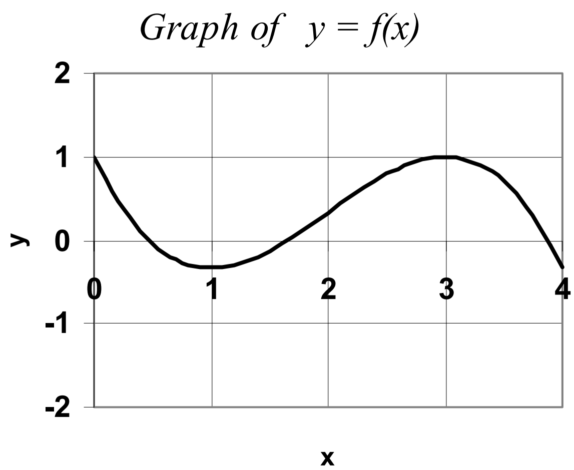
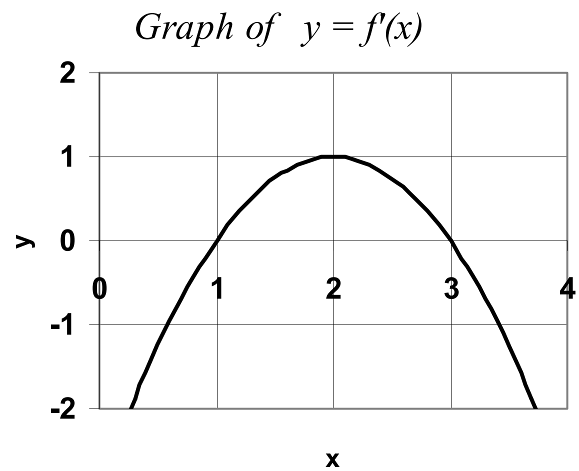
Problems 1 and 2 refer to the figures below. To the left is the graph of the function $f(x)$, while to the right is the graph of the ***derivative*** $f'(x)$.





(5) **1.** At $x = 2$, the graph of $y = f(x)$ has a

- **(a)** zero.
- **(b)** local minimum.
- **(c)** local maximum.
- **(d)** inflection point.
- **(e)** None of the above.

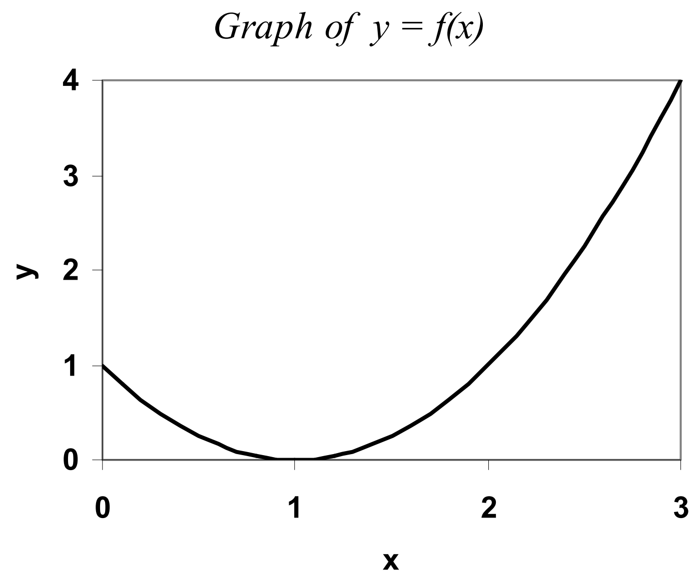
The answer is (d), an inflection point. The derivative changes at $x = 2$ from increasing to decreasing. Hence the function changes from concave up to concave down.

5) **2.** At $x = 3$, the graph of $y = f(x)$ has a

- **(a)** zero.
- **(b)** local minimum.
- **(c)** local maximum.
- **(d)** inflection point.
- **(e)** None of the above.

The answer is (c), a local maximum. At $x = 3$ the derivative is zero and decreasing, so the function is concave down and has a local maximum.

(5) **3.** To the right is a graph of the function $y = f(x)$.



Which of the following statements is correct for all x shown in the graph?

(a) $f'(x) > 0$.
(b) $f'(x) < 0$.
(c) $f''(x) > 0$.
(d) $f''(x) < 0$.
(e) None of the above.

The answer is (c). The graph is concave up, so the second derivative is positive.

(5) **4.** If $\int_0^{-1} f(x)dx = -1$ and $\int_{-1}^{2} f(x)dx = 5$, what is $\int_0^2 f(x)dx$?

(a) -6
(b) -4
(c) 4
(d) 6
(e) None of the above.

The answer is (c). $\int_0^2 f(x)dx = \int_0^{-1} f(x)dx + \int_{-1}^{2} f(x)dx = -1 + 5 = 4$.

(15) 5. Find all the asymptotes of

$$y = \frac{x^2}{x-3}.$$

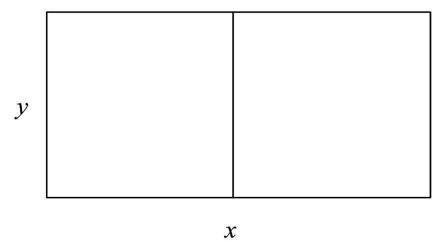
For each asymptote, state whether it is horizontal, vertical, or oblique, and justify your answer.

The denominator is zero at $x = 3$ while the numerator is 9, so the graph has a **vertical asymptote** at $x = 3$.
Now use long division to derive

$$y = \frac{x^2}{x-3} = x + 3 + \frac{9}{x-3}$$

So, as $x \to \infty$, y approaches $x + 3$, *i.e.*, the graph has an **oblique asymptote** given by $y = x + 3$.

(15) 6. Sven and Ole want to fence a rectangular plot. They also want to use additional fencing to build an internal divider parallel to two outer boundary sections, as shown in the figure below. If they have 240 linear feet of fence, what is the maximum area they can enclose? Verify that your answer yields the global maximum.



Let x be the horizontal length, and let y be the vertical length. The total length of fence is

$$2x + 3y = 240,$$

so

$$y = 80 - \tfrac{2}{3}x.$$

while the area enclosed is given by

$$A = xy = x(80 - \tfrac{2}{3}x) = 80x - \tfrac{2}{3}x^2$$

The lengths x and y must both be positive, which means that we are considering values of x in the interval $0 \le x \le 120$. The area A is zero at both ends of this interval, so the maximum cannot occur at the endpoints. Also, A is a differentiable function of x on the whole interval, so the maximum can occur only at a point in the interior where $\frac{dA}{dx} = 0$. We compute:

$$\frac{dA}{dx} = 80 - \tfrac{4}{3}x = 0$$

$$x = 60$$

So the maximum must occur at $x = 60$, which implies that $y = 80 - \tfrac{2}{3} \cdot 60 = 40$, so the maximum area is **2400 square feet.**

(15) **7.** Find the **exact value** of the definite integral $\int_{-2}^{0} e^{-2x}\,dx$. Show your work.

$$\int_{-2}^{0} e^{-2x}\,dx = \left.\frac{e^{-2x}}{-2}\right|_{-2}^{0} = \frac{e^{-2\cdot 0}}{-2} - \frac{e^{-2\cdot(-2)}}{-2} = -\frac{1}{2} + \frac{e^4}{2}$$
$$= \frac{e^4 - 1}{2}$$

(15) **8.** Set up but do not evaluate the integral for finding the area of the region bounded by curve $y = x(x-1)$ and the line $x + y = 1$.

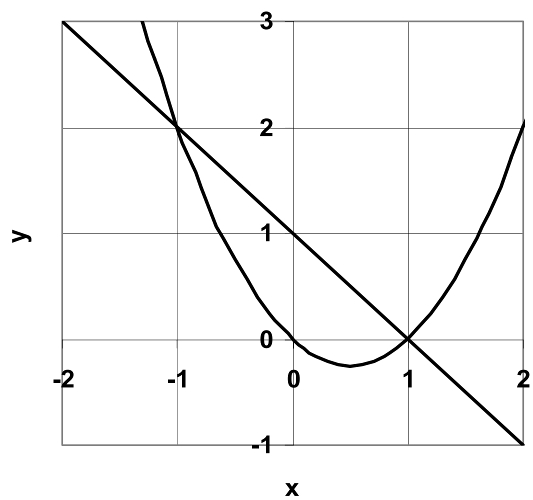
Upper curve: $y = 1 - x$.

Lower curve: $y = x(x-1)$.

Intersection points: $(x, y) = (1,0)$ and $(x, y) = (-1,2)$.

Area between the curves

$$\int_{-1}^{1} (1 - x - x(x-1))\,dx = \int_{-1}^{1} (1 - x^2)\,dx$$



(20) **9.** Find the following indefinite integrals.

(10) **a.** $\int \frac{1}{2x+3}\,dx$.

$$\int \frac{1}{2x+3}\,dx = \frac{1}{2}\int \frac{2dx}{2x+3}$$
$$= \tfrac{1}{2}\ln|2x+3| + C$$

(10) **b.** $\int \frac{3}{x^3}\,dx$.

$$\int \frac{3}{x^3}\,dx = 3\int x^{-3}\,dx = 3\cdot\frac{x^{-2}}{-2} + C$$
$$= -\frac{3}{2x^2} + C$$