of earthquakes with magnitudes $M \geq 4.5$, depths $\leq 100$ km for period 1963-2001 in the region of Japan, Kuril islands and Kamchatka and partition of region into 7 areas.
Time series of increments of Benioff's curves in areas J1-J5 and KN and KS with a time step 5 day after scaling operations in the window of adaptation of length 365 samples, i.e. 1825 days, that is equal to 5 usual years.
The sequence of aggregated wavelet coefficients for the first 5 detail levels of MRA and the result of their inverse wavelet transform (wavelet-aggregated signal).

Increments of Benioff's curves for 7 regions in Japan + Kurils + Kamchatka, $M \geq 4.5$, $H \leq 100$ km, $dt = 5$ days, 1963-2001, Haar's wavelets, length of time window = 365 samples (5 years), $L_{min} = 10$. 

<table>
<thead>
<tr>
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Robust wavelet-aggregated signal
Evolution of the product of the robust component-wise canonical correlation coefficients, estimated in the moving time window. Time marks correspond to right-hand end of the time windows of the length = 5 years.

Increments of Benioff’s curves for 7 regions in Japan + Kurils + Kamchatka, $M \geq 4.5$, $H \leq 100$ km, $dt = 5$ days, 1963-2001, Haar's wavelets, length of time window = 365 samples (5 years), $L_{\text{min}} = 10$. 

Level #1, 10-days variations

Level #2, 20-days variations

Level #3, 40-days variations

Level #4, 80-days variations

Level #5, 160-days variations
The sequences of seismic events with $M \geq 4.5$, 1963-2001, within regions Q13, Q24 and Q5678 of California.

Time series of increments of Benioff's curves with a time step 5 day after scaling operations in the window of adaptation of length 365 samples, i.e. 1825 days, that is equal to 5 usual years.
Evolution of the product of the robust component-wise canonical correlation coefficients, estimated in the moving time window. Time marks correspond to right-hand end of the time windows of the length = 5 years.

Increments of Benioff's curves for 3 regions in California $M \geq 3.0$, $dt = 5$ days, 1963-2001, Haar's wavelets, length of time window = 365 samples (5 years), $L_{\min} = 10$. 

Level #1, 10-days variations

Level #2, 20-days variations

Level #3, 40-days variations

Level #4, 80-days variations

Level #5, 160-days variations