A Survey of Max-Type Recursive Distributional Equations

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Abstract

In certain problems in a variety of applied probability settings (from probabilistic analysis of algorithms to statistical physics), the central requirement is to solve a *recursive distributional equation* of the form $X \stackrel{d}{=} g((\xi_i, X_i), i \geq 1)$. Here (ξ_i) and $g(\cdot)$ are given and the X_i are independent copies of the unknown distribution X. We survey this area, emphasizing examples where the function $g(\cdot)$ is essentially a "maximum" or "minimum" function. We draw attention to the theoretical question of *endogeny*: in the associated recursive tree process $X_{\mathbf{i}}$, are the $X_{\mathbf{i}}$ measurable functions of the innovations process $(\xi_{\mathbf{i}})$?

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