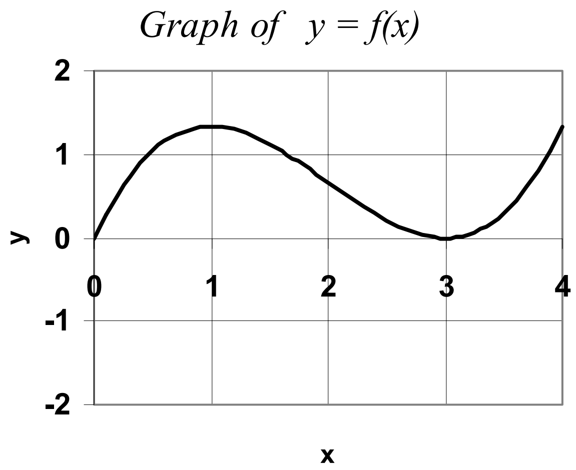
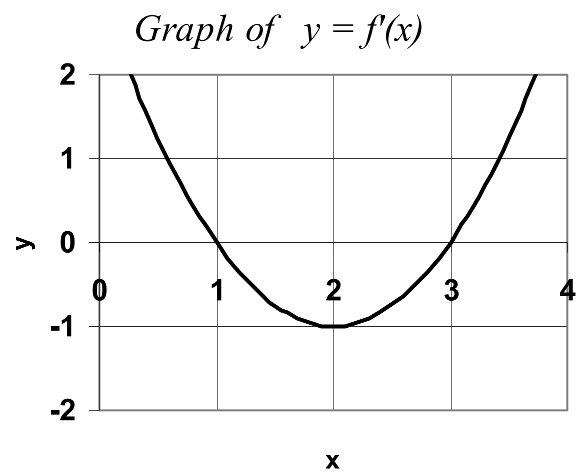
Problems 1 and 2 refer to the figures below. To the left is the graph of the function $f(x)$, while to the right is the graph of the ***derivative*** $f'(x)$.





(5) **1.** At $x = 2$, the graph of $y = f(x)$ has a

(a) zero.
(b) local minimum.
(c) local maximum.
(d) inflection point.
(e) None of the above.

The answer is (d). The derivative changes at $x = 2$ from decreasing to increasing, so the function changes from concave down to concave up, which means that $x = 2$ is an inflection point

(5) **2.** At $x = 3$, the graph of $y = f(x)$ has a

(a) zero.
(b) local minimum.
(c) local maximum.
(d) inflection point.
(e) None of the above.

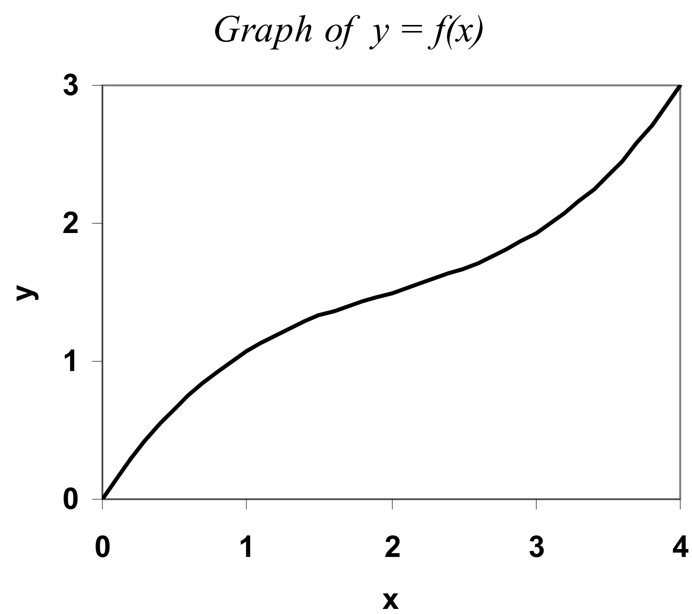
The answer is (b). At $x = 3$, the derivative changes from negative to positive, so the function changes from decreasing to increasing, which means that $x = 3$ is a local minimum.

Answer (a) is also correct.

(5) **3.** To the right is a graph of the function $y = f(x)$.

Which of the following statements is correct for all x shown in the graph?

(a) $f'(x) > 0$.
(b) $f'(x) < 0$.
(c) $f''(x) > 0$.
(d) $f''(x) < 0$.
(e) None of the above.



The answer is (a). The function is increasing for all x shown in the graph, so the derivative is positive.

(5) **4.** If $\int_{-1}^{0} f(x)dx = -1$ and $\int_{-1}^{2} f(x)dx = 2$, what is $\int_{0}^{2} f(x)dx$?

(a) -3
(b) -1
(c) 1
(d) 3
(e) None of the above.

The answer is (d).

$$\int_{-1}^{2} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{2} f(x)dx$$

$$2 = -1 + \int_{0}^{2} f(x)dx$$

$$\int_{0}^{2} f(x)dx = 3$$

(15) 5. Find all the asymptotes of

$$y = \frac{x^2}{2x+1}.$$

For each asymptote, state whether it is horizontal, vertical, or oblique, and justify your answer.

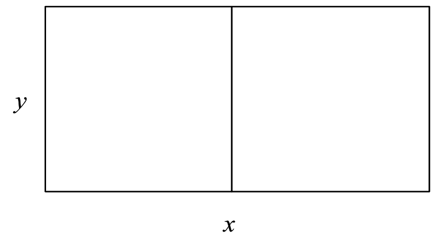
The denominator is zero at $x = -\frac{1}{2}$ while the numerator is $\frac{1}{4}$, so the graph has a **vertical asymptote** at $x = -\frac{1}{2}$.

Now use long division to derive

$$y = \frac{x^2}{2x+1} = \tfrac{1}{2}x - \tfrac{1}{4} + \frac{\tfrac{1}{4}}{2x+1}$$

So, as $x \to \infty$, y approaches $\frac{1}{2}x + \frac{1}{4}$, *i.e.*, the graph has an **oblique asymptote** given by $y = \frac{1}{2}x + \frac{1}{4}$.

(15) 6. Sven and Ole want to fence a rectangular plot of area 600 square feet. They also want to use additional fencing to build an internal divider parallel to two outer boundary sections, as shown in the figure below. What is the minimum total length of fencing that this project will require? Verify that your answer yields the global minimum.



Let x and y be as shown in the figure. The total area is $xy = 600$, so we have

$$y = \frac{600}{x}$$

The total length of fence is given by

$$p = 2x + 3y = 2x + \frac{1800}{x}.$$

Since the derivative $\frac{dp}{dx}$ is defined for all $x > 0$, the global minimum can occur only at the endpoints of the interval or at internal points where the derivative is zero. The interval of possible values of x is $(0, \infty)$, and $p \to \infty$ as $x \to 0$ or as $x \to \infty$. Therefore, the global minimum must occur at a point where the derivative is zero. We compute

$$\frac{dp}{dx} = \frac{d}{dx}\left(2x + \frac{1800}{x}\right) = 2 - \frac{1800}{x^2} = 0$$

Therefore, $x^2 = 900$, so $x = 30$, so the minimum total amount of fence is

$$p = 2 \cdot 30 + \frac{1800}{30} = 120 \text{ linear feet.}$$

(15) 7. Find the **exact value** of the definite integral $\int_0^3 e^{-2x} dx$. Show your work.

$$\int_0^3 e^{-2x} dx = \left.\frac{e^{-2x}}{-2}\right|_0^3 = \frac{e^{-2\cdot 3}}{-2} - \frac{e^{-2\cdot 0}}{-2} = \frac{e^{-6} - 1}{-2}$$

$$= \frac{1 - e^{-6}}{2}$$

(15) 8. Set up but do not evaluate the integral for finding the area of the region bounded by curve $y = x(x-1)$ and the line $y = 2x$.

We first find the points of intersection of the curve $y = x(x-1)$ and the line $y = 2x$ by setting $x(x-1) = 2x$ and solving for x.

$$x(x-1) - 2x = 0$$

$$x(x-3) = 0$$

The points of intersection occur at $x = 0$ and $x = 3$.

The line $y = 2x$ is *above* the curve $y = x(x-1)$ on the interval $0 \le x \le 3$. We can verify this by checking any point in the interior of the interval, *e.g.*, $x = 1$. At $x = 1$, the value of y on the line is $y = 2 \cdot 1 = 2$, while the value of y on the curve is $y = 1 \cdot (1-1) = 0$.

The integral for the area between the line and the curve is therefore

$$\int_0^3 \left(2x - x(x-1)\right) dx = \int_0^3 \left(3x - x^2\right) dx$$

(20) 9. Evaluate the following indefinite integrals.

(10) a. $\int \frac{5}{x+3}\, dx$.

$$\begin{aligned} \int \frac{5}{x+3}\, dx &= 5\int \frac{dx}{x+3} \\ &= 5\ln|x+3| + C \end{aligned}$$

(10) b. $\int \frac{3}{5x^2}\, dx$.

$$\begin{aligned} \int \frac{3}{5x^2}\, dx &= \frac{3}{5}\int x^{-2} dx = \frac{3}{5} \cdot \frac{x^{-1}}{-1} + C \\ &= -\frac{3}{5x} + C \end{aligned}$$