Managing High-Tech Capacity Expansion Via Reservation Contracts

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We study capacity reservation contracts in high-tech manufacturing, where the manufacturer (the supplier) shares the risk of capacity expansion with her OEM customer (the buyer). We focus on short-life-cycle, make-to-order products under stochastic demand. The supplier and the buyer are partners who enter a "design-win" agreement to develop the product, and who share demand information. The supplier would expand her capacity in any case, but reservation may encourage her to expand more aggressively. To reserve capacity, the buyer pays a fee upfront while (a pre-specified portion of) the fee is deductible from the order payment. As capacity expansion demonstrates diseconomy of scale in this context, we assume convex capacity costs. We first analyze the players' incentives as a means to evaluating the value of reservation. We show that as the buyer's revenue margin decreases, the supplier faces a sequence of three profit scenarios with decreasing desirability. We examine the effects of market size and demand variability to the contract conditions, and show that it is the demand variability that affects the reservation fee, and that the convex cost assumption leads to somewhat different insights then the linear cost cases in the literature. We propose two channel coordination contracts, and discuss additional cases when the supplier has the option not to comply with the contract, and when the buyer's (in this case, a contract manufacturer) market size is only partially known. We conclude the paper by summarizing insights useful for high-tech capacity management.

1. Introduction

Manufacturing capacity plays a significant role in high-tech industries such as semiconductors, electronics, and telecommunications equipment. This paper is motivated by our involvement with designing capacity reservation contracts for a major telecommunications device manufacturer in the US. During the upside market in the late 1990's, the manufacturer constantly suffers from capacity shortages, resulting in lost revenue, eroding their long-term market position.. Although the manufacturer had strong incentive to increase capacity so as to improve service levels and to realize higher revenue, aggressive capacity expansion may expose the firm to significant financial risk due to high capacity cost, long (capacity) lead time, and high demand volatility. For instance, a state-of-the-art semiconductor manufacturing Fab costs $500 million to $2 billion to build, but the demand volatility during a particular quarter may be as high as 80%. Moreover, the overall market size may drop sharply in reaction to economic contraction, as evident from the decline of telecommunications demand during 2001 and 2002. In this environment, the device manufacturers (the supplier) are forced to adopt a conservative capacity expansion policy, limiting their downside risk at the expense of upside potentials. Consequently, their downstream customers (e.g., OEM manufacturers) may not have adequate supplies to fill the market orders. Suppose the customer is willing to mitigate the supplier's downside risk by assuming a certain level of liability, the supplier might be willing to expand capacity more aggressively. For instance, the customer may offer early commitment on a certain portion of future capacity, in exchange, the supplier commits to increase capacity sufficient to satisfy the customer expected demands. This provides a win-win situation for the supplier and the buyer by creating additional surplus in the channel.

In this paper, we propose *capacity reservation contracts* designed for short-life-cylce, make-to-order products under stochastic demand. Such products are characteristic in the high-tech industry, but appear in many industries. We will examine reservation contracts with *deductible reservation fees:* the buyer pays a fee up-front for each unit of capacity she would like to reserve for a certain time in the future. When the buyer actually utilizes the reserved capacity (i.e., placing a firm order), the reservation fee is deducted from the order payment. However, if the reserved capacity is not fully utilized by the specified time, the reservation fee associated with unused capacity is not refundable. Since the reservation fee is typically lower than the wholesale price, the reservation contract offers the buyer flexibility as it does not ordain a firm commitment prior to demand realization. On the other hand, reservation reduces the supplier's risk of capacity expansion as the buyer is liable to a certain extend. This form of risk sharing is particularly attractive in high-tech manufacturing where the demand is volatile and the capacity is capital intensive.

Our nominal example of *capacity expansion* is for the supplier to increase her manufacturing capacity through physical expansion. Nevertheless, capacity expansion may be interpreted more generally as outsourcing, work force expansion, procurement of raw material, or pre-processing of semi-finished products, etc. In the literature, the capacity cost is typically assumed to be linear. While linear costs would be reasonable for outsourcing or the procurement of raw material, it is not realistic for physical expansion, which are typically nonlinear or discrete. While it is possible to approximate the discrete capacity expansion cost as a continuous function, it is rather difficult to do so with linear cost. Moreover, capacity expansion often demonstrates diseconomy of scale, characterizing the possibility to improve existing capacity to a certain extend, followed by a more capital-intensive expansion (e.g., building a new line). To allow a broader interpretation of *capacity expansion* without losing analytical tractability, we assume that capacity cost is a convex function increasing in capacity.

Conceptually, the reservation contract creates a win-win situation for the buyer and the supplier. However, if we consider the fact that the supplier could expand capacity based on her knowledge on the expected demand, and the buyer could count on the supplier to make some capacity available even without reservation, then there is no reason to assume that entering a reservation contact is always the best strategy for both players. We examine the *individual rationality* conditions for the supplier and the buyer and demonstrate that there are specific criteria that the players should consider before committing to the contract. We further show that while it is possible to achieve the channel coordinated solution, it could be realized only under a very special case, or under specially designed contracts which allows either *partial* payment deduction, or explicit *cost sharing* during capacity expansion.

The paper is organized as follows: in the next section, we summarize related work in the literature, which is followed by an introduction of the base model in Section 3. In Section 4, we analyze the capacity reservation contract with deductible fees, and we examine the effects of market size and demand variability to contract conditions. In Section 5 we propose two channel coordination contracts. In Section 6 we discuss two additional cases (1) when the supplier has the option not to comply with the contract, and (2) when the buyer is a contract manufacturer whose share on the demand size is only partially known. We then summarize all managerial insights in Section 8. Proofs of the main Theorems are in the Appendix.

2. Related Work in the Literature

Earlier work on capacity reservation focuses on strategies from the buyer's perspective while the terms specified in the reservation are exogenous. Silver and Jan (1994) and later Jan and Silver (1995) consider the case where the buyer pays a non-refundable premium and is guaranteed a certain level of supplies in an environment where the availability of capacity is highly uncertain. They propose methods for the buyer to determine the level of dedicated capacity to reserve, and the size of each periodic replenishment. Brown and Lee (1998) also study capacity reservations in the context of semiconductor manufacturing, in particular, they discuss "pay-to-delay" capacity reservation contracts. The authors focus their analysis from the buyer's perspective and derive optimal policies for the buyer. Our paper differs in that we consider both the buyer and the supplier's perspectives, and we assume that the supplier may not have ample capacity, while the buyer's main motivation in placing reservation is to maximize profits through increased service level.

Cachon and Larivier (2001) examine capacity contracting in the context of supplier-buyer forecast coordination. The buyer provides an initial forecast and a contract consisting of firm commitments and capacity options. After the supplier builds capacity, the buyer places order based on the up-to-date forecast. A main issue here is that the buyer may have incentive to inflate her initial forecast such that the supplier would build more capacity. Tomlin (1999) investigates a similar problem in the context of a quantity premium contract. Under a somewhat different setting, Serel *et. al.* (2001) analyze inventory reservation between a buyer and a supplier, where the buyer employs the base-stock policy in a market with uncertain demand. The buyer pays a discounted wholesale price to the supplier for each unit of inventory she reserves. In response, the supplier guarantees the delivery of the buyer's order up to the reservation amount. When the demand is realized, the buyer procures the amount beyond her reservation from the spot market. They investigate the conditions under which channel coordination is achieved using numerical analysis.

Our work differs from the above in several aspects due to the contracting environment and the industry conventions. First, entering a reservation contract is not necessarily the best strategy for the players. Long before the negotiation on capacity reservation, the OEM manufacturer (the buyer) would have entered an agreement with the supplier to jointly develop the technology (known as the "design-win" phase). At this time, the supplier would assess the expected demand, and negotiate the (wholesale) pricing. The pricing is not expected to alter later on. With or without reservation, the supplier would expand her capacity based on her assessment on the demand (we model this as the supplier's newsvendor capacity). Knowing this, the buyer would only make reservation when she fears that the supplier's newsvendor capacity is not sufficient to fulfill her revenue potentials. It is in this context the issue of capacity reservation arises. Thus, we devote part of our analysis to understand the supplier and the buyer's incentive to (or not to) enter a reservation contract in the first place. The second aspect that distinguishes our result is that we consider convex (rather than linear) capacity cost. This makes a difference in the design of coordinating contracts. We show that under linear cost it is possible to design coordinating contracts independent of the demand distribution, but this would be unacceptable under convex costs. Third, *forced compliance* is the predominate business practice in this industry as the supplier and buyer are typically long-term partners. We also show that under our contract setting even if *voluntary compliance* is allowed, the supplier would behave no differently. A final aspect that makes our work different is that the *supplier* (not the buyer) is the leader of the channel, who designs the contract and sets the reservation fee. This is consistent with the business context where the supplier would not expand more aggressively than she has to unless she believes that the buyer has strong incentive to offset her risk. We show that this belief hinges on the buyer's profit margin. In essence, as leader of the channel the supplier would set the reservation fee sufficiently high such that the buyer would only become active in reserving when her profit margin exceeds her downside risk.

Capacity reservation requires the buyer to make an early commitment toward her future orders, which provides incentive for the supplier to increase her capacity. Cvsa and Gilbert (2000) points out that it might be a good strategy for the supplier to encourage the buyers for early commitments even in the absence of capacity constraints. They consider a supplier who has no capacity constraint, and two identical competing buyers. They find the conditions under which the supplier should encourage the buyers to commit to future orders. However, they do not consider the issue of channel coordination. Burnetas and Gilbert (2001) study the buyer's trade-off between lower procurement cost, and the benefit of postponing orders when more demand information is revealed. Van Mieghem and Dada (1999) provide a comparative analysis of various postponement and early commitment strategies for capacity investment, production, and pricing under different conditions.

In our model the buyer's reservation is not a *full* commitment in that she is not required to utilize what she has reserved. In this sense, capacity reservation can be viewed as purchasing an option that may or may not be exercised. Donohue (2000) considers options as a means to directing the risk from the supplier to the buyer. The supplier designates a portion of the position inventory for the buyer before the demand is realized, and the buyer must reserve inventory beforehand. She considers a two-stage model where in the second stage the information regarding demand becomes more reliable. Return policies are employed to achieve channel coordination. Using a different approach, Barnes-Schuster *et. al.* (2000) consider a combination of committed orders and options for supplier-buyer coordination. They analyze a two-period model with correlated demand and investigate the implications of various coordination mechanisms. While the Donohue (2000) study does not investigate the value of options to the channel, Barnes-Schuster *et. al.* (2000) provides numerical studies to this respect. In this paper, we examine explicitly the value of capacity reservation to the channel, and we analyze the impact of market size and demand variability to channel efficiency.

The coordination contracts proposed in this paper are motivated by the buy-back contracts. Pasternack (1985) shows that coordination across a single-supplier, single-buyer channel can be achieved through buy-back contracts for perishable commodity. He showed that it is possible to generate a continuum of coordinating contracts specified by the wholesale, and the buy-back prices. A broader review of this type of contracts is given in Larivier (1999) and Cachon (2001). In contrast to the buy-back contrasts we consider an exogenous wholesale price and develop coordinating contracts that are characterized by only the reservation terms.

3. Model Setting

As a basic building block for the analysis of capacity reservation, we consider a one-supplier, one-buyer system in a single-period setting where the supplier faces stochastic demand. The supplier and the buyer are manufacturing partners (e.g., OEM manufacturer and their key component supplier) in the supply chain who operate in a make-to-order fashion. Before placing a firm order to the supplier, the buyer has the option to reserve future capacity in advance so as to assure availability. In the case of insufficient capacity, there will be no backorder and the unmet demand will be lost. This setting is realistic in the high-tech manufacturing environment where the product life-cycle is short while the manufacturing lead-time is long. The buyer pays a "wholesale price" w to the supplier, and sells the product for p. Unique to this environment is that the buyer pre-negotiates the wholesale pricing (w) early on during the design-win phase, and does not consider pricing a subject of further negotiation. However, it is not uncommon for the suppliers to impose other forms of price adjustments in the forms of one-time charge ("engineering fees"), or variable service charge. We assume that the supplier produces with marginal cost c, and $p > w > c$.

At the beginning of the period, the supplier has the option to expand her capacity. We use a convex function $V(k)$ to characterize the capacity cost, where $V(k)$ is increasing in capacity k and $V'(0) = 0$. We assume that the residual value of capacity is concave in the capacity amount, while the cost of building capacity increases in a convex fashion. Thus, it is possible to incorporate the residual value of capacity as part of the capacity cost $V(k)$. Without loss of generality we assume that the initial capacity is zero. Product demand x follows a continuous distribution $F(x)$ when $x \geq 0$ with density $f(x)$, both are differentiable for $x \geq 0$. We also assume that $F(x)$ is invertible and $\overline{F}(x) = 1 - F(x)$. Furthermore, we assume complete information in that the supplier has full information regarding the buyer's demand distribution and revenue function, and vice versa.

First consider a centralized supply chain where the capacity decisions are made to optimize channel efficiency. In this case, the optimal capacity is defined by the classical newsvendor solution. Let $S(k)$ be the expected sale of the channel given capacity k. Given the demand distribution $S(k)$ can be written as

$$S(k) = k - \int_0^k F(x)dx$$

For any given capacity k the integrated channel profit can be written as follows:

$$\Pi_I(k) = (p - c)S(k) - V(k) \tag{1}$$

It is straightforward to verify that $\Pi_I(k)$ is concave in capacity k and the optimal capacity solves the following equality:

$$k^o = F^{-1}(\frac{p-c-v^o}{p-c})$$

where v^o is the derivative of the capacity cost evaluated at k^o. We know that the channel efficiency is maximized when the capacity is built up to k^o. However, the supplier may not have the incentive to expand her capacity to k^o. Specifically, for any given k the supplier's profit function is as follows:

$$\Pi_{\mathrm{S}}^0(k) = (w-c)S(k) - V(k) \quad (2)$$

Thus, the optimal capacity k^* for the supplier is her own newsvendor solution:

$$k^* = F^{-1}(\frac{w-c-v^*}{w-c})$$

Since $p > w$, we know that $k^* < k^o$, i.e., the supplier will not expand her capacity to the channel optimum. If the buyer does *not* take part in the capacity expansion decision, her profit is a function of the supplier's capacity choice k^*, i.e.,

$$\Pi_B^0(k^*) = (p-w)S(k^*) \quad (3)$$

In summary, since the supplier's profit margin is less than that of the integrated channel and the buyer does not share liability for the capacity, the supplier will build less capacity than what is optimal for the channel due to *double marginalization*. This leads to revenue loss in the channel. In the following sections, we propose capacity reservation contracts that could enhance coordination between the supplier and the buyer.

4. Capacity Reservation with Fully Deductible Payments

We consider capacity reservation with the following sequence of events: (1) the supplier announces a unit reservation fee $r \leq w$; for the guaranteed use of future capacity (2) given *r,* the buyer places reservation of quantity $q(r)$, paying $r \cdot q(r)$, (3) the supplier expands her capacity to k, (4) the buyer's demand x is realized, and the buyer orders x units; (5) the supplier produces min(x, k) units with marginal cost c. The supplier deducts the amount $r \cdot$ min(x, q) from the buyer's purchasing cost, but keeps the amount $r \cdot$ max(0, $q - x$).

As described above, if the buyer's realized demand x, is less than the quantity reserved q, the supplier keeps the difference $r(q-x)$, i.e., the reservation fee for unused capacity is not refundable. However, it is possible for the buyer to order more than her reservation amount, and her orders are filled based on available capacity. We first consider the case where the supplier always expand sufficient capacity to cover the reservation amount (known as *forced compliance*). We will discuss the *voluntary compliance* case in Section 6. Note that the buyer has the option of not placing a reservation, knowing that the supplier will expand capacity to her newsvendor optimum k^*, in which case the buyer's profit $\Pi_B(k^*)$ remains positive. Thus, in order to convince the buyer to place a reservation, the supplier must offer a contract where the buyer's expected profit is no worse than $\Pi_B(k^*)$.

Note that the buyer does not order more than the realized demand since we assume that the items have no value if not sold in the current period, and no inventory can be carried to the next period. This is in general true in the high-tech environment where product specifications change frequently, and there is a high risk carrying inventory as the product will be obsolete by the following period. Under this setting we can write the supplier's profit function as follows:

$$\Pi_S(k, r) = (w-c)S(k) - V(k) + rE[max\,(0, q-x)]$$
$$= (w-c)S(k) - V(k) + r\textstyle\int_0^q (q-x)f(x)\,dx$$
$$= (w-c)S(k) - V(k) + r\textstyle\int_0^q F(x)dx \quad (4)$$

Considering the constrains on k and r, the supplier's decision problem is as follows:

$$Maximize\,\Pi_S(k, r)\; s.t.,\; k \geq q,\; r \leq w$$

The profit function consists of the expected revenue from sales, the capacity cost, and the expected revenue from the buyer's "over" reservation (the portion of reservation that exceeds the realized demand). The constraint $k \geq q$ represents force compliance for the supplier. Observe that this constraint is always binding as the buyer does not need to reserve if her reservation quantity q is less than or equal to the supplier's newsvendor capacity k^*, thus, $q > k^*$. Second, observed that the first two terms of the objective function is the supplier's newsvendor model (2) and its first order optimality condition is satisfied at k^*. Since $\Pi_S(k, r)$ is concave in k, each additional capacity beyond $q\,(q > k^*)$ will return a negative margin. As a result, the supplier would have no incentive to build capacity beyond the reservation amount. Now consider the second constraint, if $r > w$ the buyer would order the exact quantity she has reserved regardless of the realized demand so as to avoid the over reservation charge. Therefore, there is no incentive for the supplier to set r greater than w, i.e., the third term in her profit function Π_S would be zero, which is equivalent to the no reservation case. From the above analysis, and the fact that $q \equiv q(r)$, we may rewrite the supplier's decision problem as follows:

$$Maximize\,\Pi_S(r) = (w-c)S(q(r)) - V(q(r)) + r\int_0^{q(r)} F(x)dx,\; s.t.\;\; r \leq w \quad (5)$$

This implies that the supplier's capacity expansion decision is solely determined by the unit reservation fee r. In other words, by setting the reservation fee, the supplier indirectly chooses her capacity as well. Given the announced fee r, the buyer's decision problem is as follows:

$$Maximize\,\Pi_B(q) = (p-w)S(q) - r\int_0^{q} F(x)dx \quad (6)$$

The first term is the buyer's expected profit through sales, and the second term is the expected loss for unused reserved capacity. As $\Pi_B(q)$ is concave in q, the optimal reservation quantity is

$$q(r) = F^{-1}\left(\frac{p-w}{p-w+r}\right) \tag{7}$$

Note that both the reservation quantity and the buyer's expected profit decrease in r. Further, since the buyer has no incentives to reserve if her optimal reservation quantity *q(r)* is less than or equal to the supplier's new vendor capacity k^*, the following inequality should hold:

$$r \leq (p-w)\frac{v^*}{w-c-v^*} \tag{8}$$

Thus, if the supplier announces a capacity reservation fee *r* greater than the right-hand-side in (8), the buyer's best response is to reserve less than k^*, or equivalently, not reserving at all (comparing (6) and (3), it is clear that the buyer is worse off if she reserves an amount $q \leq k^*$). Hence, (8) establishes an upper bound for the reservation fee.

4.1. Individual Rationality for the Buyer and the Supplier

Of fundamental importance to contract design is to determine if the buyer and the supplier are both better off under the contract. In the following, we will examine the incentives of both parties. First consider the buyer's perspective: the main question here is "under what condition is the buyer better off making a reservation?" First, we know that the buyer's expected sales must exceed the supplier's newsvendor capacity k^*, otherwise there is no reason to reserve. To tighten this criterion further we may define a *threshold reservation quantity* q^t (where $q^t > k^*$) such that $\Pi_B(q^t) \geq \Pi_B^0(k^*)$. This *threshold quantity* is useful for the buyer in that she can negotiate a reservation fee that is lower than the *threshold reservation fee* r^t that corresponds to q^t. This is stated in the following Theorem.

Theorem 1. *It is individually rational for the buyer to reserve capacity if her optimal reservation quantity is no less than the threshold* q^t*, where* q^t *is the unique non-negative value satisfying*

$$S(k^*) = \frac{\int_0^{q^t} xf(x)dx}{F(q^t)} \tag{9}$$

The threshold reservation quantity q^t *has the following properties:*

(1) it provides a tight lower bound for the reservation quantity , i.e., $q^t > k^*$*,*

(2) it defines the threshold reservation fee r^t *(under which the buyer shall be willing to pay):*

$$r^t = (p-w)\frac{\overline{F}(q^t)}{F(q^t)} \tag{10}$$

(3) r^t *provides a tight upper bound for the reservation fee, i.e.,* $r^t < (p-w)\frac{v^*}{w-c-v^*}$.

A formal proof for the Theorem is given in the Appendix. The Theorem defines a tighter condition under which the buyer would have incentive to place capacity reservation. The Theorem requires that the buyer's expected sales *with* reservation should be no less than her expected sales *without* reservation (in the latter case the supplier offers her newsvendor capacity k^*). Thus, from the buyer's point of view, the expected increase in sales revenue must offset the expected liability from capacity reservation. Unfortunately, there is no closed form expression we can derive for the threshold value q^t. Nonetheless, since the right-hand-side of (9) is strictly increasing in q, for a given k^*, q^t can be found by a simple line search .

We now consider the *supplier's* perspective. More specifically, we would like to know if the supplier is better off accepting the buyer's *threshold reservation fee* r^t, or not offering reservation at all? As shown above, the buyer would only reserve if doing so improves her profit. However, unless the reservation induces additional surplus in the *channel*, the buyer's added profit would be the supplier's loss. This implies that for the contract to be acceptable for both, it must generate additional surplus in the channel. The following Theorem states this condition in terms of the channel profit Π_I.

Theorem 2. *Given the buyer's threshold reservation quantity* q^t*, the following are true:*
(1) the supplier should only accept the reservation if

$$\Pi_I(q^t) \geq \Pi_I(k^*) \tag{11}$$

(2) there exists a capacity level $\overline{k}$ *such that* $\overline{k} > k^0$ and $\Pi_I(k^*) = \Pi_I(\overline{k})$*, and*
(3) $k^* < q^t \leq \overline{k}$.

The first part of the Theorem is a straightforward statement from the previous observation. Part (2) can be concluded from the fact that $\Pi_I(q)$ is strictly concave in q, and Part (3) follows from (11). To analyze the supplier's incentives further, we will rewrite her profit as a function of q by substituting the reservation fee *r* with the threshold value according to (10):

$$\Pi_S(q) = (w-c)S(q) - V(q) + (p-w)\frac{\overline{F}(q)}{F(q)}\int_0^q F(x)dx \tag{12}$$

Suppose $q^w \equiv q(w)$ is the quantity the buyer is willing to reserve when the reservation fee is set at the wholesale price w. Thus, the supplier's decision problem given (12) is as follows:

$$Maximize \;\; \Pi_S(q), \; s.t., \; q \geq \text{Max}\,(q^t, q^w) \tag{13}$$

The stated constraint ensures that the supplier will choose a capacity level that is acceptable to the buyer, and the supplier cannot charge a reservation fee above the wholesale price w. We may interpret the supplier's decision as follows: knowing the highest reservation fee that the buyer is willing to pay given the demand information (10), the supplier is to choose a *capacity level* q^r that would maximize her profit. Thus, it is individually rational for the supplier to accept the buyer's reservation (knowing the threshold q^t) if the profit resulting from the optimal capacity level q^r (13) is no worse than her newsvendor profit (i.e., $\Pi_S(q^r) \geq \Pi_S^0(k^*)$). Since $q^r \geq$ Max (q^t, q^w), the resulting reservation fee will indeed fall below the buyer's *threshold* r^t.

Recall that in the environment we consider, the wholesale price w and the buyer's revenue margin p are both exogenous. Nonetheless, it is useful to know that given the pre-negotiated w, the (supplier's) individual rationality condition in (11) suggests a *threshold revenue margin* p^t*;* only when the buyer's margin is at or above this threshold would it make sense for the two parties to enter the reservation contract. This is stated in the following Theorem:

Theorem 3. *It is individually rational for the supplier and the buyer to enter the reservation contract provided that the buyer revenue margin p is no less than the threshold* p^t *as follows:*

$$p^t = c + \frac{V(q^t) - V(k^*)}{S(q^t) - S(k^*)} \tag{14}$$

From (11), it is straightforward to verify that only when the revenue margin $p \geq p^t$ would there be sufficient surplus generated in the channel to benefit both players. In essence, p^t is the *effective marginal cost* of expanding capacity from k^* to q^t. As would be intuitive, p^t is increasing in the marginal cost for capacity expansion (i.e., $V(q^t) - V(k^*)$), and decreasing in expected additional sales (i.e., $S(q^t) - S(k^*)$). To answer the earlier question if "the supplier is better off accepting the buyer's *threshold reservation fee* r^t, or not offering reservation at all?" we may conclude that the supplier's decision should be based on the potential surplus that could be generated from a coordinated channel, i.e., $\Pi_I(q^t) - \Pi_I(k^*)$. Only when the buyer's profit margin is sufficiently high would she be able to (adequately) offset the supplier's risk of expanding beyond her newsvendor capacity. In reality, the buyer may not have a sufficiently lucrative margin to support such a win-win outcome, in which case the supplier should reject the reservation offer and stick with her newsvendor expansion. With this understanding, we continue our analysis on the supplier's optimal strategy.

A closer inspection of (12) reveals that the supplier's profit function may have multiple local maxima, that is, if one exists at all. The shape of the profit function $\Pi_S(q)$ is primarily driven by the demand distribution. To derive some insights, we focus our analysis on increasing failure rate (IFR) distributions, in which case the first order condition for $\Pi_S(q)$ has a unique solution. For any IFR distribution, the ratio $f(x)/\overline{F}(x)$ (known as the *failure rate*) increases in x. IFR distributions represent a rather general class of which many widely used distributions (e.g., Uniform, Weibull, Gamma and Normal) belong. The following theorem shows that the supplier's profit function $\Pi_S(q)$ is well behaved if the demand follows any IFR distributions. For notational convenience, we denote $q^m \equiv Max\ (q^t, q^w)$.

Theorem 4. *The supplier's decision problem specified in (13) has the following properties:*
(1) The profit function $\Pi_S(q)$ *is strictly decreasing in* $[k^o, \infty)$,
(2) If the demand distribution is IFR, then $\Pi_S(q)$ *is either* ***decreasing*** *or* ***unimodal*** *on* $[q^m, k^o)$, *i.e., the decision problem has an unique optimal solution.*
(3) To find the unique optimal reservation quantity, q^r, *we have*

(i) If $\Pi_S(q)$ *is* ***decreasing*** *in* $[q^m, k^o)$, *then* $q^r = q^m$, *and*

(ii) if $\Pi_S(q)$ *is* ***unimodal*** *in* $[q^m, k^o)$, *then* q^r *is the unique point in* $[q^m, k^o)$ *that satisfies the first order optimality condition for* $\Pi_S(q)$.

A formal proof of the theorem is given in the Appendix. As pointed out in the theorem, should both the supplier and the buyer choose to initiate a reservation (i.e., $q^t < \overline{k}$) the supplier's best action is to set the reservation price at $r(q^r)$, and the buyer will respond by reserving q^r. Recall that for technical convenience, we rewrote the supplier's profit as a function of q (by substituting the reservation fee *r)*. To provide some practical insights for the Theorem, we convert the supplier's decision back as a function of the reservation fee *r*. Figures 1a-1c depict a sequence of three scenarios faced by the supplier as the buyer's revenue margin *p* decreases: (1) the supplier's optimal reservation fee r^* is strictly less than the threshold reservation fee. This occurs when the buyer's revenue margin *p* is sufficiently large. This is the best-case scenario for the supplier and the buyer where they both benefit. (2) the supplier's optimal reservation fee r^* coincide with r^t. At r^t the buyer is indifferent between reserving or not reserving, (3) the supplier's optimal strategy is to reject the reservation. This is the case when the buyer's margin is too thin to offset the supplier's risk for expansion.

4.2. The Effects of Market Size and Demand Variability

The above results only require that the demand distribution is IFR. If additional information is known about the market that drives the demand, we may be able to derive additional insights. First, consider a demand distributions where given a market size parameter θ there exists an increasing function $\tau(\theta)$ such that $F(x|\theta) = F(\frac{x}{\tau(\theta)}|1)$. The demand distribution is said to be in the *scaled family*, e.g., Uniform [0,θ], Exponential, Weibull and Gamma are in this family. Similarly, we define distribution from the *shifted family* as $F(x|\theta) = F(x-\theta|0)$. Normal distribution. is known to be in the *shifted family.* The following Theorem specifies additional properties when the demand distribution is in the *shifted family.*

Theorem 5. *If the demand distribution $F(x|\theta)$ is IFR and is in the shifted family, then*
(1) The supplier's optimal reservation price is $r^* = min(r^t, w)$.
(2) If the capacity cost v is linear then neither the optimal reservation fee, r^* nor the supplier's surplus ($\Pi_S(q^r) - \Pi_S^0(k^*)$) are dependent on the market size θ, however, the threshold reservation quantity q^t does increase in θ.
(3) If the capacity cost $V(k)$ is strictly convex, then the threshold reservation fee r^t is increasing in market size θ. Furthermore, there exists a unique market size threshold, θ^t, above which the capacity reservation is no longer favorable for the channel.

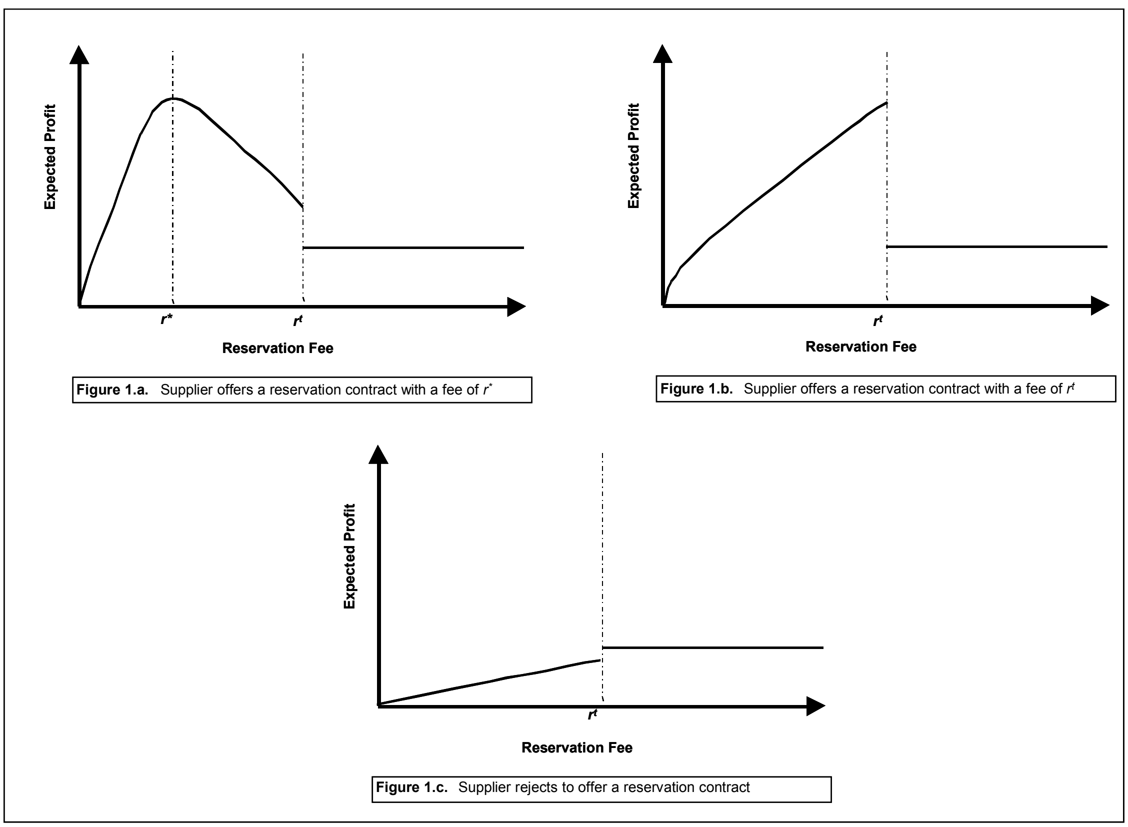

Figure 1.a. Supplier offers a reservation contract with a fee of r^*

Figure 1.b. Supplier offers a reservation contract with a fee of r^t

Figure 1.c. Supplier rejects to offer a reservation contract

Figure 1. Possible outcomes of supplier's expected profit function

A formal proof of Theorem 5 is given in the Appendix. The first part of the Theorem states that if the demand distribution is in the *shifted family,* the supplier's optimal reservation price should be set as $min(r^t, w)$, this follows the same intuition as Theorem 4, *Part (3)-(i)*. In the second part of the Theorem, we see that if the cost for capacity expansion is linear, the supplier has incentive to increase her capacity to match the growing market size, resulting in a higher threshold reservation quantity. Interesting, in this case the increase in market size neither affects the optimal reservation fee, nor the supplier's surplus (for accepting reservation). This is because for any demand distribution of *shifted family* the coefficient of variation decreases as the market size (the mean) increases, which in turn reduces the supplier's risk exposure. In essence, the increase in the capacity cost due to market size is balanced with the reduction in risk. However, this does not suggest that the profits are independent of the market size. In fact, the supplier and the buyer's profits increase with a factor of $(w-c-v)$ and $(p-w)(w-c-v)/(w-c)$ with the market size, respectively. As a result while the profits increase with the market size, the value of reservation remain the same. One may thus conclude if the capacity cost is linear, as the market size increases, the merit of reservations does not change in an absolute sense, but its relative benefit diminishes. Contrary to this observation, Part (3) of the Theorem shows that if the capacity cost is *convex*, an increased market size indeed affects the reservation fee and the supplier's surplus. This is rather intuitive since the supplier needs more assurance from the buyer when the capacity cost demonstrates diseconomy of scale. In this case, the decrease in the coefficient of variation due to market size is not sufficient to offset the risk of expansion. We explore the effect of capacity cost further in the following Theorem.

Theorem 6. *If the demand is Normally distributed, then the threshold reservation fee r^t increases in standard deviation. If the capacity is costly, i.e., $V'(k^*) \geq 2(w-c)$, then the value of capacity reservation (the channel's surplus) increases in standard deviation.*

The above two theorems reveal that *demand variability* and the *cost structure of capacity* are two main factors determine the value of capacity reservation. Clearly, as the variability increases the supplier is exposed to more risk, and would be reluctant to build more capacity unless the buyer raises her threshold reservation fee. When the capacity cost is high relative to the supplier's profit margin, it is better for her (and for the channel) to take reservation. This is consistent with the intuition that the higher the demand variability, the more appealing it is to take capacity reservation. Under the setting of a buyer-lead channel with *linear* capacity cost, Cachon and Larivier (2001) also observe that market size does not affect the pricing decision. We make the additional observations that (1) it is the demand variability (standard deviation) that affects the reservation fee (Theorem 6), (2) the reservation *quantity* (thus the total profit) does increase with market size, and (3) when the capacity cost is *convex*, the reservation fee should indeed increase with market size. In the following Theorem, we explore the case when the demand distribution is in the *scaled family*.

Theorem 7. *Suppose the demand distribution is IFR and is in the scaled family, and the capacity cost is linear, then*
(1) the threshold reservation fee r^t *increases in market size* θ.
(2) if taking reservation creates positive surplus for the channel under market size θ*, then the surplus increases as* θ *increases.*

For demand distributions in the *scaled family* the coefficient of variation does not change with the market size scaled by $\tau(\theta)$. This implies that the *demand variability* (as characterized by the standard deviation) increases with the market size. Thus, from the above analysis, it should follow that the threshold reservation fee increases with the market size (i.e., it is the demand variability that affects the reservation fee). Note that in this case even with *linear* capacity cost, the increase in total capacity cost (due to market size) is *not* balanced by the reduction in risk.

5. The Coordination Mechanisms

Our analysis so far reveals that capacity reservation could provide real benefit for the buyer, the supplier, and the supply channel. This is especially the case when the capacity is costly, and market is volatile, both are characteristics of the high-tech industry. The contract we have considered allows the buyer to *fully deduct* the reservation fee from the final payment. Under this setting, the channel profit generated from capacity reservation may not be optimal $q^t = k^0$. For later reference, we state this more formally as follows:

Remark. *In a capacity reservation contract with fully deductible reservation fee, the channel profit is suboptimal unless the threshold reservation quantity* $q^t = k^0$*, i.e.,* $\Pi_I(q^t) < \Pi_I(k^0)$.

A useful insight from the remark is that if we restrict ourselves to capacity reservation with *fully deductible* reservation fee, channel coordination (achieving maximal channel profit) is only possible under a very restrictive case. Similarly, it can be shown that if the contract has non-deductible reservation fee, the channel would be equally difficult to coordinate. Precisely for this reason, in the buy-back contract literature (c.f.., Larivier, 1999), channel coordination is achieved by adjusting the buy-back price and the wholesale price. In the following, we propose two different types of contracts to coordinate the channel.

5.1. Capacity Reservation with Partially Deductible Payments

We first propose a *partially deductible reservation* contract where only a portion of the reservation fee *r* paid by the buyer is deductible from the final payment, where the deductible portion r_2 is pre-defined as a contract parameter. The reservation contract has the following sequence of events:

(1) the supplier announces the reservation fee r for each unit of capacity reserved, and the refund rate r_2 for each unit of capacity actually utilized, where

$$r_2 = \frac{p-c}{v^o} r - (p-w) \tag{15}$$

(Recall from Section 3 that v^o is the marginal capacity cost evaluated at k^o.)

(2) given *r* and r_2, the buyer places reservation of quantity q, paying $r \cdot q$,

(3) the supplier expands her capacity to q,

(4) the buyer's demand x is realized, and the buyer orders x units;

(5) the supplier produces min$(x,\ k)$ units with marginal cost c. The supplier deducts the amount $r_2 \cdot$ min$(x,\ q)$ from the buyer's purchasing cost.

With the added dimension (r_2) in partially deductible contract, the supplier has more flexibility in setting the reservation fee (*r*). In order for the contact (r, r_2) to be implementable, it must satisfy $r > r_2 > 0$, from which we may derive the range for the reservation fee:

$$(p-w)\tfrac{v^o}{p-c} < r < (p-w)\tfrac{v^o}{p-c-v^o} \tag{16}$$

The following theorem shows that the *partially deductible* contract could successfully coordinate the channel.

Theorem 8. *Under the partially deductible contract specified by (15) and (16), the buyer will reserve the channel optimal capacity, i.e., q(r)* $= k^o$. *Moreover,*

(1) The buyer's profit is increasing in r*,*

(2) The supplier's profit is decreasing in r*,*

(3) The supplier would only offer such contract if $v^o \leq 2w - c$

While the *partially deductible* contract allows a range of possible reservation fees, the actual fee corresponds to a specific split of the channel profit between the supplier and buyer. The Theorem shows that as the reservation fee decreases the supplier's share rises. This may seem counter intuitive at first, but a quick reflection would reveal that this is exactly to be expected: each unit of increase in the reservation fee *r* corresponds to a higher rate of increase in the refund r_2 as $\partial r_2 / \partial r = (p - c)/v^o > 1$ since $p > v^o + c$. Thus, the penalty charged for the buyer's unused reservation, $r - r_2$, actually decreases in r. This observation is consistent with that of Pasternack (1985) under the return policies. The Theorem also shows that the *partially deductible* contract is only feasible (for the supplier) if $v^o \leq 2w - c$.

Note that the above discussion is only relevant when the capacity cost is strictly convex. A majority of the work in the literature assumes linear capacity cost. With linear cost, v, the supplier only stays profitable if w is greater than $v + c$. In such case it is trivial to justify that the buyer and supplier's split (of channel profit) should be $r/v \cdot \Pi_I^o$ and $(v - r)/v \cdot \Pi_I^o$, respectively. In this case, both parties will receive positive profit for any r within the range defined by (16). Linear capacity cost also allows the design of coordination contracts independent of the buyer's demand distribution. Thus the contract designed for one buyer can be used for any other buyer with a similar cost structure (but possibly with a different demand pattern). However, with convex capacity cost, computing v^o independent of the demand distribution would be unacceptable. The contract would be useful in the situation that often arise in practice where the supplier does not have complete information regarding the buyer's demand distribution, but is aware that (1) the distribution is from a scaled or shifted family, and (2) all parameters of the distribution, except for the scale parameter θ, are known. In other words, the demand pattern is known (e.g., from historic data), but the market size is not. In this case, the supplier may request a forecast from the buyer as a means to determining the market size. Without a coordination contract, the buyer has the incentive to understate the market size (Theorem 5) as an attempt to lower the reservation fee. Essentially, the *partially deductible reservation* contract changes the buyer's incentive by including a penalty for misrepresenting the true market size.

The *partially deductible* contract demands accountability on capacity reservation not only from the buyer but also from the supplier, which leads to higher profits for the channel. However, if one is to broaden the consideration in a practical setting, the coordination contract may not always work. For instance, the buyer may have outside options (other suppliers) that is more attractive then the coordinating contracts currently offered by the supplier. In this case, the supplier may need to increase r beyond the interval given in (16). This results in full deduction *and* a discount for the buyer, since $r_2 > r$. This may be viewed as a price correction mechanism as the supplier's pricing may be less than competitive in the first place. Similarly, if the supplier's capacity is in high demand, she may wish to decrease r below $(p - w)v^o/(p - c)$ since the foregoing contract coordinates the channel with regard to a particular buyer, but is suboptimal for the supplier in general. This corresponds to a no-deduction policy with additional fee linear to the expected sale, since r_2 would be negative. The additional charge, r_2, can be interpreted as the "engineering fee" exercised in the semiconductor industry. This, again, may be viewed as a price correction mechanism as the supplier's product may be under priced in the first place.

5.2. Coordination via Cost-Sharing Contracts

We now consider a *cost-sharing contract* where the buyer pays for a portion of the *capacity cost* associated with her reservation. Depending on her demand realization, the buyer either receives a refund, or makes an additional payment for the utilized capacity. The *cost-sharing contract* is similar in concept to the *option contracts* where the buyer offers the supplier a contract consisting of firm orders and options. Option contracts can be reduced to *buy-back contracts* (Pasternack 1985) and *profit-sharing contracts* (Cachon and Larivier 2000). In the following we will describe how the contract coordinates the channel in the particular setting of high-tech capacity reservation. The contracts has the following sequence of events:

(1) the supplier announces a reservation charge $\alpha V(q)$, specified by parameter α, and the refund rate r_2 for each unit of capacity actually utilized, where

$$r_2 = \alpha(p-c) - (p-w), \ 0 \leq \alpha \leq 1 \tag{17}$$

(2) given α and r_2, the buyer places reservation of quantity q, paying $\alpha V(q)$,

(3) the supplier expands her capacity to q,

(4) the buyer's demand x is realized, and the buyer orders x units;

(5) the supplier produces min(x, q) units with marginal cost c. The supplier deducts the amount $r_2 \cdot$ min(x, q) from the buyer's purchasing cost.

Under this contract setting the buyer's profit is as follows:

$$\Pi_B(\alpha, r_2) = (p - w + r_2)S(q) - \alpha V(q)$$

whereas the supplier's profit is

$$\Pi_S(q) = (w - c - r_2)S(q) - (1-\alpha)V(q)$$

The following theorem shows that channel coordination can be achieved under the contract.

Theorem 9. *Suppose the supplier offers a cost-sharing contract* (α,$r_2(\alpha)$) *with* r_2 *specified* by (17), *then the buyer will reserve the channel optimal capacity, i.e.,* $q = k^o$*, furthermore,*

(1) The buyer's profit is increasing in α*, and*

(2) The supplier's profit is decreasing in α*.*

To prove the above theorem, first observe that under the *cost-sharing contract* the critical fractile of the buyer is equal to that of the integrated channel. Hence, the buyer would indeed reserve the system-optimal capacity. Further, the derivative of the buyer's profit with respect to r_2 is $\Pi_I^o/(p-c)$ which is strictly positive, implying that buyer's profit is increasing in r_2. Thus, the supplier's profit is decreasing in r_2. We may conclude *(1)* and *(2)* from the fact that r_2 is increasing in α in (17).

This theorem depicts a continuum of contracts where both the buyer's profit, $\alpha\Pi_I^o$, and the supplier's profit, $(1-\alpha)\Pi_I^o$, are determined by α. Note that depending on α (i.e., for $\alpha < \frac{p-w}{p-c}$), r_2 may assume negative values, implying that the buyer has the option of taking less up-front responsibility in capacity investment (smaller α), but shares her revenue with the supplier (instead of receiving a refund), as in the *profit sharing* contract. One may also interpret this case as a variant of the option contract where the buyer pays $\alpha V(q)$ to purchase the option of utilizing q units of capacity, then pay r_2 for each unit of option exercised. Observe that this coordination contract is independent of the demand distribution and can be employed for different buyers. If we take for granted the contract will be accepted by both parties then the supplier does not need knowledge of the demand distribution when preparing the contract. As this is true for convex capacity cost in general, the *cost-sharing* contract is more robust then the *partially-deductible* contract. Moreover, as a contract parameter, α offers more flexibility for the buyer and supplier to split the channel profit. However, it should be noted that α must be greater than the ratio $\Pi_B^0(k^*)/\Pi_I(k^o)$ so that the buyer does not opt out by not reserving. In general, if the buyer has an outside option, α should be selected to counter the competition. Comparing to the *partially-deductible* contracts, a main drawback of the *cost-sharing* contracts is that the supplier must transfer her capacity cost information (i.e., $V(q)$) to the buyer.

6. Discussion and Additional Analysis

Throughout the paper, the capacity reservation contracts are considered under two main assumptions: (1) *forced compliance:* once the supplier enters the reservation contract, she is committed to provide the reserved capacity to the buyer regardless of market conditions, and (2) *complete information:* the supplier and the buyer are partners who share the demand information. Both assumptions are reasonable in the context of high-tech capacity reservation: it is indeed the industry practice to adhere to contracted reservations, and the OEM manufacturer indeed share their demand information with their suppliers (e.g., Cisco's e-Hub). Nonetheless, we are interested to know if the contract properties would change significantly when these assumptions are altered. This will be the subject of the following discussion.

6.1. Voluntary Compliance

There are situations in practice where the supplier may not be able (or willing) to provide the capacity reserved. For instance, due to short-term demand surge, it may not be possible for the supplier to provide the committed amount at the right time. Moreover, in some cases the supplier may choose to "overbook" (i.e., under expand) so as to ensure high utilization of her capacity. In specific, we consider the "voluntary compliance" case (the term is introduced by Cachon and Lariviere, 2001) where the supplier may choose to expand her capacity below the committed amount, but is subject to non-compliance penalties. To streamline the analysis, we will focus on the capacity reservation contract with *fully deductible* payments (Section 4).

Suppose the supplier has to pay a non-compliance penalty u for each reserved capacity unit that she fails to provide. Knowing that the supplier's capacity expansion may be less than q, the buyer can no longer make *her* reservation decision based on revenue p and wholesale price w as before (specified in (7)), but will have to base her decision on the reservation fee r and the non-compliance penalty u. More specifically, for the buyer's reservation quantity, q, for given fees, r and u, we replace (7) with the following:

$$q(r,u) = F^{-1}(\frac{u}{u+r})$$

Note that if $u < p - w$, the buyer will reserve less than she otherwise would under the force compliance case. On the other hand, the supplier's expected profit is as follows:

$$\begin{aligned}\Pi_S(\,k, q|k \leq q) &= (w-c)S(k) - V(k) + u\frac{\overline{F}(q)}{F(q)}\int_0^q F(x)dx \\ &- u\int_k^q (x-k)f(x)dx - u\overline{F}(q)(q-k)\end{aligned} \tag{18}$$

Observe that the first derivative of (15) with respect to q is strictly negative, implying that the supplier's profit is decreasing in q. This suggests that in fact $k = q$ is the dominant strategy for the supplier, i.e., the supplier would behave no differently from the *forced compliance* case. Interestingly, offering the supplier *voluntary compliance* does not change her capacity expansion strategy, but at the same time the buyer may reserve less. In order to prevent this situation, the supplier will need to assure the buyer that she will build enough capacity to cover the reservation. This could be accomplished, for instance, by offering a penalty rate no less than the buyer's margin.

Remark *If the supplier offers a voluntary compliance contract where* $u \geq (p-w)$*, then the buyer's dominant strategy is to reserve while the reservation quantity would be no different from the forced compliance case*.

This simply states that the supplier will fully compensate the buyer for her lost sales associated with the reserved (but not the delivered) capacity. The remark is straightforward to confirm by observing that if $u \geq (p-w)$, the first derivative of (18) with respect to q is strictly negative, either the buyer makes her decision based on *u,* or on $p-w$. In other words, the supplier's expected profit is decreasing in q anyways. Thus, the buyer can be assured that the supplier will voluntarily comply with the reservation contract. On the other hand, the buyer would not benefit from over reserving as the supplier will build her capacity to the reserved amount, therefore no penalty would be charged. Thus the buyer should determine her reservation quantity based on her own margin $(p-w)$, as in the forced compliance case.

6.2 Contract Manufacturer with Two Demand Types

A trend emerging in high-tech manufacturing is for the OEM manufacturer to outsource their operations to contract manufacturers (CMs). In this case the supplier has to take capacity reservation from the contract manufacturer(s) rather than the OEM customer. Typically the demand is split among more than one contract manufacturers, but the actual split is unknown to the supplier until later. This creates a situation where the market demand information is known but the actual demand size for a particular buyer (CM) is not. We will derive some insights for this situation as a simple extension to our base model: the supplier knows that the buyer's demand distribution is from the scaled or shifted family, but is uncertain about the scale parameter. The supplier is given the information that the buyer's demand size can be either high (as specified by scale parameter θ_H), or low (θ_L), $\theta_H > \theta_L$. Since the supplier is the leader of the channel, she must determine the reservation fee by guessing the buyer's demand type. We will focus our analysis in the context of *partially deductible reservation contract* (see Section 5.1).

Suppose the supplier offers a *partially deductible* contract with two options, namely $C_H = (r_H^1, r_H^2)$ and $C_L = (r_L^1, r_L^2)$, where r^1 and r^2 are the reservation fee and the deductible portion of the reservation fee, respectively. Similar to (15), the deductible portion of the fee is specified as follows:

$$r_i^2 = r_i^1 \frac{p-c}{v_i^o} - (p-w), \textit{for } \; i = H, L$$

Denote q_{ij} the best response (reservation quantity) for buyer type $i\,(i = H, L)$ in reaction to contract option C_j $(j = H, L)$, e.g., q_{HL} denotes the optimal reservation quantity of a type H buyer for (r_L^1, r_L^2). Based on this definition $q_{HH} = k_H^o$ (and $q_{LL} = k_L^o$), for type H (L) buyer, contract option C_H (C_L) coordinates the system, as implied by the results from *partially deductible contracts*.

Remark. *Type H and Type L buyers will choose contract option C_H and C_L, respectively, if*

$$r_H^1\left(\frac{p-c}{v_H^o}S(q_{HH}) - q_{HH}\right) \geq r_L^1\left(\frac{p-c}{v_L^o}S(q_{HL}) - q_{HL}\right) \tag{19}$$

and

$$r_H^1\left(\frac{p-c}{v_H^o}S(q_{LH})-q_{LH}\right)\leq r_L^1\left(\frac{p-c}{v_L^o}S(q_{LL})-q_{LL}\right) \tag{20}$$

Note that the left- and right-hand sides of inequalities (19) correspond to the optimal expected profits for type H buyer under contract options, C_H and C_L, respectively. Similarly, in (20), the two sides of the inequality correspond to the optimal expected profit for type L buyer under contract options C_H and C_L, respectively. Obviously, the statement concerning contract choices holds if both inequalities hold. In the following we will show that it is always possible for the supplier to devise contract options C_H and C_L such that (19) and (20) hold simultaneously. To this end, it is sufficient to show that if the supplier chooses r_H^1 and r_L^1 to satisfy the *equality* constraint in (20), the same $r_H^1,\ r_L^1$ pair will always satisfy (19). First, for notational convenience, we replace the terms within the parenthesis as π_{ij}, thus

$$\pi_{ij}=\frac{p-c}{v_j^o}S(q_{ij})-q_{ij}, \text{ for } i=H,L \text{ and } j=H,L$$

e.g., π_{HH} represents the term within the parenthesis of the left hand side in (19).

Suppose that the supplier chooses reservation fees r_H^1 and r_L^1 such that

$$r_L^1=r_H^1\frac{\pi_{LH}}{\pi_{LL}}$$

Observe that with the same $r_H^1,\ r_L^1$ pair, inequality (19) holds if the following holds:

$$\frac{\pi_{LH}}{\pi_{LL}}\leq\frac{\pi_{HH}}{\pi_{HL}} \tag{21}$$

To see that (21) holds it is sufficient to show that π_{iH}/π_{iL} increases in the scale parameter θ. Define $D_i(\theta)$ as follows:

$$D_i(\theta)=S_\theta(q_{iH})\pi_{iL}-S_\theta(q_{iL})\pi_{iH}$$

where $S_\theta(.)$ denotes the partial derivative of $S(.)$ with respect to the scale parameter θ. Hence, from the envelop theorem the partial derivative of π_{iH}/π_{iL} with respect to θ is $D_i(\theta)/\pi_{iL}^2$. Note that at optimality $\overline{F}(q_{iL})=v_L^o/(p-c)$ and $\overline{F}(q_{iH})=v_H^o/(p-c)$. Thus

$$\pi_{ij}=\frac{\int_0^{q_{ij}}xf_i(x)dx}{\overline{F}_i(q_{ij})}\ i=H,L;j=H,L \tag{22}$$

where f_i and $\overline{F}_i$ denote the density and tail distributions for the demand of type i buyer. We now consider the cases when the demand distribution belongs to the shifted vs. the scaled families, and show that the buyer's incentive differs in these cases. First, consider the shifted distribution case. From (22) and the envelop theorem, we have

$$\frac{D_i(\theta)}{\pi_{iL}^2}=\frac{F_i(q_{iH})\int_0^{q_{iL}}xf_i(x)dx-F_i(q_{iL})\int_0^{q_{iH}}xf_i(x)dx}{\overline{F}_i(q_{iH})\overline{F}_i(q_{iL})\pi_{iL}^2} \tag{23}$$

From the fact that $\int_0^q xf(x)dx/F(q)$ increases in q and $q_{iL}>q_{iH}$, the RHS of (23) will return a strictly positive value, implying that π_{iH}/π_{iL} increases in θ. Hence, (19) holds with strict inequality, i.e., if the equality in (20) holds, a strict inequality holds in (19). Thus, the buyer has the incentive to choose the contract option designed for her type.

Now, consider the case of scaled distributions. Again from (22) and the envelop theorem, $D_i(\theta)/\pi_{iL}^2$=0, implying that equality holds for (19) as well as (20). Thus, the buyer is indifferent between the two contract options. It remains true that the supplier may devise contract options C_H and C_L such that (19) and (20) hold simultaneously, thus convince the buyer to choose the option that optimize the channel profit.

7. Conclusions

This paper examines capacity reservation contracts in the context of high-tech manufacturing. Specifically, we propose reservation contracts with *deductible reservation fees:* the supplier announces a fee for capacity to be reserved for a certain time in the future. The buyer places her reservation for capacity based on the reservation fee. Later, the supplier determines how much capacity to build. The reservation contracts are designed for short-life-cylce, make-to-order products under stochastic demand. In the following, we summarize the main conclusions of the paper.

When is Capacity Reservation Beneficial? Without the presence of capacity reservation, the supplier would simply determine her capacity based on her knowledge of the demand, i.e., she would find the optimal capacity level based on a newsvendor-type decision model. We show that the supplier and the buyer could benefit from early commitment contracts under a specific set of conditions. First of all, the buyer's expected order size should be larger than the supplier's newsvendor capacity. Second, the buyer should be able to negotiate a threshold reservation fee (Theorem 1) below which her expected profit justifies her added liability. We also identify the conditions when it is beneficial for the supplier to accept the buyer's threshold offer (Theorem 2). The basic insight here is that reservation is beneficial when the buyer's revenue margin is sufficiently high (beyond a certain threshold value, as specified in Theorem 3). We show that as the buyer's revenue margin decreases, the supplier will face a sequence of three scenarios with decreasing level of attractiveness (Figure 1 and Theorem 4).

The Effects of Market Size and Demand Variability. How does the benefit of capacity reservation vary by the market size and the demand variability (as characterized by standard deviation)? To tackle this question we first examine demand distributions in the shifted families (e.g., Normal). Our analysis shows that the assumption on capacity cost (linear vs. convex) has a significant impact to the answer (Theorem 5). If the capacity cost is linear (e.g., outsourcing) neither the reservation fee nor the supplier's surplus are dependent on the market size. Intuitively, the increase in the capacity cost due to market size is balanced with the reduction in risk. However, if the capacity cost is convex (e.g., physical expansions), the reduction in risk is not sufficient to justify the increase in capacity cost. Thus, the threshold reservation fee is increasing in market size. It should not be surprising that there exists a market-size threshold, above which capacity reservation (as defined in this context) is no longer favorable. While the linear cost results are consistent with what is known in the literature, we make the additional observations that (1) it is the demand variability that affects the reservation fee (Theorem 6), (2) the reservation *quantity* (thus the channel profit) does increase with market size, and (3) the convex capacity cost assumption described above. Some aspects of these insights can be generalized to demand distributions in the scaled family (Theorem 7). As the market size increases, the increase in capacity costs (even in the *linear* case) is not balanced with the reduction in risk, thus the supplier must raise the reservation fee.

Designing Contracts to Improve Supply Chain Efficiency. The next research question is the design of contract mechanisms that would realize the potential benefits of capacity reservation. We first show that, except for a special case, contracts with *fully deductible* or *non-deductible* fees both generate surplus that is suboptimal (for the channel). We propose two coordination contracts: first, the *partially deductible reservation* where a pre-specified portion of the reservation fee is deductible from the final payment. The deductible fee (refund) is a derived function of the reservation fee to achieve channel coordination (Theorem 8). We show that *partially deductible* contract allows a range of possible reservation fees, each corresponding to a split of the channel surplus between the supplier and buyer. An interesting finding here is that the supplier's share actually *decreases* in the reservation fee. This is due to fact that each unit of increase in the reservation fee corresponds to a higher rate of increase in the refund, thus, the penalty charged for the buyer's unused reservation decreases in the reservation fee. We then discuss the fact that without a coordination contract, the buyer may have incentive to understate the market size as an attempt to lower the reservation fee. But the *partially deductible* contract changes the buyer's incentive by including a penalty for misrepresenting the true market size. This analysis provides another example where the assumption of linear vs. convex capacity cost matters (see the discussion in Section 5.1). The second coordination mechanism is the *cost-sharing contract,* where the buyer pays for a portion of the *capacity cost* associated with her reservation. Depending on her demand realization, the buyer either receives a refund, or makes additional payment for the capacity utilized. This contract provides a greater level of flexibility to the supplier as she may specify the contract term independent of the buyer's demand distribution, and she could generate all possible allocation of optimal surplus using the contract term.

Both coordination contracts investigated in this paper are similar to the *buy-back* contracts in concept. Specifically, the *reservation fee* corresponds to the difference between the *wholesale price* and the *buy-back price*. In contrast to *buy-back* we device the coordination contracts via reservation fees assuming an exogenous wholesale price, and more importantly, we assume convex capacity cost which capture the diseconomy of scale in physical expansions.

Additional Considerations. First, we consider the case where the supplier can choose not to comply with the contract. We show that in the proposed contract setting the supplier will not behave very differently in the situation. Second, we examine a situation arise in contract manufacturing where the buyer's (now a contract manufacturer) demand size can be one of two types. We show that it is possible for the supplier to design contract options ex ante such that the buyer will choose the option right for her type while coordinating the channel.

APPENDIX

Proof of Theorem 1: It is individually rational for the buyer to make a reservation if $\Pi_B(q) \geq \Pi_B^0(k^*)$. The buyer's optimal reservation quantity q given reservation fee r is given in (7). Thus, r expressed in terms of the reservation quantity q is:

$$r = (p-w)\frac{\overline{F}(q)}{F(q)} \tag{A1}$$

From (6) and (A1), we may express the buyer's profit after reserving q as follows:

$$\Pi_B(q) = (p-w)S(q) - (p-w)\frac{\overline{F}(q)}{F(q)}\int_0^q F(x)dx$$

which can be reduced to

$$\Pi_B(q) = (p-w)\frac{\int_0^q xf(x)dx}{F(q)} \tag{A2}$$

First consider the definitions of $\Pi_B^0(k^*)$ and $\Pi_B(q)$ in (3) and (A2), respectively. We know that $\Pi_B(q) = \Pi_B^0(k^*)$ when the equality in (9) holds. Thus, if the buyer's optimal reservation quantity is at the threshold q^t, she is indifferent between reserving and not reserving. Since $\Pi_B(q)$ is strictly increasing in q, she indeed has incentive to reserve when $q > q^t$.

To prove property (1), observe that when $q^t = k^*$ the right-hand-side of equality (9), $\Pi_B(q^t)/(p-w)$, is less than the left-hand-side $S(k^*)$. Again, since $\Pi_B(q)$ is strictly increasing in q, this implies that $q^t > k^*$. Property (2) follows directly from (A1) above.

Now consider Property (3). Recall that we derived the upper bound for the reservation fee (8) from the observation that the buyer would reserve no less than the supplier's newsvendor quantity k^*. Since property (1) states that $q^t > k^*$, and the fact that (A1) is decreasing in q, r^t must be a tighter upper bound, i.e., $r^t < (p-w)\frac{v^*}{w-c-v^*}$. □

Proof of Theorem 4. Let's first show that $\Pi_S(q)$ is decreasing in $[k^o, \infty)$. The first derivative of $\Pi_S(q)$ with respect to q can be written as follows:

$$\frac{\partial \Pi_S(q)}{\partial q} = \Pi_S'(q) = (p-c)\overline{F}(q) - V'(q) - (p-w)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx$$

Note that the foregoing function is equivalent to:

$$\Pi_S'(q) = \Pi_I'(q) - (p-w)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx \tag{A3}$$

Since $\Pi_I(q)$ is concave in q and k^o is the unique maximum, $\Pi_I'(q) \leq 0$ if $q \geq k^o$, and the second term is positive, thus for any q in $[k^o, \infty)$, the right hand side returns a negative value, i.e., $\Pi_S(q)$ is strictly decreasing in q in $[k^o, \infty)$.

From the supplier's decision problem (13), we know that $q^r \geq$ Max $(q^t, q^w) \equiv q^m$. If $q^m \geq k^o$ then $q^r = q^m, \Pi_S(q)$ is strictly decreasing in q in $[k^o, \infty)$. If $q^m < k^o$, the maximum is in $[q^m, k^o)$. Since $\Pi_S(q)$ is compact and continuous within this interval, from Weierstrass' Theorem we know there exists a maximum solution. If there is no stationary point in $[q^m, k^o)$, then $q^r = q^m$. If there is a stationary point, q^*, such that $q^m \leq q^* < k^o$, q^* would be the unique maximum for $\Pi_S(\cdot)$. We will prove this in the following: first, q^* satisfies the first order optimality condition:

$$(p-c)\overline{F}(q^*) - V'(q^*) = (p-w)\frac{f(q^*)}{F^2(q^*)}\int_0^{q^*} F(x)dx \tag{A4}$$

The second derivative for $\Pi_S(q)$ is as follows:

$$\frac{\partial^2 \Pi_S(q)}{\partial q^2} = \Pi_I''(q) - (p-w)\frac{1}{F(q)}\left(f(q) + \frac{\int_0^q F(x)dx}{F(q)}\left(f'(q) - 2\frac{f^2(q)}{F(q)}\right)\right) \tag{A5}$$

Since $F(x)$ is an IFR distribution, we have

$$f'(x) \geq -\frac{f^2(x)}{\overline{F}(x)} \tag{A6}$$

Define a new function $H_1(q)$ by replacing $f'(q)$ in (A5) with $-f^2(q)/\overline{F}(q)$. After some manipulation $H_1(q)$ can be written as follows:

$$H_1(q) = \Pi_I''(q) - (p-w)\frac{f(q)}{F(q)}\left(1 - \left(\frac{1+\overline{F}(q)}{\overline{F}(q)}\right)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx\right)$$

From (A6) we notice that $H_1(q) \geq \Pi_S''(q)$ and thus if $H_1(q) < 0$ at q, the second derivative of $\Pi_S(q)$ is negative. Using (A4) we can write $H_1(q^*)$ as

$$H_1(q^*) = \Pi_I''(q^*) - \frac{f(q^*)}{F(q^*)}\left((p-w) - \left(\frac{1+\overline{F}(q^*)}{\overline{F}(q^*)}\right)\Pi_I'(q^*)\right)$$

The foregoing function can be reduced to

$$H_1(q^*) = \frac{f(q^*)}{F(q^*)}\left(\Pi_I'(q^*) + (p-c)\overline{F}(q^*) - \frac{V'(q^*)}{\overline{F}(q^*)} - (p-w)\right) - V''(q^*) \tag{A7}$$

Let $H_2(q^*)$ denote the term inside the parenthesis in (A7). It is sufficient to show that q^* is a local maximum (i.e., $\Pi_S''(q^*) < 0$) if $H_2(q^*)$ is non-positive. Since $q^* \geq q^m > k^*$, from (2), $V'(q^*)/\overline{F}(q^*) > (w-c)$. Define $H_3(q^*)$, by replacing $V'(q^*)/\overline{F}(q^*)$ with $w-c$ in $H_2(q^*)$, thus

$H_3(q^*) = (p-w)\overline{F}(q^*) - (p-c)F(q^*).$

Note that $H_3(q^*) > H_2(q^*)$. Obviously if

$$(p-c) \geq (p-w)\frac{\overline{F}(q^*)}{F(q^*)} \tag{A8}$$

then $H_3(q^*) \leq 0$ implying that $H_1(q^*)$ is negative, and q^* is a local maximum. Note that the right hand side of the foregoing inequality is $r(q^*)$. Since the capacity reservation price decreases in q, if (A8) holds, any $q^* \geq q^m$ has to be a local maximum, i.e., there exists at most one local maximum in (q^m, ∞). Suppose that inequality in (A8) does not hold, *i.e.*, $(p-c) < r(q^*)$, observe from (A4) that $(p-c)\overline{F}(q^*) > V'(q^*)$ implying that $r(q^*) > V'(q^*)$ as well. In other words at q^* the reservation price should be strictly greater than the marginal cost of capacity. Define $H_4(q^*)$ by replacing the last term in $H_2(q^*)$ with $V'(q^*)\dfrac{F(q^*)}{\overline{F}(q^*)}$ and then $V'(q^*)/\overline{F}(q^*)$ with $(w-c)$. Thus, $H_4(q^*) = 2\big((p-c)\overline{F}(q^*) - (w-c)\big)$. Since $r(q^*) > V'(q^*)$ and $w - c < V'(q^*)/\overline{F}(q^*)$, $H_4(q^*) > H_1(q^*)$. The constraints in (13) enforces that $w \geq r(q^*)$ implying that $w/p \geq \overline{F}(q^*)$. Thus, obviously $(w-c)/(p-c) \geq \overline{F}(q^*)$ as well. Therefore if $(p-c) < r(q^*)$ then $H_4(q^*) \leq 0$. Since $H_4(q^*) > H_1(q^*)$, the second derivative at q^* must be negative. This shows that the second derivative of $\Pi_S(q)$ in $[q^m, k^o)$ at any stationary point is negative, i.e., there exists at most one stationary point in $[q^m, k^o)$, and if one exist it is the only local maximum. □

Proof of Theorem 5. To prove Part (1) we need to show that $\Pi'_S(q^t) < 0$. To do this we will use the relationship between q^t and k^* defined in (9). Define $L(q^t, k^*)$ as follows:

$$L(q^t, k^*) = S(k^*) - \frac{\int_0^{q^t} x f(x)dx}{F(q^t)}$$

From (9) we know that $L(q^t, k^*) = 0$. Thus, the derivative of this function with respect to k^* must be 0 as well. Hence, from the chain rule

$$\frac{\partial L(q^t, k^*)}{\partial k^*} = \overline{F}(k^*) - \frac{f(q^t)}{F(q^t)}\int_o^{q^t} F(x)dx \cdot \frac{\partial q^t}{\partial k^*} = 0$$

Note also that derivative with respect to θ should be 0. That is,

$$\frac{\partial L(q^t, k^*)}{\partial \theta} = \overline{F}(k^*)\left(1 - \frac{\partial k^*}{\partial \theta}\right) - \frac{f(q^t)}{F(q^t)}\int_o^{q^t} F(x)dx \cdot \left(1 - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \theta}\right) = 0 \tag{A9}$$

A straightforward analysis of (A9) will show that

$$\overline{F}(k^*) = \tfrac{f(q^t)}{F(q^t)}\textstyle\int_o^{q^t} F(x)dx \tag{A10}$$

and $\partial q^t/\partial k^* = 1$ for any distribution that is from a shifted family. Thus,

$$\overline{F}(q^t) < \tfrac{f(q^t)}{F(q^t)}\textstyle\int_o^{q^t} F(x)dx \tag{A11}$$

since $k^* < q^t$. We may write the first derivative of the supplier's profit function evaluated at q^t:

$$\Pi_S'(\mathrm{q}^t) = \big((w-c)\overline{F}(q^t) - V'(q^t)\big) + (p-w)\left(\overline{F}(q^t) - \frac{f(q^t)}{F(q^t)}\int_o^{q^t} F(x)dx\right)$$

Note that the term inside the first parenthesis is negative since $q^t > k^*$. From (A11), the second parenthesis is negative as well, thus $\Pi_S'(\mathrm{q}^t) < 0$. Recall from Theorem 4 that $\Pi_S(q)$ is strictly decreasing in $[q^t, \infty)$, thus it is decreasing in $[q^m, k^o)$. Thus, the optimal quantity q^r should be at the boundaries of the feasible region, i.e., $q^r = Max\ (q^t, q^w)$, as shown in Theorem 4, *Part (3)-(i).* From (10), we may conclude that $r^* = min(r^t, w)$.

To prove Part (2), first consider the threshold reservation fee with respect to θ, we have :

$$\frac{\partial \mathrm{r}^t}{\partial \theta} = (p-w)\frac{f(q^t)}{F(q^t)}\left(1 - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \theta}\right) \quad \text{(A12)}$$

When the capacity cost is *linear*, it can be verified using (2) that $\partial k^*/\partial\theta = 1$ for any distribution in the *shifted family*. From Part (1), we have $\partial q^t/\partial k^* = 1$. Thus, $\frac{\partial \mathrm{r}^t}{\partial\theta}$*=0.* Thus, we can conclude that r^* is independent of the market size. However, from (10) we can write that

$$F(q^t - \theta|0) = \frac{p-w}{p-w+r^t}$$

Since r^t and thus the foregoing ratio does not change in θ, clearly the quantity q^t must be increasing in market size θ.

To show that the supplier's surplus is not influenced by the market size, look at the first derivative of $(\Pi_S(q^\mathrm{r}) - \Pi_S^0(k^*))$ with respect to θ. Let $\Psi_\theta(q^t(k^*), k^*)$ denote this derivative for given q^t and k^*. Then, from the envelop theorem and the chain rule we can write the following;

$$\Psi_\theta = \frac{\partial\left(\Pi_S(q^r) - \Pi_S^0(k^*)\right)}{\partial\theta} = (w-c)\overline{F}(k^*) - (p-c)\overline{F}(q^t) + (p-w)\frac{f(q^t)}{F(q^t)}\int_o^{q^t} F(x)dx + \Pi_S'(q^t)\frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial\theta}$$

Observe that since both $\partial q^t/\partial k^*$ and $\partial k^*/\partial\theta$ are equal to 1, thus $\Psi_\theta(q^t(k^*), k^*)$=0, i.e., the supplier's surplus does not change with market size θ.

Now consider Part (3). From (2), it is easy to see that if the capacity cost is strictly convex $\partial k^*/\partial\theta < 1$, but $\partial q^t/\partial k^* = 1$. From (A12), it follows that the reservation fee is increasing in θ. Further, from the envelop theorem, we can write the marginal change in surplus (as induced by the reservation) as follows;

$$\Psi_\theta(q^w, k^*) = (w-c)\overline{F}(k^*) - V'(q^w)$$

From the first order optimality condition we know that if $q^w > k^*$, then $\Psi_\theta < 0$, i.e., the value of reservations decreases in θ. Since the increment in k^* is less than the increment in q^w when θ increases, q^w will exceed k^* eventually, i.e., there is a market size threshold, above which the surplus generated by reservation decreases. □

Proof of Theorem 6. From (2) we know that $\overline{F}\ (k^*) - \dfrac{V'(k^*)}{w-c} = 0$

Differentiate the foregoing equality with respect to the standard deviation, σ, we have:
$f(k^*)\left(\frac{k^*-\mu}{\sigma} - \frac{\partial k^*}{\partial \sigma}\right) = \frac{V''(k^*)}{w-c}\frac{\partial k^*}{\partial \sigma}$
This implies that k^* increases (decreases) in σ when $k^* > \mu$ ($k^* < \mu$). Thus,
$\left|\frac{\partial k^*}{\partial \sigma}\right| < \left|\frac{k^*-\mu}{\sigma}\right|$ (A13)
Using the chain rule, we have $\frac{\partial r^t}{\partial \sigma} = (p-w)\frac{f(q^t)}{F^2(q^t)}\left(\frac{q^t-\mu}{\sigma} - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \sigma}\right)$
As Normal distribution is from the *shifted family*, from Theorem 5, we have $\partial q^t/\partial k^*$=1. Thus, from (A13) and the fact that $q^t > k^*$, the term within the parenthesis in the above expression returns a positive value, i.e., the threshold reservation fee r^t increases in σ.
Now consider the channel surplus at q^t, which can be written as follows using (9)

$$\Pi_I(q^t) - \Pi_I(k^*) = (p-c)\frac{\overline{F}(q^t)}{F(q^t)}\int^{q^t} F(x)dx - V(q^t) + V(k^*)$$

Using (A10), the partial derivative of $\Pi_I(q^t) - \Pi_I(k^*)$ with respect to σ can be written as:
$-(p-c)\int^{k^*}\frac{x-\mu}{\sigma}f(x)dx + \left((p-c)\left(\overline{F}(q^t) - \frac{f(q^t)}{F^2(q^t)}\int^{q^t}F(x)dx\right) - V'(q^t) + V'(k^*)\right)\frac{\partial k^*}{\partial \sigma}$
The first term is positive. Since $V'(k^*) \geq 2(w-c)$, $\partial k^*/\partial\sigma$ is negative. Thus, from (A11) and $q^t > k^*$ the second term is positive. In other words, the surplus created through reservations increases in the standard deviation. □

Proof of Theorem 7. First; we have

$$\frac{\partial r^t}{\partial \theta} = (p-w)\frac{f(q^t)}{F^2(q^t)}\left(\frac{q^t}{\theta} - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \theta}\right)$$

Similar to that of Theorem 5, we have $\partial k^*/\partial\theta \leq k^*/\theta$ and $\partial q^t/\partial\theta \leq q^t/\theta$. Hence, the term within the parenthesis is positive, i.e., r^t increases with the market size θ. Now consider Part (2). Suppose that $q^r = q^*$, then the *marginal change* in surplus can be reduced to the following

$$\Psi_\theta(q^*, k^*) = \tfrac{1}{\theta}\left(\Pi_S(q^*) - \Pi_S^0(k^*)\right)$$

Notice that the term within the parenthesis is the surplus generated by reserving q^* which is positive. Now let $q^r = q^t$ and suppose that capacity reservation is a preferred strategy for the supplier in the first place (*i.e.*, $\Pi_S(q^t) > \Pi_S^0(k^*)$). Then it can be verified that

$$\Psi_\theta(q^t, k^*) \geq \tfrac{1}{\theta}\left(\Pi_S(q^t) - \Pi_S^0(k^*)\right)$$

Since the right hand side of the expression is positive, the left hand side is positive also, i.e., the surplus increases in market size θ. □

Proof of Theorem 8.
Since the deductible reservation fee is $r_2 = \frac{p-c}{v^o}r - (p-w)$, for all r the critical fractile faced by the buyer is: $\frac{r-p+w-r_2}{r} = \frac{p-c-v^o}{p-c}$

Observe that the critical fractile faced is the same as that of the integrated channel. Thus, if the contract is accepted by the supplier, the channel is coordinated. Now consider the buyer and the supplier's profit as a function of the reservation fee as follows:

$$\Pi_B(r,\, r_2) = (p - w + r_2)S(k^o) - rk^o = r\Big(\frac{p-c}{v^o}S(k^o) - k^o\Big) \le \frac{r}{v^o}\Pi_I^o.$$

$$\Pi_S(r,\, r_2) = (p-c)\frac{v^o - r}{v^o}S(k^o) - V(k^o) + rk^o \ge \frac{v^o - r}{v^o}\Pi_I^o.$$

The derivative with respect to r is positive for Π_B, and negative for Π_S, indicating that the buyer's profit increases in r whereas the supplier's profit decreases in r (Parts (1) and (2)).

Now consider Part (3), it is clear that the buyer profits is positive for any r chosen within the range defined by (16), and the minimum profit that the buyer will receive is $(p-w)\int_0^{k^o} xf(x)dx$. However, the maximal reservation fee the supplier can charge is $\underline{r} = (p-2w)\dfrac{v^o}{p-c-v^o}$. If $v^o > 2w - c$, it is easy to verify that Π_S is negative with $\underline{r}$ and the contract will be unacceptable to the supplier. Thus, the contract is only acceptable to the supplier when $v^o \le 2w - c$. ☐

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